A novel approach for composite non-linear constitutive materials. Long fibre-reinforced laminates

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Abstract

A novel computational method is proposed in order to modelling the material non-linearity of Fibre Reinforced Plastic structures. The enhanced method explained in this paper is the so-called serial-parallel theory (S-P) which consists on dividing the composite behaviour into two mechanical behaviours, parallel for fibre directions and serial for orthogonal directions. This division is introduced by means of two 'closure equations' which make assumptions of iso-deformation across fibre directions and iso-tension across orthogonal directions to the fibres. In the present work, the classical theory is combined with serial-parallel method in order to model multi-direction laminates. The method is validated via different benchmark tests and experimental data obtained from [Hinton MJ, Soden PD. Predicting failure in composite laminates: The background to the exercise. Comp Sci Technol 1998; 58:100110].

Keywords: `FRP, Long fibre laminates, Composite failure, Non-linear modelling, FEM`

1. Introduction

During the last 5 decades, the use of Long Fibre Composite (LFC) has been broadly developed in the naval, aeronautical and automotive industries due to the fact of their excellent mechanical properties and densities. This is the major reason which lead researchers to create a mathematical basis description of the macro-mechanical and micro-mechanical behaviour of such materials and thus many different models can be used nowadays. However, this vast range of models is not in concordance to failure prediction theories which have not been under an extensive development since 2 decades ago.

This research has been motivated by the increasing use of this products in the industry and the lack of an efficient computational solution to modelling the degradation of mechanical properties of the composites - being able to capture the heterogeneity of the material in micro-scale without an extremely high computational cost - and at the same time being able to be attached or recycled onto many different FEM codes without undertaking enormous changes to the code framework.

Such technology will grant a huge advantage since it

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could reuse currently FEM code for homogeneous materials into heterogeneous materials or composites. This fact is the major motivation of this work which seeks to obtain a sufficient efficient code which can provide quite an accurate model together with a fast and reliable simulation. This model will help to reduce the expensiveness of the nowadays composite tests in order to validate composite structures.

The development of such framework for a constitutive composite model uses a laminate approach and since the non-linear behaviour of component per layer obligates the use of a sequence approach[1]. Puck[2, 3] have also remarked that, in the analysis of fibre-reinforced laminates, it is essential to distinguish between fibre failure and matrix failure as well as between fibre degradation and matrix degradation. Which means that models which only use internal/state variables of the equivalent homogenized material, will not be accurate.Furthermore, as shown by Oller et al. [4, 5], the computational cost of a complete double scale approach for a large scale non-linear structural analysis is still not affordable with ordinary computers, even including parallel computations.

Inside what is called mean-field methods (MFM) which assume an average of stress and strain tensor fields to represent the equivalent fields of the composite by means of a volumetric participation of each component. Voigt[6] and Reuss[7] proposed what in the present literature is the so-called rule of mixtures (RoM) and inverse rule of mixtures (iRoM) to compute the elastic constants of the composite.

Classical mixing theory (CMT), whose simpler expression is the ROM, was firstly studied in 1960 [8] and lead to the establishment of the basis of subsequent developments [9, 10, 11, 12]. CMT takes into account the volumetric participation but no its morphological distribution sequence (assuming pure parallel behaviour). Few modifications to this theory were developed [13, 14]. At last, previous research on the field lead the authors [15, 16] to achieve the methodology explained in this paper.

In the current work, the serial-parallel theory is used together with a MFM method. The novel goal of this research is to developed a serial-parallel rules of mixtures and relate the laminate properties depending on the constitutive laws of the component materials taking into account their volumetric participation and layer distribution sequence.

In order to validate the numerical simulation data. Comparison with the experimental data obtained from benchmark tests [17] is used.

2. Numerical model development

The serial-parallel rule of mixture is a model firstly developed by Rastellini and Oller[18] in order to assess plasticity and damage of the elastic stiffness tensor. The remarking work of these two author lead to an unconstrained non-linear constitutive model of the component (meaning any kind of constitutive non-linear model can be used).

The serial-parallel model considers two closure equations, the first one is what normally is used in most FEM codes and is called iso-strain hypothesis in fibre direction (parallel) and the second closure equation which is governed by an iso-stress hypothesis into the orthogonal directions to the fibre direction (serial).

The aim of serialparallel (SP) models is to help quickly and accurately in the assessment of the non-linear behaviour of composite structures due to material degradation. The consistency of the results is assured by the appropriate election of component material models.

2.1. Basic notations and definitions

In the development of this theory a two layer components model is employed. It is assume that there exists a periodical sequence of a combination of these two components inside the representative volume element (RVE). This two components will be addressed as 'matrix' (m) and 'fibre' (f). The RVE domain is denoted by Ω and it is related to the subdomains as $\Omega =^m \Omega \cup^f \Omega$. The volumetric participation is defined as ${}^f k$ and ${}^m k$ and they fulfil ${}^f k +^m k = 1$.

The composite equivalent properties arise from the average component properties with the relation:

$$\begin{aligned} {}^c\sigma &=^m k^m\sigma +^f k^f\sigma \\ {}^c\varepsilon &=^m k^m\varepsilon +^f k^f\varepsilon \end{aligned} \tag{1}$$

From a strain driven formulation, it is assumed that the current state of deformation is defined by strain in a particular point and set of internal variables (${}^i\beta$), where i denotes any of the two components. The constitutive law is stated by the system of differential equations:

$$\begin{aligned} {}^i\dot{\sigma} &=^i g({}^i\varepsilon,{}^i\beta,{}^i\dot{\varepsilon}) \\ {}^i\dot{\beta} &=^i h({}^i\varepsilon,{}^i\beta,{}^i\dot{\varepsilon}) \end{aligned} \tag{2}$$

2.2. Serial/parallel decomposition

The approach used in this paper is the use of projector tensors to define the component serial-parallel directions. From tensor algebra, a projector tensor (4^{th} order tensor), can be defined by means of a change of basis:

$$\begin{aligned} &N_{ij} = e_i \otimes e_j \\ &P_{P\,i,j,q,p} = N_{i,j} \otimes N_{q,p} \\ &P_S = P_P - \mathcal{I} \end{aligned} \tag{3}$$

where $\mathcal{I}$ is the identity tensor. Now each projected tensor field can be described as:

$$\begin{aligned} \sigma_i &= P_i : \sigma \\ \varepsilon_i &= P_i : \varepsilon \end{aligned} \tag{4}$$

where i is serial or parallel subscript. And since both projectors add to the identity:

$$\begin{aligned} \sigma &= \textstyle\sum_{i=S}^{P} \sigma_i \\ \varepsilon &= \textstyle\sum_{i=S}^{P} \varepsilon_i \end{aligned} \tag{5}$$

2.3. Closure equation

The closure equation for LFC are:

$$\begin{aligned} {}^m\varepsilon_P &=^f \varepsilon_P =^c \varepsilon_P \\ {}^m\sigma_S &=^f \sigma_S =^c \sigma_S \end{aligned} \tag{6}$$

These two assumptions were first used by Dvorak and Bahei-El- Din[19] to define anisotropic plasticity model and later by Rastellini et al. [15, 16, 18, 20].

2.4. Algorithm

Recapitulating on the different aspects of the algorithm.

2.4.1. Governing Equations

The governing equations for the composite model are:

1. Constitutive laws:

$$\begin{aligned} &{}^i\dot{\sigma} =^i g({}^i\varepsilon,{}^i\beta,{}^i\dot{\varepsilon}) \\ &{}^i\dot{\beta} =^i h({}^i\varepsilon,{}^i\beta,{}^i\dot{\varepsilon}) \\ &i = m, f \end{aligned} \tag{7}$$

2. Rule of mixtures:

$$\begin{aligned} &\sigma = \textstyle\sum_{i=m}^{f} {}^i k^i\sigma \\ &\varepsilon = \textstyle\sum_{i=m}^{f} {}^i k^i\varepsilon \\ &i = m, f \end{aligned} \tag{8}$$

3. Closure equations:

$$\begin{aligned} {}^m\varepsilon_P &=^f \varepsilon_P =^c \varepsilon_P \\ {}^m\sigma_S &=^f \sigma_S =^c \sigma_S \end{aligned} \tag{9}$$

2.4.2. Algorithm's Flow

Considering the model as a strain-driven problem.

Known Variables.

At load step n:

$$ {}^{n,i}\varepsilon, {}^{n,i}\beta, {}^{n,c}\varepsilon \quad i = m, f \tag{10}$$

At load step $n+1$:

$$ {}^{n+1}\varepsilon \tag{11}$$

Unknown Variables.

At load step $n+1$:

$$\begin{aligned} &{}^{n+1,i}\sigma, {}^{n+1,c}\sigma \quad i = m, f \\ &{}^{n+1,i}\varepsilon, {}^{n+1,i}\beta \quad i = m, f \end{aligned} \tag{12}$$

Kernel Steps.

1. Evaluate the component tensional state using the constitutive laws in equation (7):

$$ [{}^{n+1,i}\sigma, {}^{n+1,i}\varepsilon] = f({}^{n,i}\varepsilon, {}^{n,i}\beta, {}^{n+1,i}\varepsilon) \tag{13}$$

   where $i = m, f$.

2. Evaluate the constitutive tangent tensor:

$$\begin{aligned} &{}^k\mathcal{C}_{i,j} = \frac{\partial^k \sigma_i}{\partial^i \varepsilon_j} \\ &{}^k\mathcal{C}_{i,j} = \mathcal{P}_i : {}^k\mathcal{C} : \mathcal{P}_j \\ &k = m, f \quad , \quad i, j = P, S \end{aligned} \tag{14}$$

3. Evaluate the serial state tension residual:

$$ \Delta\sigma_S = {}^m\sigma_S - {}^f\sigma_S \tag{15}$$

4. Assume ${}^m\varepsilon$ to be the independent variable for the N-R scheme:

$$ {}^f\varepsilon_S({}^m\varepsilon_S) = \frac{1}{{}^f k}\varepsilon_S - \frac{{}^m k}{{}^f k}{}^m\varepsilon_S \tag{16}$$

5. Use equation (16) into equation (15) and then optimising the objective function:

$$ {}^r\mathcal{J} = {}^r\left(\frac{\partial(\Delta\sigma)_S}{\partial^m \varepsilon_S}\right) = {}^r\left({}^m\mathcal{C}_{SS} + \frac{{}^m k}{{}^f k}{}^f\mathcal{C}_{SS}\right) \tag{17}$$

   where r denotes the iteration loop.

6. Then update of the iterative independent variable:

$$ {}^{r+1}({}^m\varepsilon_S) = {}^r({}^m\varepsilon_S) - {}^r(\mathcal{J}^{-1} : \Delta\sigma_S) \tag{18}$$

Flow.

The flow would be $1 \rightarrow 4$ and whether convergence of the residual has been achieved or not, GoTo 1 *again* or Repeat 5, 6, 4 .

Flow Diagram.

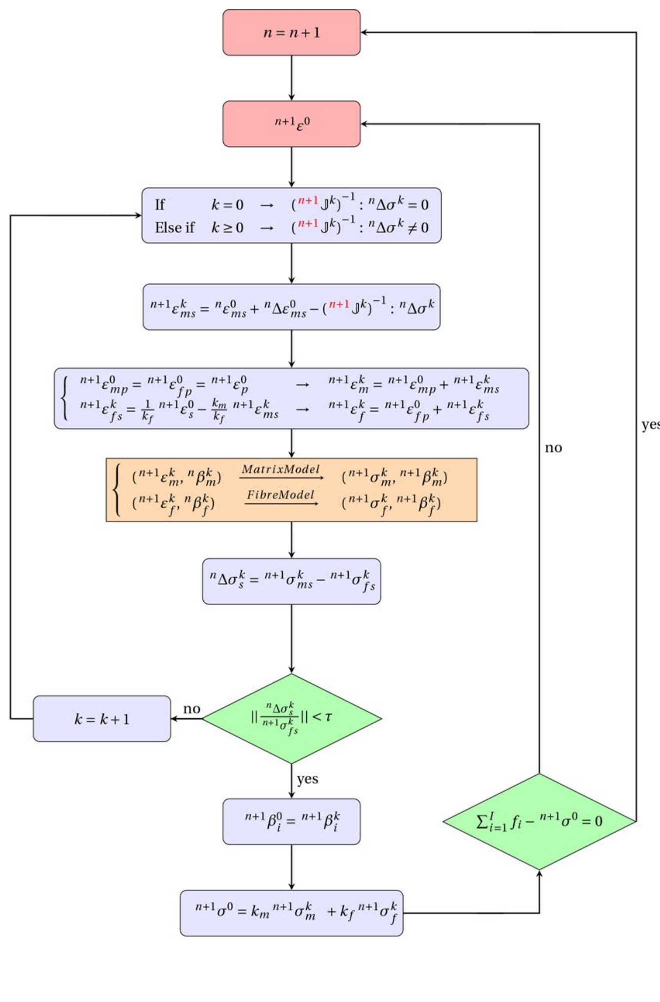

Figure 1: Diagram of the code flow.

3. Numerical Simulation

3.1. Close equations fulfilment

3.1.1. Parallel loading

To test the proposed constitutive model under parallel loading, a loadunload controlled longitudinal deformation in fibre direction is applied to a Discrete Kichhoff Triangle element with no other constrains. The mechanical properties of component materials are shown in 1.

	Matrix	Fibre
Constitutive law	J2 plasticity	J2 plasticity
Young modulus (MPa)	40000	80000
Elastic limit (MPa)	1000	3480
Poisson ratio	0.0	0.0
Volume fraction	0.58	0.42

Table 1: Mechanical properties for parallel loading

Observe that the σ_P is different for each component due to the iso-strain hypothesis, but the strains are equal.

Note that at the beginning the composite stiffness is intact, then the matrix yields and finally the fiber yields as well leading to a composite yielding as well. At 10% the load is reversed, initial elastic stiffness is shown in all materials during unloading. At complete unload, residual stresses remain in the components due to plasticity. These residual stresses are auto-equilibrated since the resultant stress in the composite is zero.

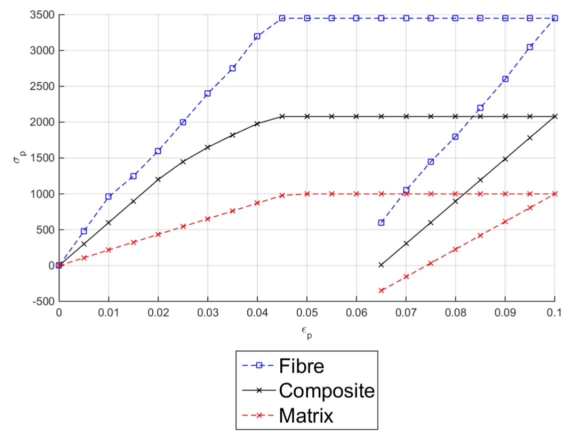

Figure 2: Parallel stress (MPa) vs. parallel strain curves for the composite and component materials under parallel deformation-controlled loadunload testing.

3.1.2. Transversal loading

In order to validate the serial behaviour of the model, a DKT composite element is subjected to pure transversal loading. The test is performed by applying a load unload transversal controlled deformation up to 5% strain.

	Matrix	Fibre
Constitutive law	J2 plasticity	Damage
Young modulus (MPa)	3000	2000
Elastic limit (MPa)	60	40
Poisson ratio	0.0	0.0
Volume fraction	0.5	0.5

Table 2: Mechanical properties for transversal loading

Note that ν is equal to 0 to avoid coupling effects from parallel behaviour. Observe that the ε_S is different for each component due to the iso-stress hypothesis, but the stress are equal (it is represented by dashed horizontal black lines in Figure(3)).

In the first elastic branch (0 to $\sigma_y = 40$ MPa), the composite transversal stiffness, given by SP model, is in accordance with the inverse ROM. When the matrix reaches the yielding threshold, this material experiments plastic deformations, but keeps on incrementing its stress due to its hardening law. This fact also brings about a reduction in the composite stiffness along the branch ($\sigma_y = 40$ MPa to $\sigma_y = 60$ MPa), while the fibre remains elastic up to point ($\sigma_y = 60$ MPa), when its damage begins. The fibre damages along branch ($\sigma_y = 60$ MPa to $\sigma_y = 50$ MPa), and thus causes all stresses to decrease; as a consequence, the matrix experiments elastic unload. From point ($\sigma_y = 50$ MPa) on, the sign of the applied deformation is reversed (unloading), consequently all materials experiment elastic unload. Note that the fibre unloads with a reduced stiffness due to internal damage. Note also that the matrix unloads with the initial elastic stiffness, and at complete unload retains residual plastic strains.

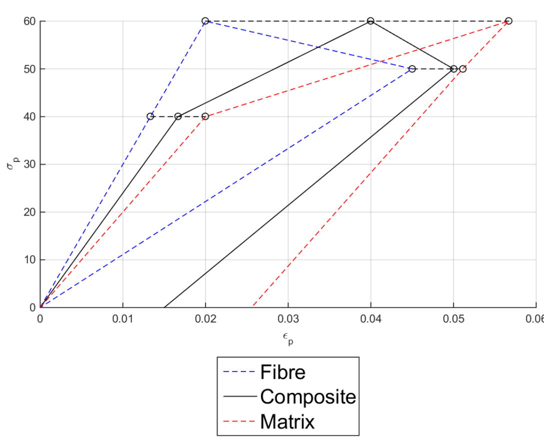

Figure 3: Serial stress (MPa) vs. serial strain curves for the composite and component materials under parallel deformation-controlled loadunload testing.

3.1.3. *Transversal stiffness vs. fibre volume fraction*

For this validation, empirical formulas and experimental data is used to validate the SP-ROM theory. The validation consists in subjecting a DKT laminate element to pure transversal loading at different fibre volume fractions V_f. The mechanical properties for this numerical simulation are:

	Matrix	Fibre
Material	Epoxy	Glass
Young modulus (MPa)	105950	5000
Poisson ratio	0.22	0.38
Volume fraction	0.6	0.4

Table 3: Mechanical properties for transversal stiffness test for a E-Glass composite.

The experimental data has been obtianed from Barbero[21] and the Halpin-Tsai equations from[22].

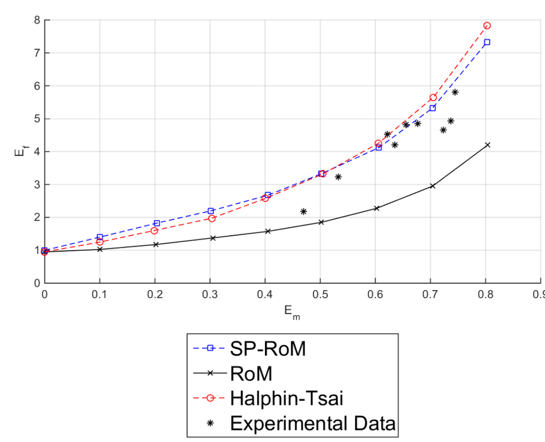

Figure 4: Relative transversal stiffness E_f/E_m vs. fibre volume fraction V_f. Comparison between the results given by the proposed method, experimental data, ROM and HalpinTsai equation.

Note that Figure (4) for SP-RoM model, with a $\nu = 0$, it would recover the RoM model since no Poisson effect would be taken into account. It can be seen that the transversal stiffness for the fibre is not underestimated as for the RoM model and clearly fits better the experimental data than the RoM theory and Halpin-Tsai model.

4. Concluding Remarks

The serial-parallel (SP) model is combined with classical lamination theory (RoM) to describe laminates consisting of unidirectional continuously reinforced layers. Its relative simplicity and efficiency make the serial-parallel approach well suited for implementation as a material model in finite element programs for studying the elasto-plastic response of structures or components made of long fibre-reinforced laminated composites. In addition, it requires relatively small computational resources when implemented into a structural FE code. It still presents a drawback regarding underestimation of the transversal stiffness.

Quadratic convergence is achieved on local nodes for well-posed constitutive models. Also obtaining a more realistic equivalent constitutive tangent operator for the composite allows to obtain a quicker convergence for the global equilibrium.

Comparison between experimental and numerical testing carried out on material samples enables us to state that the methodology presented here is very promising for non-linear analysis of composite materials and structures.

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