Master in Numerical Method in Engineering

Computational Mechanics Tools

Assignment 2
PDE-Toolbox

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Excersice 1

1. Consider $t_{end} = 10$, solve the problem, and refine the initial mesh up to 4 times. Verify that the theoretical convergence order holds.

   (a) Obtained Solution
   In order to compute the convergence was needed to refine the initial mesh four times as figure 1 shows. The initial mesh is located on top left of figure 1.

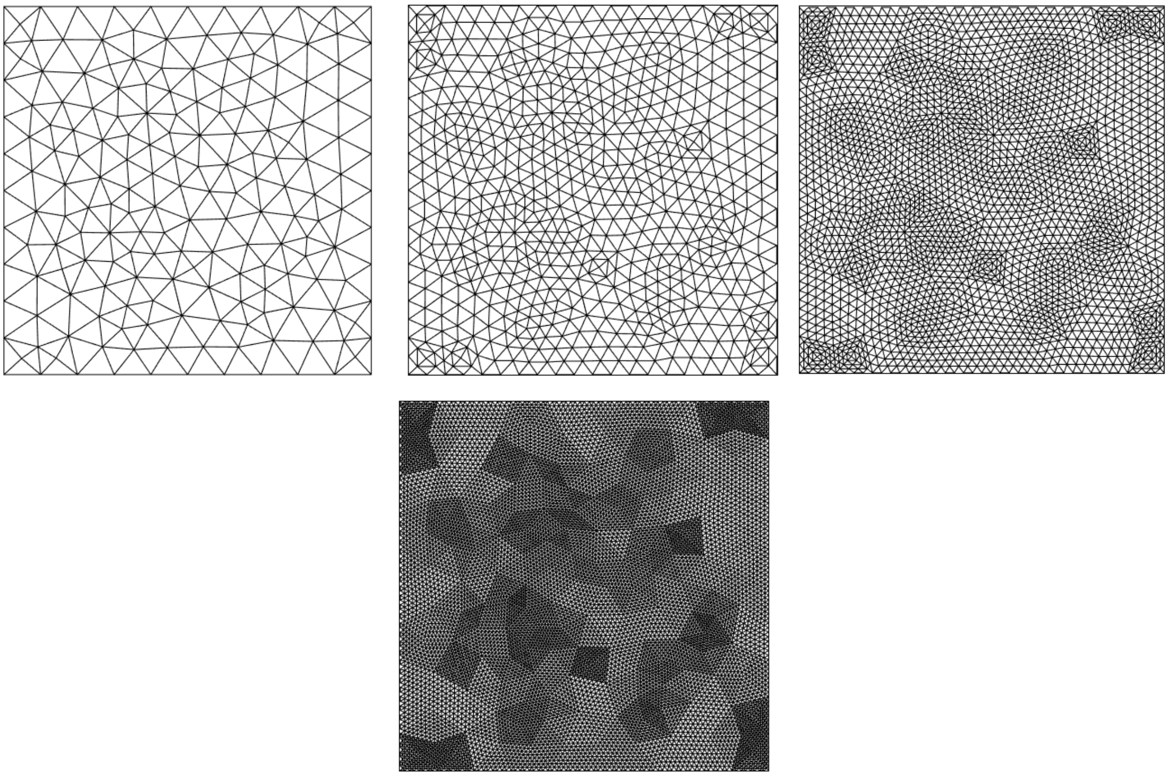

Figure 1: Meshes. Top left: Initial mesh. Top Centre: first refinement. Top Right: Second refinement. Bottom: Last refinement.

In figure 2 can be observed the numerical and analytical solution at time t = 10. The numerical solution that shows up below, was obtained using a second refinement mesh (On top left of figure 1).

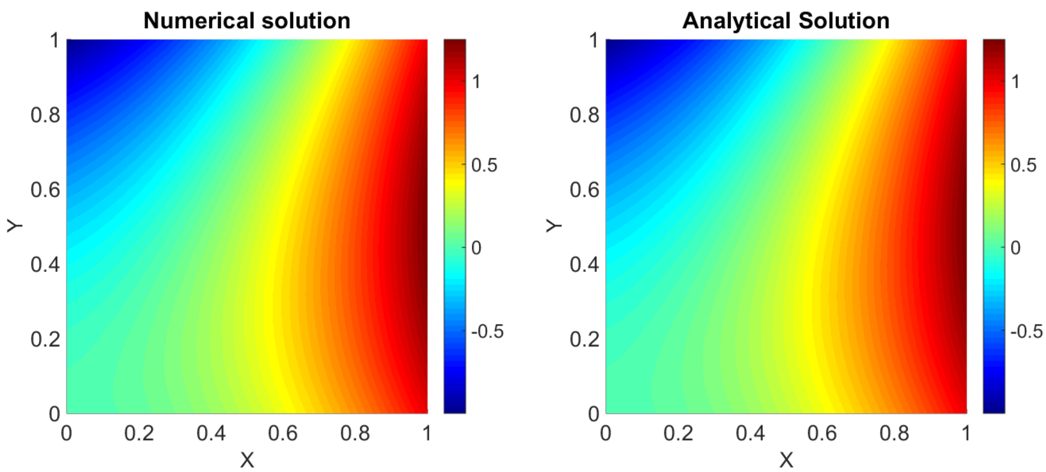

Figure 2: Left: Numerical solution at time t = 10. Right: Analytical solution at time t = 10. B

Figure 3 is showing an error map. It can be noted how the absolute error is distributed over the chosen mesh. It can also be seen that the absolute error is very low everywhere.

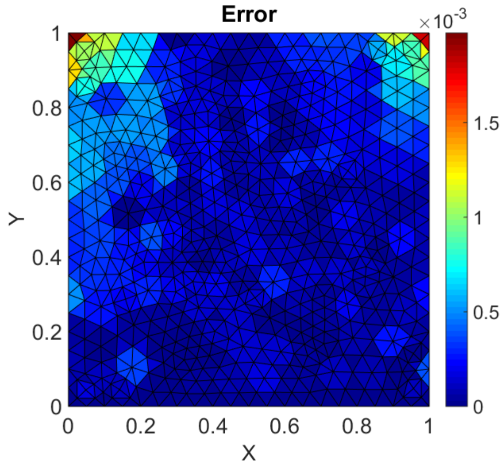

Figure 3: Max. absolute error at time t = 10.

(b) Verification of convergence order against theoretical one

It can be observed, in figure below, the maximum absolute error plot.
The maximum absolute error was computed as the absolute value between the numerical solution minus the analytical solution, and the size mesh was computed using the expression written below.

$$h = \frac{\sqrt{2 \cdot Area}}{N}$$

Where Area = physical domain area, N = Elements number.
In order to compute the slope of the straight line, a matlab tool, whose name is basic fitting tool, was used. According to the calculation, the slope value was equal to $1.79 \approx 1.8$ as theory says.

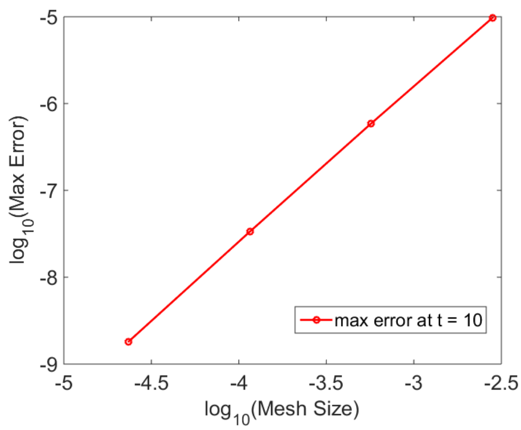

Figure 4: Convergence plot.

2. How is the solution affected when we modify the final time.

In order to see how the solution is affected when the final time is modified, it was performed a new calculation using a time lapse between zero and 1 $t = [0:1]$. After solving this problem, the computed error was compared against the calculated one of the previous problem.

Figure 5 shows two convergence curves. One is corresponded to the previous problem and the other one corresponds to the current one. It can be noted how this two curves converge one each other while size mesh decreases. On the other hand, it can be observed while the mesh size grows, the current problem error increases, and a significant error difference between the previous problem and the current one appear. The latter remark can be noted if a large size mesh is chosen.

As a conclusion. It could be observed that the solution is affected when the final time is modified. While time lapse becomes short, the maximum absolute error increases, for e given size mesh. On the other hand, while size mesh decreases, then, the maximum absolute error decreases too, for a given time lapse.

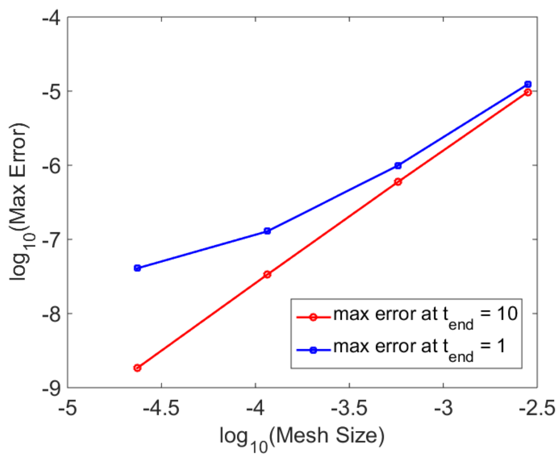

Figure 5: Convergence plot. Blue curve represents the max absolute error for the current problem ($t = [0:1]$). Red line represents the max absolute error for the previous problem ($t = [0:10]$)

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3. We are interested in obtaining the solution at time tend = 50. Find a more efficient manner to solve this problem. You do not need to prove the equivalence mathematically, but you need to provide numerical evidence of the new method.

   A time response analysis was performed in order to analyse how the transient behaviour of the solution is.

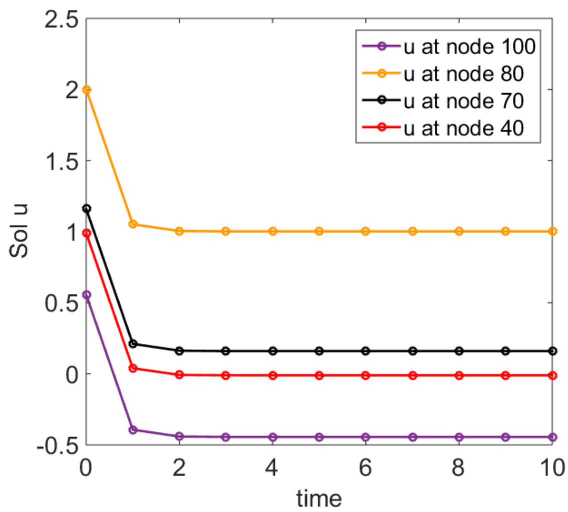

Figure 6: Time response analysis.

In figure 6 can be seen the numerical solution "u" respect time, obtained in different nodes. It is important to note that the solution at time t = 2 reaches a steady state. Therefore, any solution "u" will be constant respect to time, for any time t ¿ 2. In particular, the solution "u" at time t = 50 will be equal to any solution at time $t > 2$.