# FEKETE POINTS

The Fekete Points is a method to discretize a surface S in a set of points $w_n = \{p_1, p_2, \dots p_n\}$ with $p_i \in S$, $\forall i$ which guarantees that your last configuration will be well-distributed on your surface.

The method is based in the Physical interpretation of the behaviour of a system of particles when they search for a minimal energy configuration and it is an implication of the second law of thermodynamics.

It determines the position of n points of a compact $S \subset \mathbb{R}^3$ which maximizes their Euclidean distances product. It is easy to see that in this situation, the set of points $w_n = \{p_1, p_2, \dots, p_n\}$ must minimice the functional:

$$\Rightarrow \Gamma(w_n) = \sum_{1 \leq i < j < n} \frac{1}{|p_i - p_j|^s} \text{ with } s > 0$$

These functional are the potential energy of a system if we consider unitary mass in each point.

$$\Rightarrow V_i(w_n) = \sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{1}{|p_i - p_j|^s}$$

It is shown that when this parameter $s \to \infty$ the final configuration is perfectly equidistributed on any surface. In our case of interest, $S \subset \mathbb{R}^3$ and with s=3 we will obtain a final distribution almost equally spaced.

Once the method has compute the energy of the system, each point is moved in the direction of the repulsive force that this point receives by the other ones, computed as:

$$\Rightarrow F_i = -\nabla V_i(p_i)$$

At the end of the iterations, the repulsive forces have only normal component to the surface and the points are stable on the surface. The energy of our final configuration of points is lower than the previous ones.

To end up, I would like to program this method into any kind of surfaces and obtaining these set of points which can be used to modelling any kind of structural problem or to design some kind of triangulation through these points.