Assignment 2 - Presentation

Álvaro Rodríguez Luis

Comunication Skills I

January 2019

INTRODUCTION

GOVERNING EQUATION - TENSION TERM

$$-(EI\mathbf{r}'')'' + [(T - EI\kappa^2)\mathbf{r}']' + [GJ\tau(\mathbf{r}' \times \mathbf{r}'')]' + \mathbf{f} = \left(\frac{1}{4}\pi d \rho_c \mathbf{I}\right)\ddot{\mathbf{r}}$$

$$\rho_0 \frac{\partial^2 \mathbf{r}(t,s)}{\partial t^2} = \frac{\partial}{\partial s}\left(T(t,s)\boldsymbol{t}(t,s)\right) + \mathbf{f}(t,s)(1 + e(t,s))$$

$$\boldsymbol{t}(t,s) = \frac{1}{1+e}\frac{\partial \mathbf{r}}{\partial s},$$

$$T(t,s) = EA_0\left(e + \beta\frac{\partial e}{\partial t}\right)$$

$$e = \left|\frac{\partial \mathbf{r}}{\partial s}\right| - 1.$$

GOVERNING EQUATION - EXTERNAL FORCES

$$\rho_0 \frac{\partial^2 \mathbf{r}(t,s)}{\partial t^2} = \frac{\partial}{\partial s}\left(T(t,s)\boldsymbol{t}(t,s)\right) + \mathbf{f}(t,s)(1+e(t,s))$$

$$\mathbf{f} = \mathbf{f}_{hg} + \mathbf{f}_{dt} + \mathbf{f}_{dn} + \mathbf{f}_{mn}$$

$$\begin{aligned}
\mathbf{f}_{hg} &= \rho_0 \frac{\rho_c - \rho_w}{(1+e)\rho_c}\mathbf{g} \\
\mathbf{f}_{dn} &= -\frac{1}{2} C_{dn} d \rho_w |\mathbf{v}_n| \mathbf{v}_n \\
\mathbf{f}_{dt} &= -\frac{1}{2} C_{dt} d \rho_w |\mathbf{v}_t| \mathbf{v}_t \\
\mathbf{f}_{mn} &= -C_{mn} \frac{\pi d^2}{4} \rho_w \mathbf{a}_n
\end{aligned}$$

WEAK FORMULATION

$$\mathbf{w} \in V = \{f \in C_1[0, L] \,:\, f(0) = f(L) = 0\}$$

$$\int_0^L \left(\rho_0 \frac{\partial^2 \mathbf{r}}{\partial t^2} - \frac{\partial}{\partial s}\left(T(t,s)\frac{\partial \mathbf{r}}{\partial s}\right) - \mathbf{f}(1+e)\right)\mathbf{w}\,ds = 0$$

$$\int_0^L \left(\rho_0 \frac{\partial^2 \mathbf{r}}{\partial t^2}\mathbf{w} + \left(T(t,s)\frac{\partial \mathbf{r}}{\partial s}\right)\frac{\partial \mathbf{w}}{\partial s} - \mathbf{f}(1+e)\mathbf{w}\right)ds - \left[\left(T(t,s)\frac{\partial \mathbf{r}}{\partial s}\right)\mathbf{w}\right]_0^L = 0$$

$$\int_0^L \left(\rho_0 \frac{\partial^2 \mathbf{r}}{\partial t^2}\mathbf{w} + \left(T(t,s)\frac{\partial \mathbf{r}}{\partial s}\right)\frac{\partial \mathbf{w}}{\partial s} - \mathbf{f}(1+e)\mathbf{w}\right)ds = 0$$

CHOOSING A BASIS FOR A FINITE DIMENSION SUBSPACE

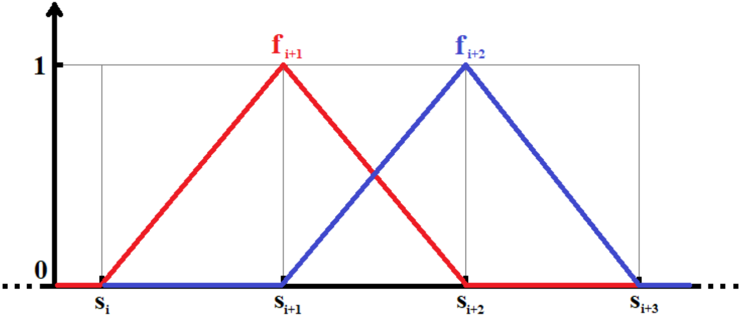
$$V_h = < B > \subset V, \qquad \#(B) = N \in \mathbb{N}$$

$$B = \{\varphi_1(s),\ \varphi_2(s),\ \dots,\ \varphi_i(s),\ \dots\}$$

$$\mathbf{r}(s,t) = \sum_{i=0}^{N} r_i(t)\varphi_i(s) \qquad \frac{\partial \mathbf{r}}{\partial s}(s,t) = \sum_{i=0}^{N} r_i(t)\frac{\partial \varphi_i}{\partial s}(s).$$

$$\sum_{i=0}^{N} \rho_0 \int_0^L \frac{\partial^2 r_i}{\partial t^2}\varphi_i\varphi_k\, ds + \sum_{i=0}^{N} \int_0^L \left(T(t,s) r_i \frac{\partial \varphi_i}{\partial s}\right) \frac{\partial \varphi_k}{\partial s} ds - \int_0^L \mathbf{f}(1+e)\varphi_k ds = 0 \qquad \forall \varphi_k \in B$$

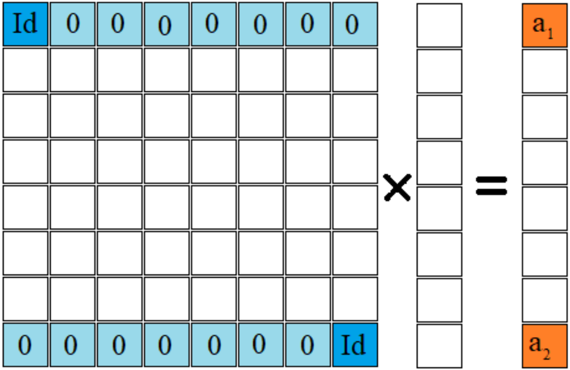
NODE-CENTERED SCHEME



$$\dot{\mathbf{r}}_{k-1} \approx \dot{\mathbf{r}}_k \quad \dot{\mathbf{r}}_{k+1} \approx \dot{\mathbf{r}}_k \quad \ddot{\mathbf{r}}_{k-1} \approx \ddot{\mathbf{r}}_k \quad \ddot{\mathbf{r}}_{k+1} \approx \ddot{\mathbf{r}}_k$$

SYSTEM OF EQUATIONS AND BOUNDARY CONDITIONS

$$\mathbf{M} \cdot \ddot{\mathbf{r}}(t + dt) = \mathbf{F}(t) - \mathbf{K}(t) \cdot \mathbf{r}(t) = \mathbf{F}_{total}(t)$$



SPHERICAL BODY TOWING BOUNDARY CONDITIONS (I)

$$\mathbf{a}_{sphere} = \frac{\mathbf{F}_M + \mathbf{F}_{gb} + \mathbf{F}_T}{m + C_a}$$

$$\mathbf{F}_M = \rho C_m V \cdot \dot{\mathbf{u}} + \frac{1}{2}\rho C_d A \cdot \mathbf{u}|\mathbf{u}|$$

$$\mathbf{F}_{gb} = (V_u \cdot \rho - m) \cdot g \cdot \mathbf{e}_z$$

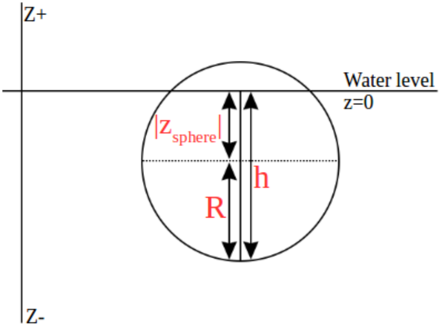
$$\mathbf{F}_T = EA_0 \cdot (e_1 + \beta \cdot \dot{e}_1) \cdot \frac{\mathbf{r}_1 - \mathbf{r}_0}{|\mathbf{r}_1 - \mathbf{r}_0|}$$

$$\mathbf{F}_T = EA_0 \cdot \left[(e_i + \beta \cdot \dot{e}_i) \cdot \frac{\mathbf{r}_{i-1} - \mathbf{r}_i}{|\mathbf{r}_{i-1} - \mathbf{r}_i|} + (e_{i+1} + \beta \cdot \dot{e}_{i+1}) \cdot \frac{\mathbf{r}_{i+1} - \mathbf{r}_i}{|\mathbf{r}_{i+1} - \mathbf{r}_i|}\right]$$

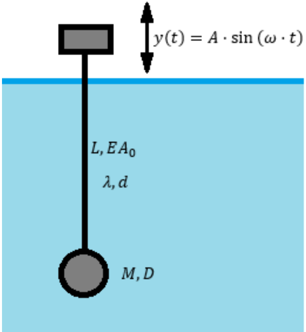
SPHERICAL BODY TOWING BOUNDARY CONDITIONS (II)

$$V_u^{sphere} = \frac{\pi h^2}{3}(3R - h)$$

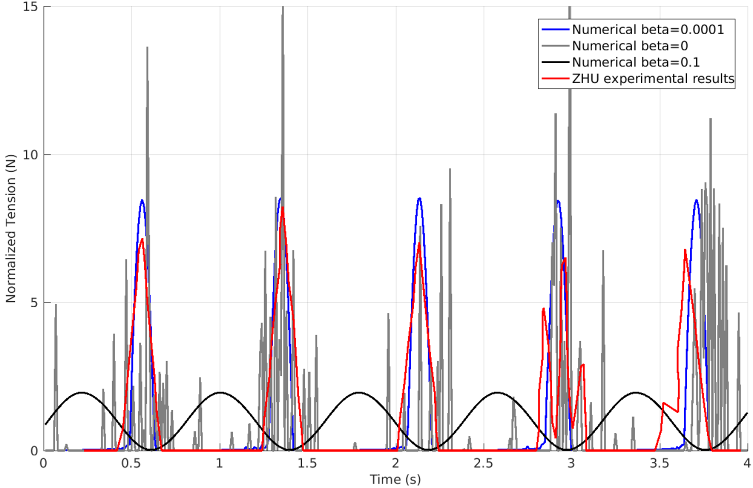
$$h = min\{2R, max\{0, R - z_{sphere}\}\}$$



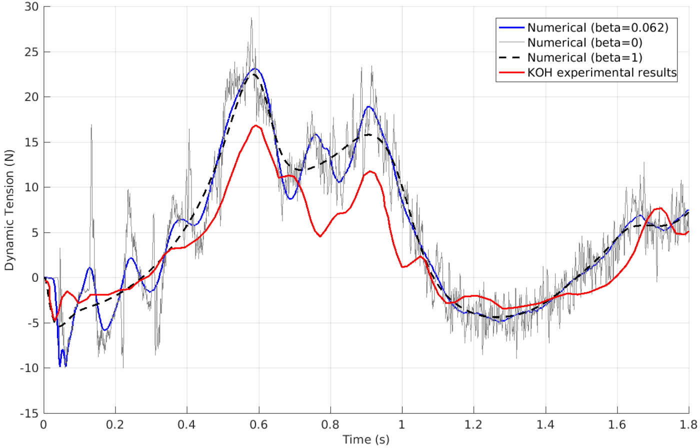
VALIDATION - ZHU'S EXPERIMENT (I)



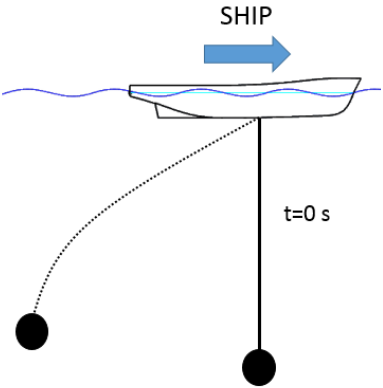
VALIDATION - ZHU'S EXPERIMENT (II)



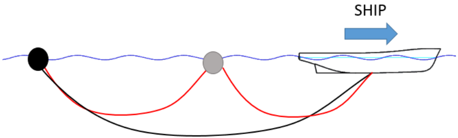
VALIDATION - KOH'S EXPERIMENT

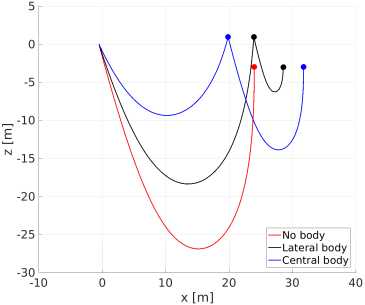


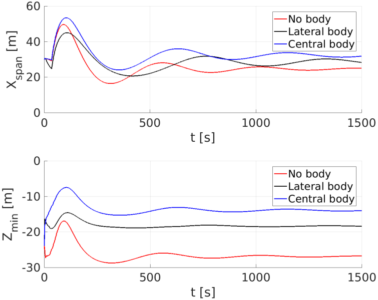
APPLICATION CASES - SUBMERGED



APPLICATION CASES - FLOATING







CONCLUSIONS AND FUTURE RESEARCH

- Previous numerical models were improved using the Rayleigh springs model for the tension term.
- The proposed model is a useful tool in towing system design, study or optimization.
- Future Research
  - Develop a higher order method.
  - Consider bending and torsion effects.

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