AN IMPLICIT MATERIAL POINT METHOD

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Keywords: Material Point Method, Finite Element Method, Solid Mechanics.

Abstract: Granular materials, during the last few decades have been object of study by a great number of researcher at academic and industrial level. Granular matters can be found in every aspect of the human life and its use is fundamental for its sustenance. Either, most of the products commonly used can be considered made of 'particles' (e.g. thinking to the pharmaceutical or food industry) or originally, before being processed, the raw material they are composed of was granular. This explains why particulate materials are interesting and deep knowledge of them is demanded.

Sometimes the lack of information and knowledge of material's behaviour is cause of waste of time and money. For this reason laboratory tests are performed and many researcher team are focused on the material modelling. However it is not always possible to solve every problem conducting real tests, because they are too expensive or the scale effect makes them extremely difficult to reproduce. Nowadays, in most of the cases, the study of the material's behaviour at a macroscale is performed by means of appropriate computational methods and constitutive relations.

The main objective of this work lies in the development of a numerical technique designed to be applied to practical engineering problems, involving dense granular flows, such as, for example, particles moving inside silos or hoppers, where a flowing and a static regime coexist. After an accurate review of the state of the art, the Material Point Method (MPM) has been identified as a suitable numerical method to achieve such objective. The Material Point Method (MPM) is one of the latest developments of particle-in-cell(PIC) methods [Harlow, 1956], originally used to model highly distorted fluid flow, proposed for the first time by [Harlow, 1964] . During the last decades different variants of PIC method have been proposed. Recently, the method has been applied to solid mechanics too, where the ability of material points to advect naturally Lagrangian state variables has been exploited in MPM. Several studies have shown the successful application of MPM to solid mechanics. Lagrangian material points allow easy storage of internal variables representing the history-dependent state and version of hyperelasticity, hypoelasticity, plasticity and viscoelasticity have been

implemented.
The MPM has been applied to solve a wide variety of problems as:

- silo discharge [Wickowski et al., 1999]
- landfill settlement [Zhou et al., 1999]
- elastic vibration [Sulsky et al., 1994]
- response of granular material [Bardenhagen et al., 2000] [Bardenhagen et al., 2001]
- impact/contact problem
- penetration and perforation with history-dependent internal state variables [Sulsky et al., 1994, Sulsky et al., 1995, Sulsky, D., Schreyer, 1996].

It presents several advantages in simulating cases where large displacement/large deformation and a history dependent behaviour of the material are involved. In the MPM two types of description are used, as it can be observed in Figure 1. The first one is represented by the material domain composed by a set of points, termed as particles or material points, which following the deformation of the material track the position and the state variables. The second one by a computational background grid, that could be fixed or not, employed to solve the governing equations.

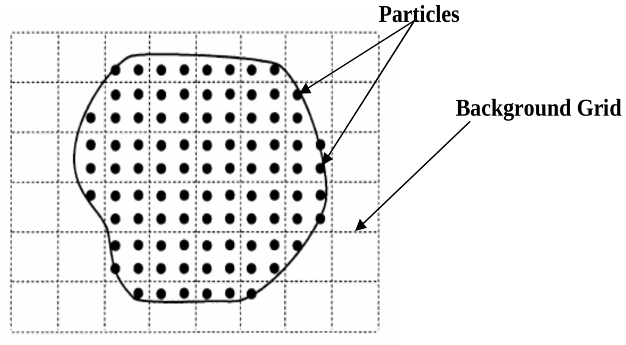

Figure 1: Material points and background grid, [V. Karuppiah, 2004]

A Lagrangian description is used to track with more accuracy the deformation of the continuum avoiding the element distortion, typical drawback of other Lagrangian techniques which requires a frequent remeshing.

As written in [Sulsky et al., 1994], many features of the MPM are connected to the Finite Element Method's ones. MPM uses particles that carry their own mass and velocity and internal state variable and solves the governing equations in a FEM-like manner on a background mesh. The material points essentially function as integration points for the assembly in finite element calculation. The implicit time integration strategy exploits similarities between the function of material points in MPM and integration points in FE calculations to adapt implicit time integration for use with MPM.

MPM naturally respects mass and momentum balance, an important feature which makes it really useful in several engineering applications. This feature is guaranteed by means of a first mapping of information in terms of mass and momentum from particles to nodes, performed before the resolution of the system of linearised equations. This step (step a of Fig.2) is fundamental for the evaluation of the initial conditions on the nodes of the computational mesh. When a solution, which satisfies the convergence criterion, is obtained (step b of Fig.2), the kinetic variables on the material points in terms of displacement, velocity and acceleration are updated (step c of Fig.2). Finally, before initializing the next time step, the new position of the material point is evaluated (step d of Fig.2).

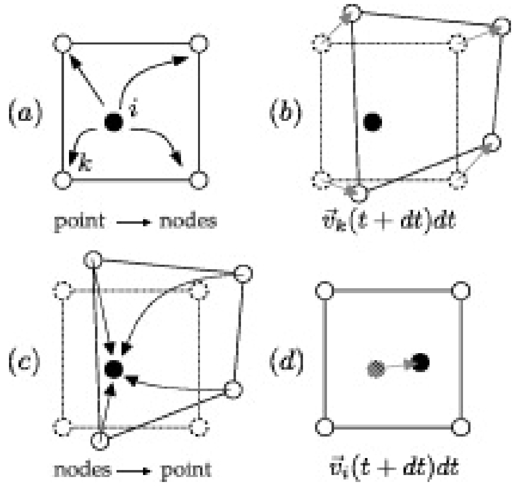

Figure 2: Algorithm of MPM divide in four steps: a) mapping of the previous step informations from particles to nodes; b) solving the balance of linear momentum on the nodes; c) mapping of the solution from nodes to particles; d) update of the new particle's position, see [Gilabert, F. A., V. , Cantavella, E., Sánchez and G., 2011]

Moreover the MPM it is naturally parallelizable ensuring a high efficiency in large dimension problems.

In the present work an implicit Material Point Method implemented in Kratos Multiphysics is presented.

The results of some examples used for the verification of the code, two static and one dynamic, respectively are shown.

Regarding the static cases, the bend of a 2D cantilever subjected to a point load on the free edge, in the first case, and to the self-weight, in the second one is considered. In both cases a good agreement has been achieved between the analytical solution and the MPM code results in terms of vertical displacement and σ_{xx} Cauchy stress component. Moreover, a mesh convergence study is performed and the same quadratic order of convergence of a FEM code is found, as can be observed in Fig.3.

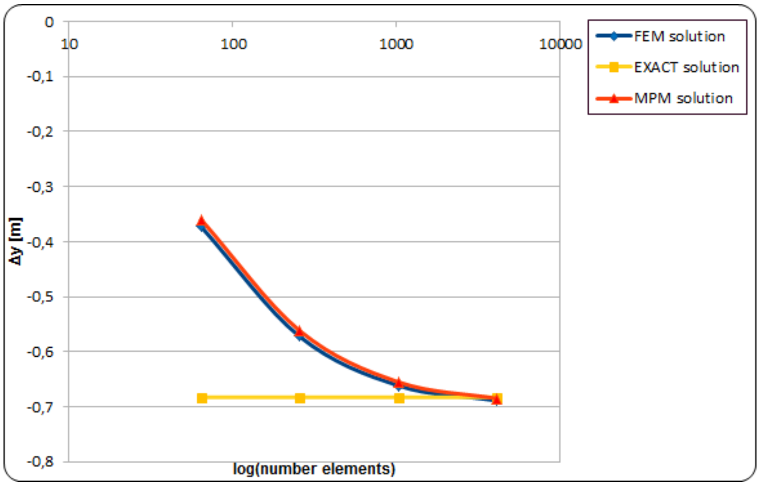

Figure 3: Convergence study

The dynamic case is represented by the impact of two elastic bodies. Through this representative numerical simulation the accuracy for problems involving large deformation, contact and dynamics is demonstrated. Unlike a traditional FEM code, a MPM code doesn't need the involvement of a contact algorithm for contact without friction. This is possible because contact occurs when information from the two bodies is interpolated to common nodes, as can be observed in Fig.4.

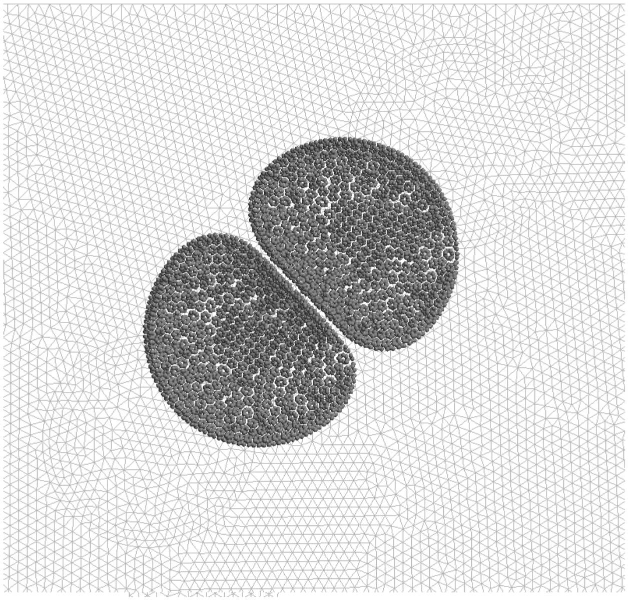

Figure 4: snapshot of the impact of two elastic bodies example

Further benchmarks are needed to conclude the verification of the code.

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