

Computational Solid Mechanics

Assignment 1

Report on Continuum Damage Models

Author Name: Anil Bettadahalli Channakeshava

07/04/2016

1 Introduction

In civil engineering, it is important to be able to predict the damage encountered by a structure after an accidental event. The continuum damage models are used where the cracks are not reproduced but considered through a variable that describes the degradation of the elastic properties. Moreover, these models are phenomenological models which are appropriated for geo-materials, materials which are not metals. In this assignment, we are dealing with two parts, the first part includes *Rate independent model (Inviscid case)* and the second part includes *Rate dependent models (Viscid or viscous case)*. We are analysing only in the plane strain case that means our study in this assignment is limited to only plane strain case.

2 Rate Independent Models (Inviscid case)

2.1 Non-symmetric tension-compression damage model :

The implementation of the MATLAB codes for this case has been demonstrated in *Appendices* and post process results of appropriate curves of stress space and stress-strain curve have been analysed, considering hard soft modulus as 0.5 and yield stress ($100N/m^2$).

Case1:

For loading Paths,

$$\Delta\bar{\sigma}_1^{(1)} = \alpha = 300; \quad \Delta\bar{\sigma}_2^{(1)} = 0 \quad (\text{uniaxial tensile loading})$$

$$\Delta\bar{\sigma}_1^{(2)} = -\beta = -500; \quad \Delta\bar{\sigma}_2^{(2)} = 0 \quad (\text{uniaxial tensile unloading/compressive loading})$$

$$\Delta\bar{\sigma}_1^{(3)} = \gamma = 400; \quad \Delta\bar{\sigma}_2^{(3)} = 0 \quad (\text{uniaxial compressive unloading/ tensile loading})$$

As we can see from Figure 1, for uniaxial loading case up to yield stress, material is in elastic region but when it passes through the yield stress point there is tensile loading. But in the case of compressive loading the material is completely in the elastic regime as in compression loading our elastic regime has increased for non symmetric case by parameter 'n'. Hence our implementation is correct and behaves as that of analytical approach or theoretical approach.

Case2:

For loading Paths,

$$\Delta\bar{\sigma}_1^{(1)} = \alpha = 300; \quad \Delta\bar{\sigma}_2^{(1)} = 0 \quad (\text{uniaxial tensile loading})$$

$$\Delta\bar{\sigma}_1^{(2)} = -\beta = -250; \quad \Delta\bar{\sigma}_2^{(2)} = \beta = -250 \quad (\text{biaxial tensile unloading/compressive loading})$$

$$\Delta\bar{\sigma}_1^{(3)} = \gamma = 200; \quad \Delta\bar{\sigma}_2^{(3)} = \gamma = 200 \quad (\text{biaxial compressive unloading/ tensile loading})$$

The Figure 2 shows us the variation in stress space and stress-strain curve for case 2, i.e. uniaxial and biaxial loading. Up to yield stress, material is in elastic regime but when loaded beyond the point of yield stress ($100N/m^2$) there is tensile loading. And when compressive loading starts, the elastic regime for non symmetric case is enlarged and hence in compressive loading the material is completely in elastic domain. Our apparent stress is not beyond the damage surface as seen from the stress space curve from Figure 2 and we can conclude that we are dealing with inviscid case that is rate independent models.

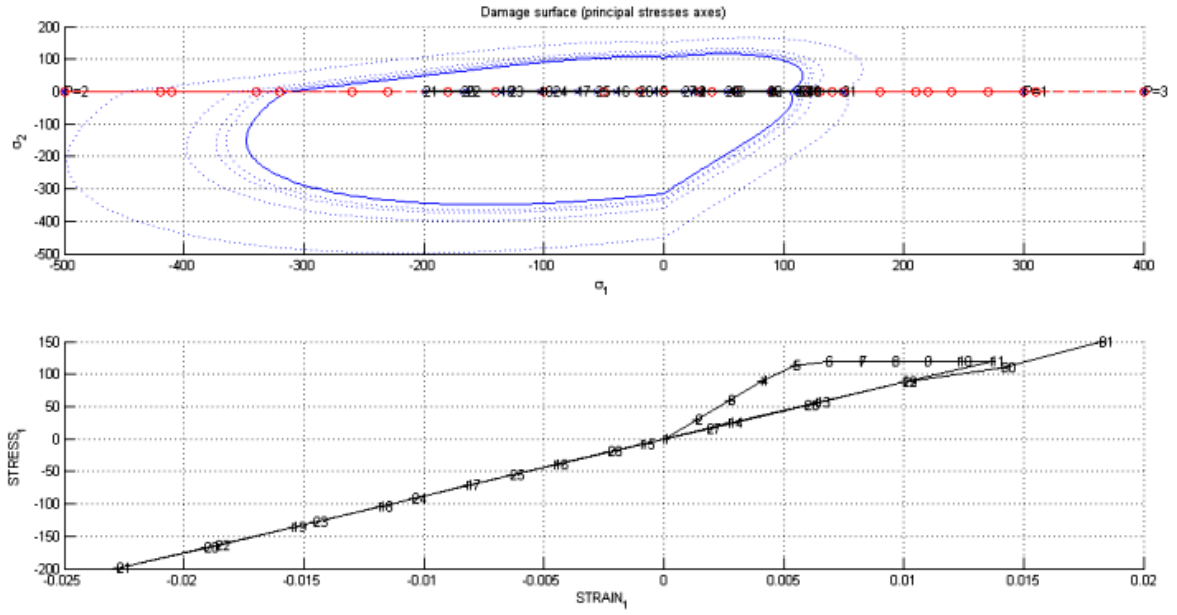


Figure 1: Path of Stress space and Stress strain curve for case 1 (Non-symmetric model).

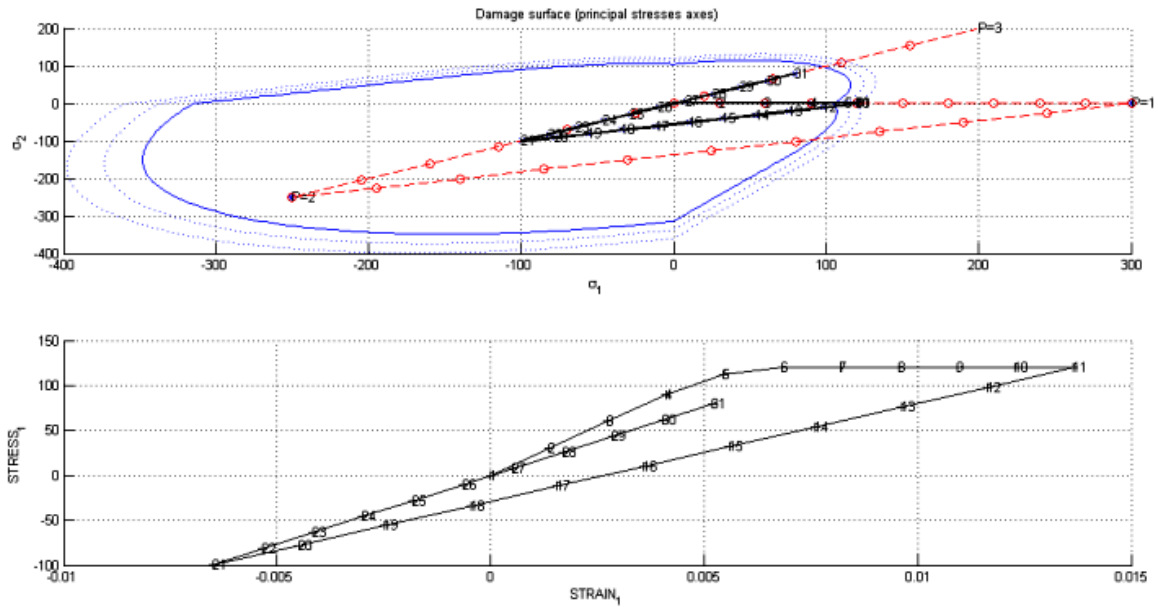


Figure 2: Path of Stress space and Stress strain curve for case 2 (Non-symmetric model).

Case3:

For loading Paths,

$$\Delta\bar{\sigma}_1^{(1)} = \alpha = 200; \quad \Delta\bar{\sigma}_2^{(1)} = \alpha = 200 \quad (\text{biaxial tensile loading})$$

$$\Delta\bar{\sigma}_1^{(2)} = -\beta = -250; \quad \Delta\bar{\sigma}_2^{(2)} = \beta = -250 \quad (\text{biaxial tensile unloading/compressive loading})$$

$$\Delta\bar{\sigma}_1^{(3)} = \gamma = 150; \quad \Delta\bar{\sigma}_2^{(3)} = \gamma = 150 \quad (\text{biaxial compressive unloading/ tensile loading})$$

Case 3 for non-symmetric model behaves as that of case 2 during biaxial tensile loading. The material will be in elastic regime until the yield stress point and later the tensile loading starts during compression loading and hence it behaves similarly as that of case 2 of non-symmetric damage model which is well depicted in Figure 3. Hence we can conclude that our implementation is correct and working as usual.

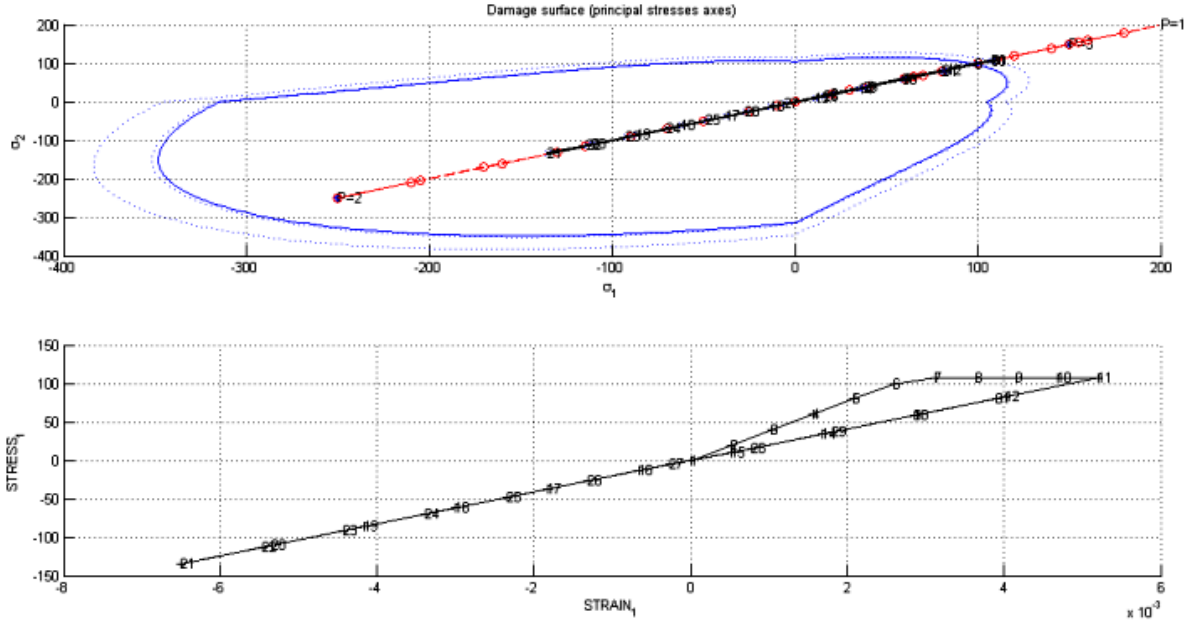


Figure 3: Path of Stress space and Stress strain curve for case 3 (Non-symmetric model).

2.2 Tension only damage model:

The implementation of the MATLAB codes for this case has been demonstrated in *Appendices* and post process results of appropriate curves of stress space and stress-strain curve has been analysed, considering hard soft modulus as -0.5 .

Case1:

For loading Paths,

$$\Delta\bar{\sigma}_1^{(1)} = \alpha = 300; \quad \Delta\bar{\sigma}_2^{(1)} = 0 \quad (\text{uniaxial tensile loading})$$

$$\Delta\bar{\sigma}_1^{(2)} = -\beta = -500; \quad \Delta\bar{\sigma}_2^{(2)} = 0 \quad (\text{uniaxial tensile unloading/compressive loading})$$

$$\Delta\bar{\sigma}_1^{(3)} = \gamma = 400; \quad \Delta\bar{\sigma}_2^{(3)} = 0 \quad (\text{uniaxial compressive unloading/ tensile loading})$$

For uniaxial tensile loading and compressive loading the material behaves as expected that is for uniaxial tensile loading, the material is in the elastic regime until the yield stress ($100N/m^2$). But when load path passes the yield stress limit the tensile loading starts and the deformation of the material occurs and hence the blunt curve numbering from 5 to 11 in Figure 4 represents the same. But when the compression loading comes to picture, even though the load path passes through yield stress, there is no compression loading because the material will be in elastic regime even though the threshold value of yield stress has been passed. Therefore the Figure 4 depicts the same and hence we can conclude that our implementation is correct compared with analytical approach.

Case2:

For loading Paths,

$$\Delta\bar{\sigma}_1^{(1)} = \alpha = 300; \quad \Delta\bar{\sigma}_2^{(1)} = 0 \quad (\text{uniaxial tensile loading})$$

$$\Delta\bar{\sigma}_1^{(2)} = -\beta = -250; \quad \Delta\bar{\sigma}_2^{(2)} = \beta = -250 \quad (\text{biaxial tensile unloading/compressive loading})$$

$$\Delta\bar{\sigma}_1^{(3)} = \gamma = 200; \quad \Delta\bar{\sigma}_2^{(3)} = \gamma = 200 \quad (\text{biaxial compressive unloading/ tensile loading})$$

From the stress space curve from Figure 5, we can say that our apparent stress is within the damage surface and it does not go beyond the damage surface. From the Figure 5, we can also say that for uniaxial tensile load path loading the material remains in elastic regime until the yield stress ($100N/m^2$) but as it passes the limit there is a tensile loading. For compression loading for given load paths the material is purely in the elastic regime even though the load paths crosses the yield stress. And hence it behave normally as expected and we can conclude that our implementation is correct.

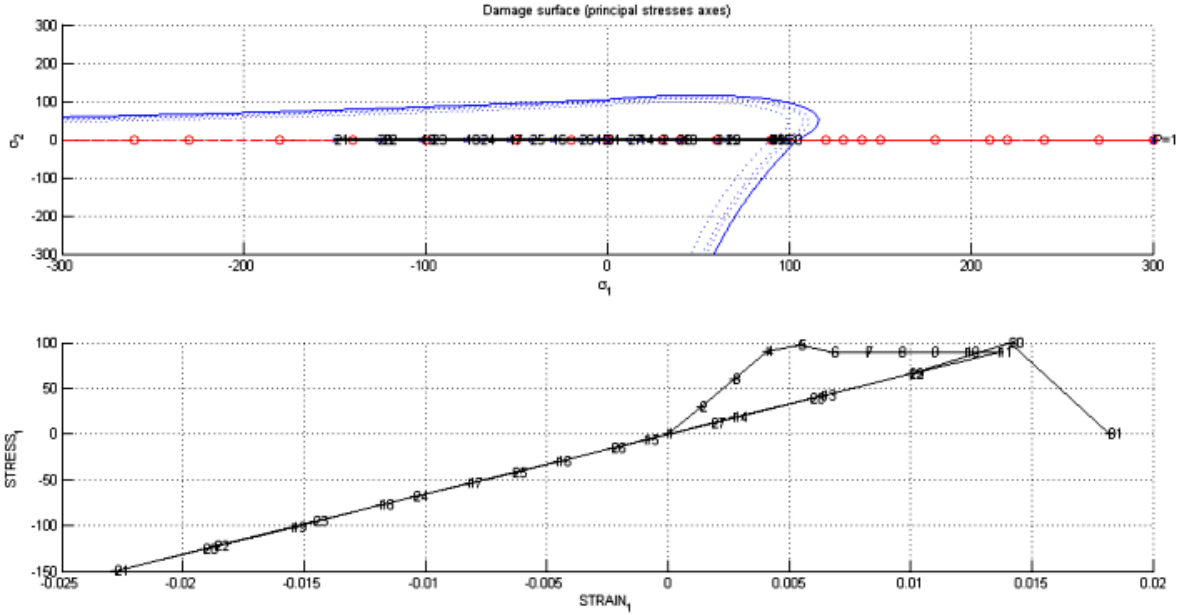


Figure 4: Path of Stress space and Stress strain curve for case 1 (Tension only damage model).

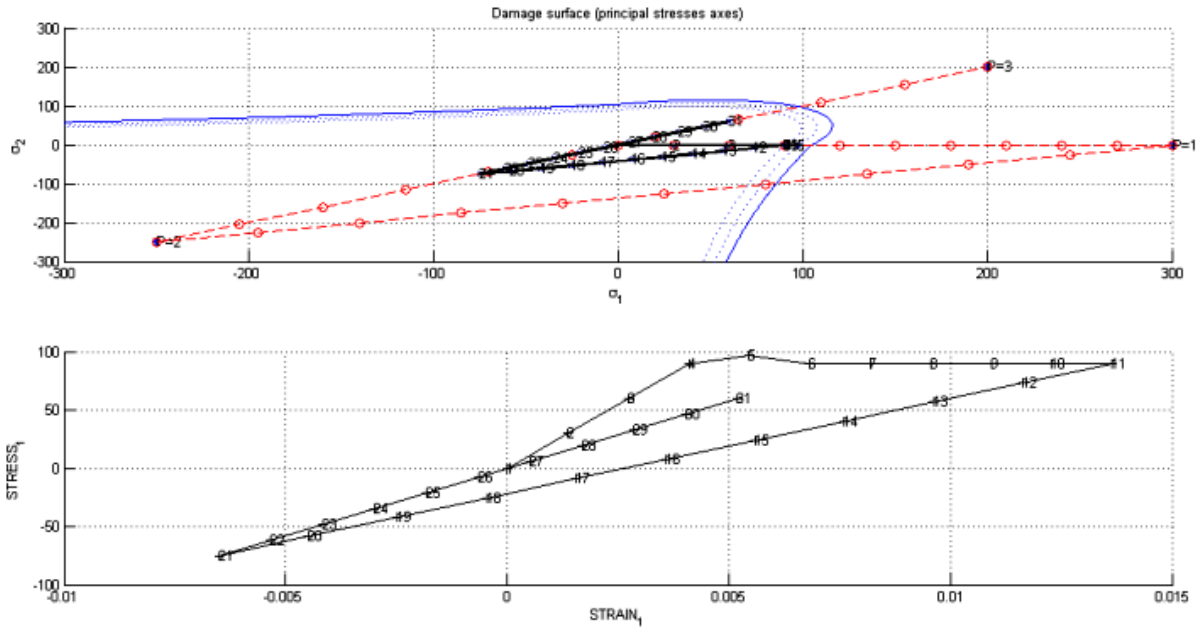


Figure 5: Path of Stress space and Stress strain curve for case 2 (Tension only damage model).

Case3:

For loading Paths,

$$\Delta\bar{\sigma}_1^{(1)} = \alpha = 200; \quad \Delta\bar{\sigma}_2^{(1)} = \alpha = 200 \quad (\text{biaxial tensile loading})$$

$$\Delta\bar{\sigma}_1^{(2)} = -\beta = -250; \quad \Delta\bar{\sigma}_2^{(2)} = \beta = -250 \quad (\text{biaxial tensile unloading/compressive loading})$$

$$\Delta\bar{\sigma}_1^{(3)} = \gamma = 150; \quad \Delta\bar{\sigma}_2^{(3)} = \gamma = 150 \quad (\text{biaxial compressive unloading/ tensile loading})$$

Case 3, behave as usual for tension damage model. It remains in elastic regime upto yield stress and as it passes the threshold there is tensile loading as we it can be seen in the Figure 6. But for compressive loading the material is completely in elastic regime even after passing the yield stress. Hence our implementation is correct.

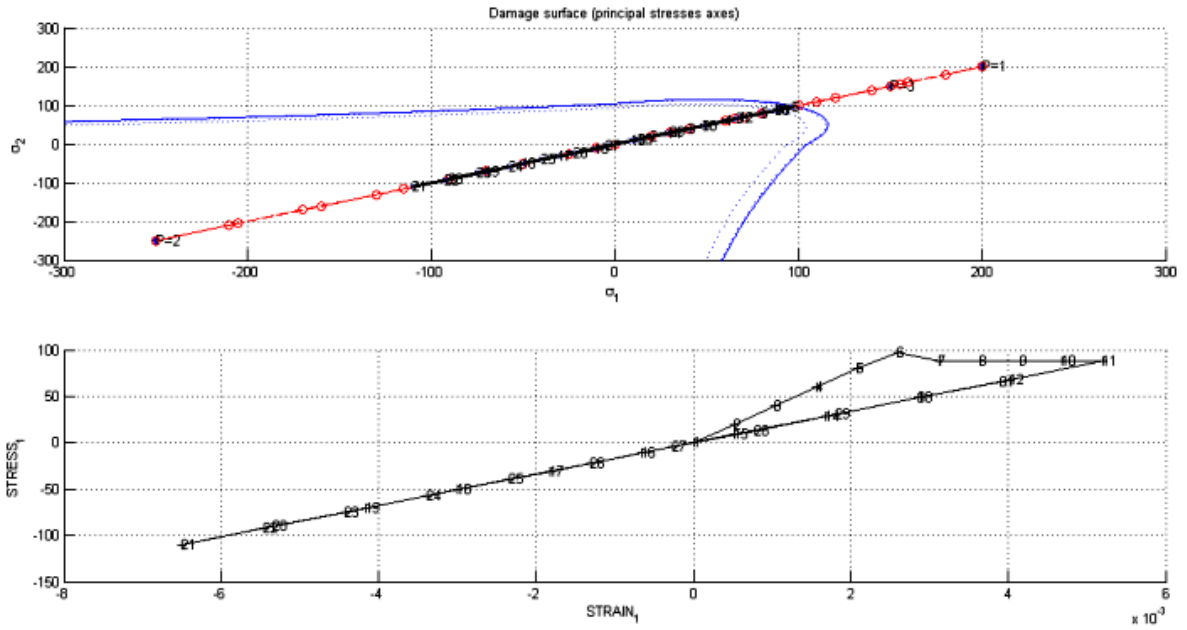


Figure 6: Path of Stress space and Stress strain curve for case 3 (Tension only damage model).

2.3 Linear hardening/softening($H > 0/H < 0$):

The hardening and softening are the material property and here we are analysing how stress like internal variable q varies with strain like internal variable r . The initial values of q_0 and r_0 are set same in order to satisfy the analytical approach, hardening and softening modulus H as constant. We know that if hardening modulus $H > 0$ we have hardening and if hardening modulus $H < 0$, then we have softening. In case of hardening the elastic domain expands and in the case of softening it shrinks. Symmetric tension-compression model is used to show the implementation.

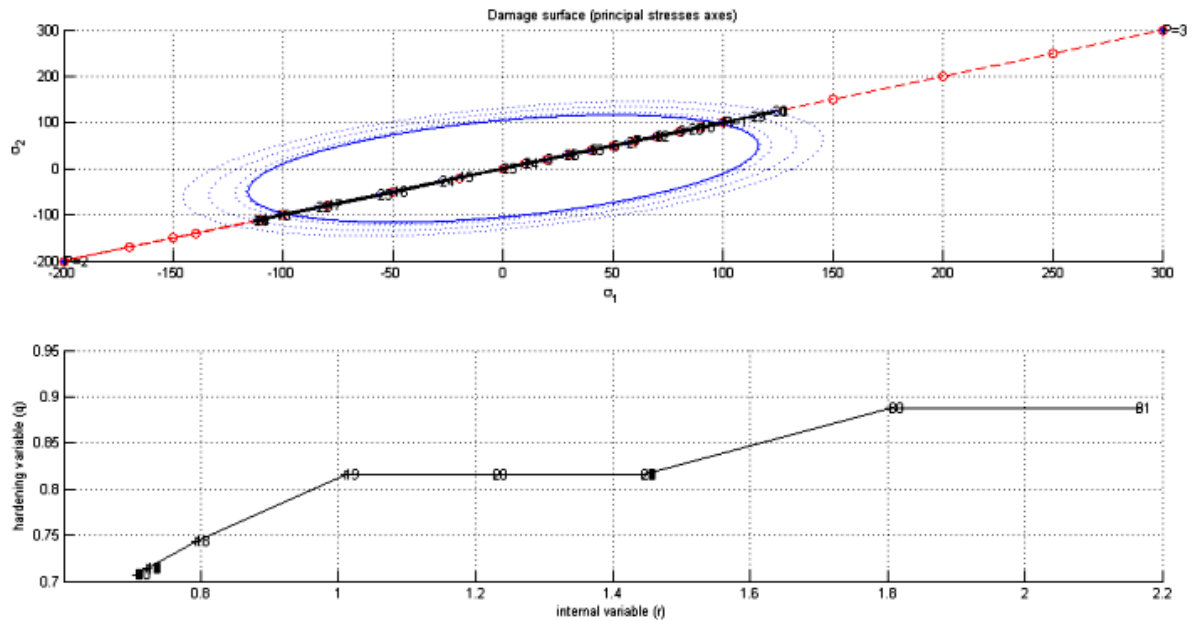


Figure 7: Linear Hardening with $q_\infty > r_0$

From Figure 7, we can conclude that for different load paths, the hardening parameter (H) is considered to be 0.5. Since it is a positive value the elastic domain has to expand which can be clearly seen from the stress space curve. From the plot of r v/s q , we can say that slope (H) comes into the picture multiple times because of the different load paths and hence it behaves as required and our implementation is correct. Here we should be very careful while dealing with q_∞ , because in order to get hardening the value of this should be always greater than r_0 .

But from Figure 8 we have assumed that hardening parameter (H) as -0.5 , a negative value hence the elastic domain has to shrink which can be seen in the stress space curve. From the plot of r v/s q , we can conclude that slope (H) has increased multiple times because of the different load paths which were required for our implementation and hence it is correct. In case of linear softening q_∞ should always be less than r_0 . If not then there is an error in the algorithm that is we get hardening.

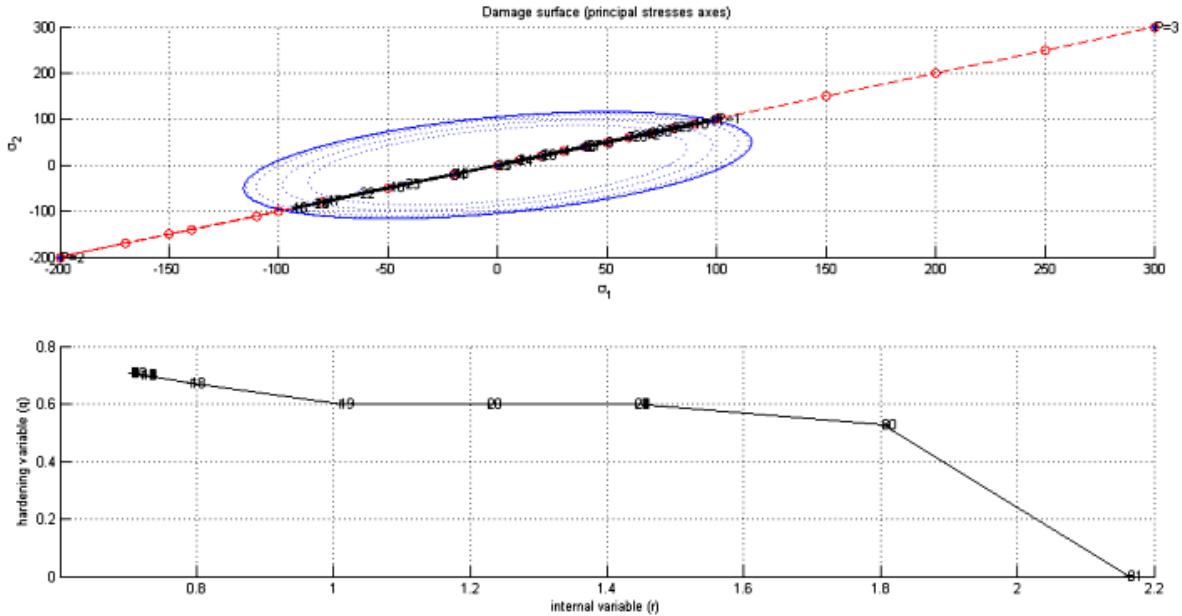


Figure 8: Linear Softening with $q_\infty < r_0$

To conclude we can say that, in linear Hardening and softening the variation in the plot of r v/s q we have linear variation and it behaves the same for change in load paths and slope (H). Therefore the implementation of linear hardening and softening is correct.

2.4 Exponential hardening/softening ($H > 0/H < 0$):

The other way of explaining the hardening and softening law is continuous hardening and softening. Here the r_0 and q_∞ are the parameters that have to be considered and rest are same as that of linear hardening and softening law, but the slope H does not play a major role in this section. We can see from Fig 9 and Fig 10, the variation of exponential hardening and softening for different load paths along with the plot of (internal variable) r v/s q (hardening variable) and stress space as well. Symmetric tension-compression model is used to show the implementation.

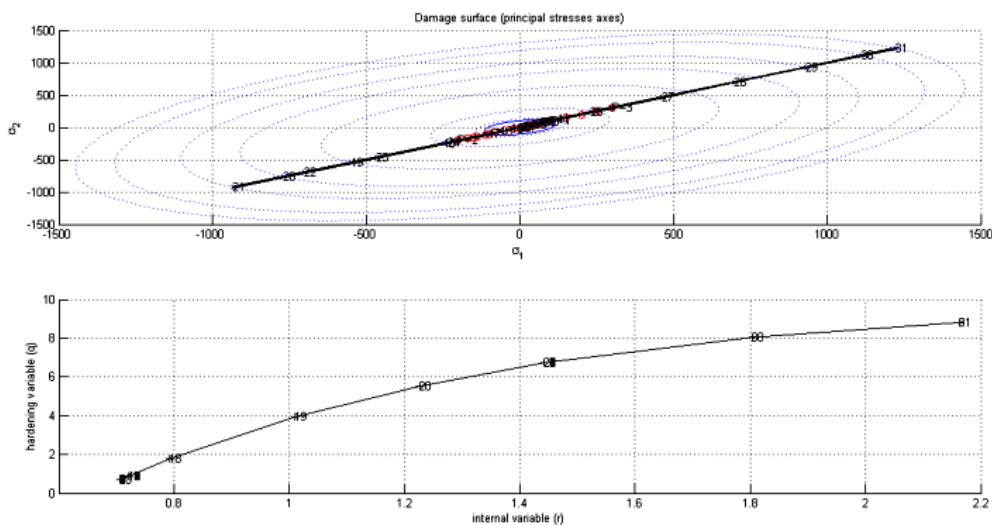


Figure 9: Exponential Hardening with $q_\infty > r_0$

From the Figure 9 we can see the exponential hardening, here the curve is continuous and blunt in shape and the exponential function is asymptotic to q_∞ . In order to get exponential hardening, the value of q_∞ should always be greater than r_0 . The stress space curve shows us that there is an expansion in it because of hardening and also even though we have multiple loads the curve is varying exponentially.

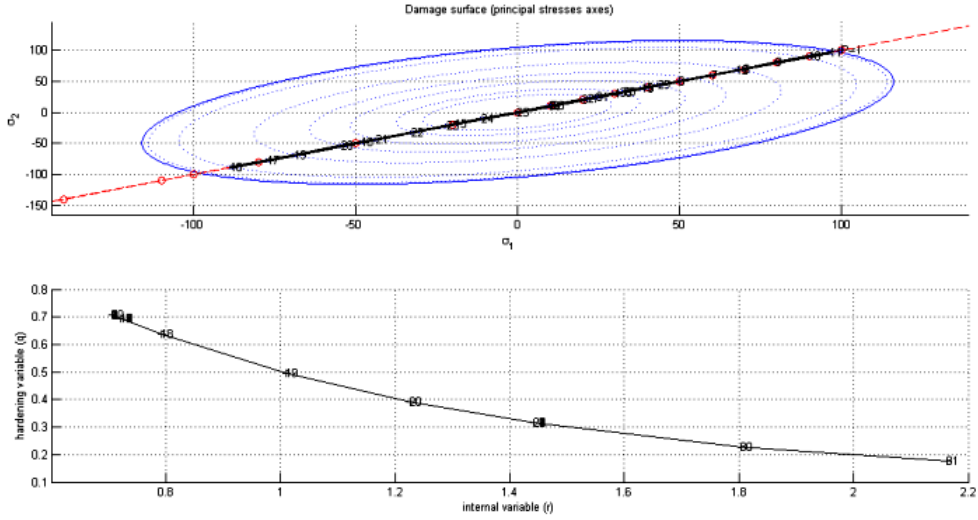


Figure 10: Exponential Softening with $q_\infty < r_0$

From Figure 10, we can see the exponential softening. Here the curve behaves same as that of the hardening and in order to get exponential softening, the value of q_∞ should always be lesser than r_0 . The value of q_∞ was varied with respect r_0 and the implementation was done and analysed, therefore it behaved as expected. If we increase the value of q_∞ to a very large value than r_0 , there will be an elastic model and the minimum value of q_∞ is zero. At this point the model completely shrunk up to a point domain.

3 Rate dependent Model (Viscous case)

The modified MATLAB codes for viscous damage model has been given in the *Appendices* for reference. For the analysis of the viscous case we have kept the following parameters: poisson's ratio, $\nu = 0.3$ and linear hardening/softening parameter, $H = 0.5$. The assessment of correctness of the implementation is done by varying the values of (η) , for different strain rates $(\dot{\epsilon})$ and for different α values. The variation of the above parameters is analysed with the help of stress-strain curves as shown in below figures.

3.1 Different viscosity parameters, η :

Viscosity parameter (η) is the important parameter when we model the viscodamage model. The ratio of $\frac{1}{\eta}$ should always be greater than or equal to zero. When we have $\eta = 0$, we obtain the inviscid damage model. But increasing the value of η increases the stress irrespective of variation in strain. The below stress-strain curves in figures below show us the variation of η for different values with constant linear hardening parameter.

From Figure 11 shown below we can see that as and when the η parameter is increased, the damage surface increases as well as the apparent stress also increases and it goes beyond the elastic regime for the yield stress = $100N/m^2$ and for the following loading paths,

$$\begin{aligned} \Delta\bar{\sigma}_1^{(1)} &= 300; & \Delta\bar{\sigma}_2^{(1)} &= 400 \\ \Delta\bar{\sigma}_1^{(2)} &= 500; & \Delta\bar{\sigma}_2^{(2)} &= 600 \\ \Delta\bar{\sigma}_1^{(3)} &= 700; & \Delta\bar{\sigma}_2^{(3)} &= 800 \end{aligned}$$

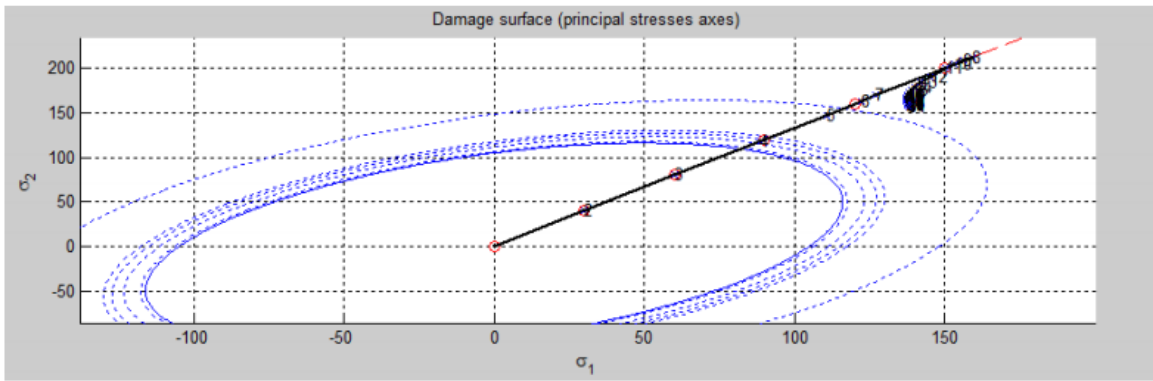


Figure 11: Variation of stress space for viscous paramter $\eta = 1$

The main criterion to check for the correctness of the implementation is by taking the values of $\eta = 0$ and $\alpha = 1$. When this implementation is done we retain the rate independent model (Inviscid damage model). For constant time steps, i.e for TimeTotal=10 and $\alpha = 0.5$, the values of η was varied in order to check for the changes in the stress-strain curve and following points were drawn.

For $\eta = 0.5$, the stress strain curve in Figure 12 gives idea that with the increase in the value of the stress, and if we further increase $\eta = 1$, the value of stress is also increased and so on for $\eta = 2$ and $\eta = 5$ which can be seen in below Figures 13,14,15.

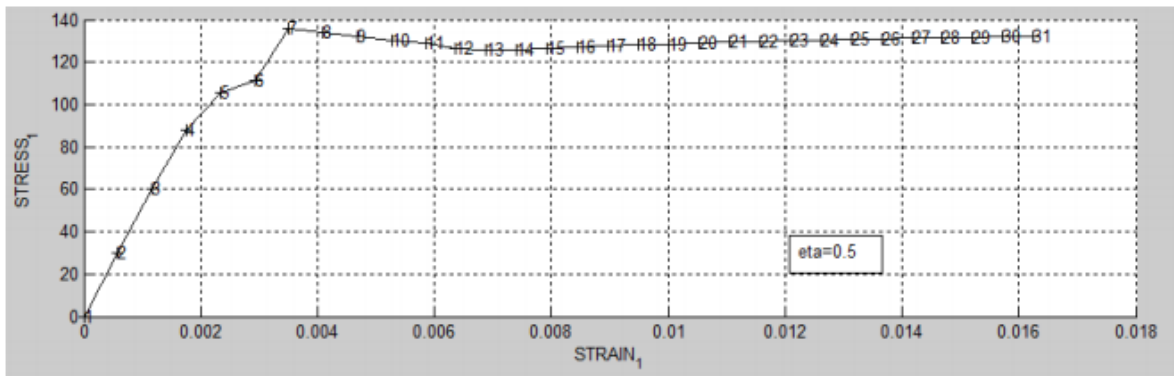


Figure 12: Variation of stress-strain curve for viscous paramter $\eta = 0.5$

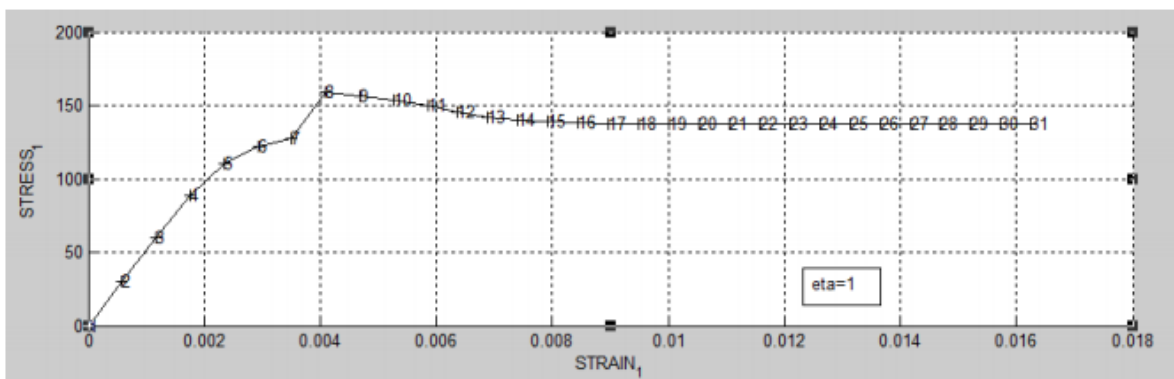


Figure 13: Variation of stress-strain curve for viscous paramter $\eta = 1$

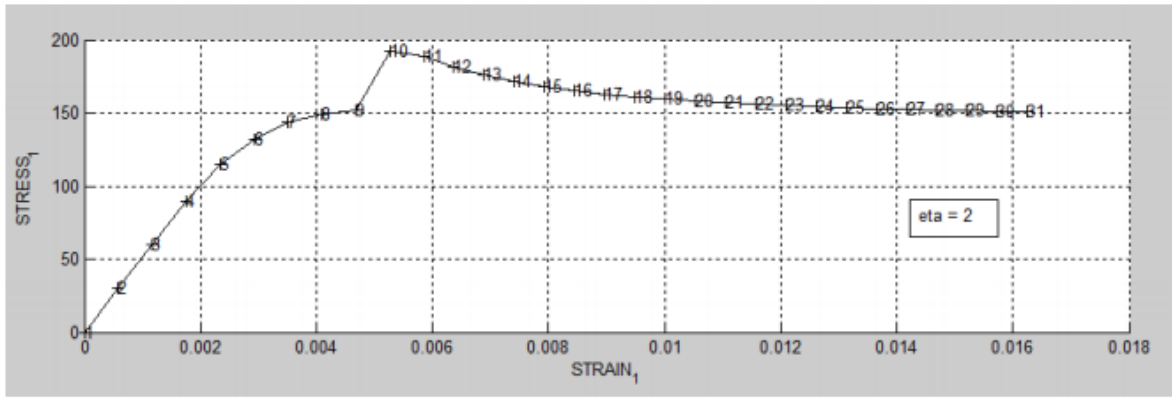


Figure 14: Variation of stress-strain curve for viscous parameter $\eta = 2$

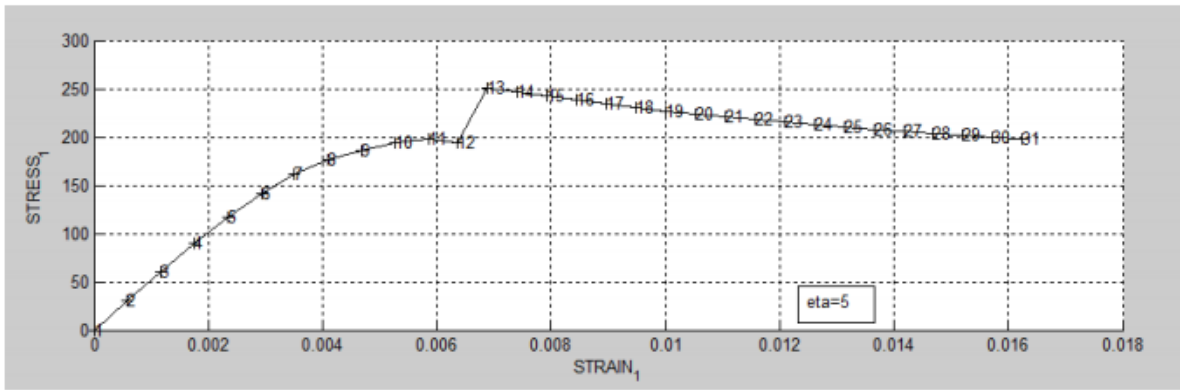


Figure 15: Variation of stress-strain curve for viscous parameter $\eta = 5$

The main reason for this behaviour is that as and when the η parameter was increased, the damage surface is increased and the apparent stress which is in picture also increases in a manner such that, it exceeds beyond the elastic regime and as it passes through the elastic regime and near the damage surface the value of stress in the material under study increases for variation in the value of η . The damage surface just expands for changes in the value of η and the apparent stress in the viscous case tends to cross the elastic regime, which was the expected behaviour for different viscosity parameter η and hence our implementation works as expected.

3.2 Different strain rate, $(\dot{\epsilon})$:

The main idea behind the variation in strain rate, $(\dot{\epsilon})$ is as when the value of $(\dot{\epsilon})$ was increased, that is we increase the value of 'Time Total' in *mainointeractive.m* file in order to study the variation of $(\dot{\epsilon})$. This parameter behaves same as that of variation or different values in the viscosity parameter. That is the stress increases for change in the value of $(\dot{\epsilon})$.

From Figure 16 below, we can see that as the TimeTotal was increased to 100 for constant $\eta = 1.5$, the stress space is expanded and because of this variation the points of apparent stress does not cross the damage surface and will be well posed in the elastic regime. Here the increase in $(\dot{\epsilon})$ which tends to catch all the points of apparent stress within the elastic regime and these points never expels out, but there is slight increase in the stress-strain curve which is explained below.

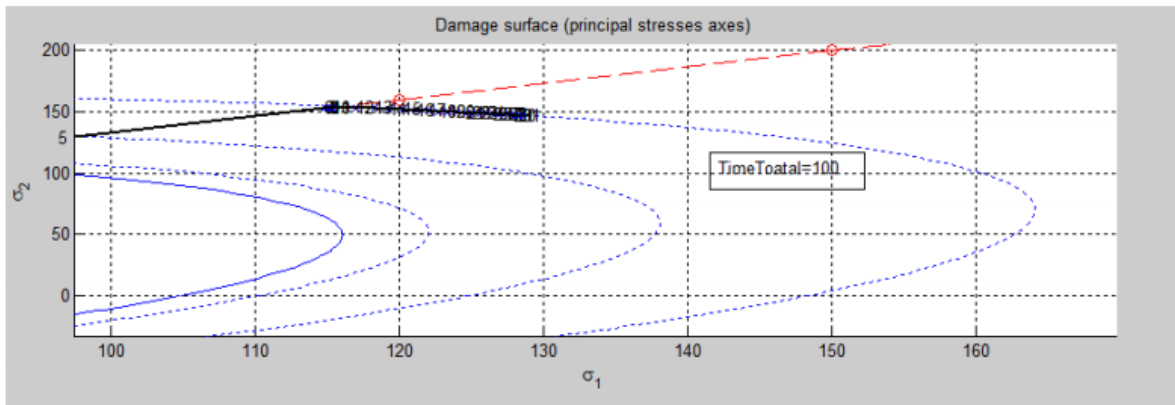


Figure 16: Variation of stress space for viscous case for TimeToal=100

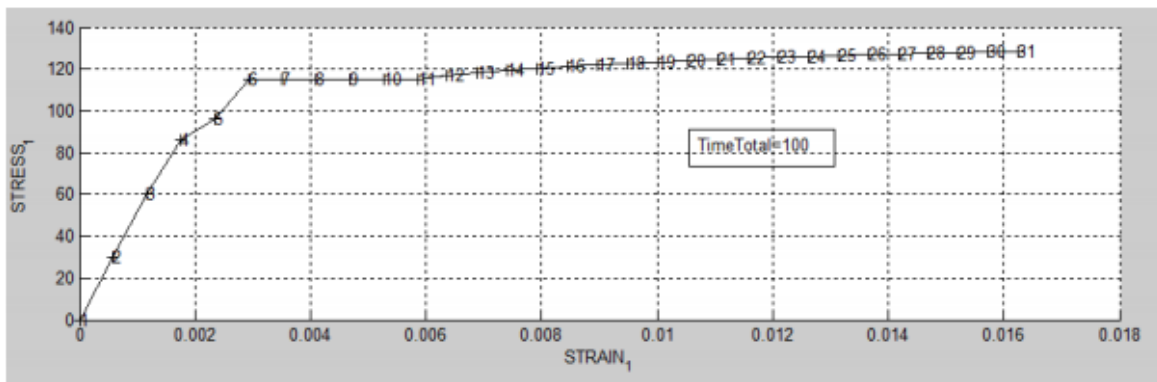


Figure 17: Variation of stress-strain curve for viscous case for TimeToal=100

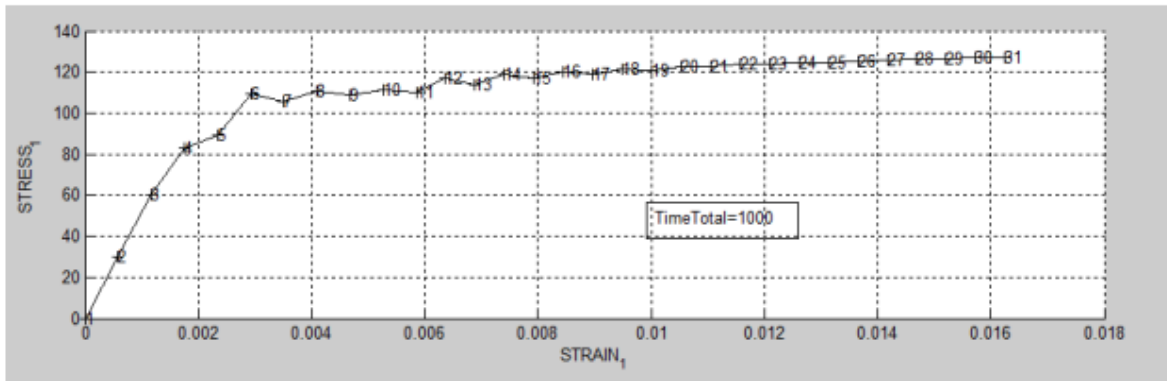


Figure 18: Variation of stress-strain curve for viscous case for TimeToal=1000

The stress variation is similar as that of change in η parameter, but as explained above, the damage surface increases with increase in $(\dot{\epsilon})$ because of this the stress also increases as it passes the yield stress that has been considered and initiates the tensile loading for the load paths applied same as that of different viscosity parameters, η .

The Figure 17 and 18 depicts the minor increase in the value of stress. This was the behaviour that was expected when we change the value of $(\dot{\epsilon})$. And hence our implementation is working fine for different range of values of $(\dot{\epsilon})$.

3.3 Different α values :

In order to check for the correctness of the α ,time-integration method we test for the following cases of $\alpha = 0, 1/4, 1/2, 3/4$ and 1. We are assessing how the stress-strain curves varies for different values of α and we have to comment on the stability criterion for different values of α by keeping the η and $(\dot{\epsilon})$ as constant values. Hence assessed the value of $\eta = 1.5$ and TimeTotal=10. The variation of α is explained with the help of stress-strain curves.

The Figure 19, 20 and 21 below depicts the variation of α along the stress-strain curve. For $\alpha = 0$ the variation of curve in Figure 19 is similar as when $\alpha = 0.5$ and 1. The main reason for this because within the range of $\alpha = 0, 0.25, 0.5, 0.75$ and 1, the 'alpha method' is first order accurate and there is no oscillation in the stress-strain curve for all the cases of α . Hence we can conclude that the Viscodamage model behaves perfectly as expected from the analytical or theoretical point of view, hence therefore our implementation is behaving as expected.

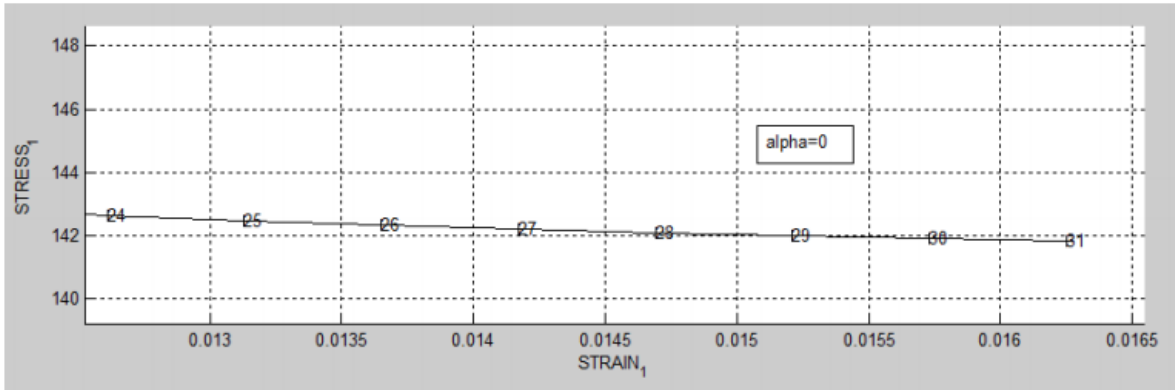


Figure 19: Variation of stress-strain curve for viscous case for $\alpha = 0$ (zoomed view)

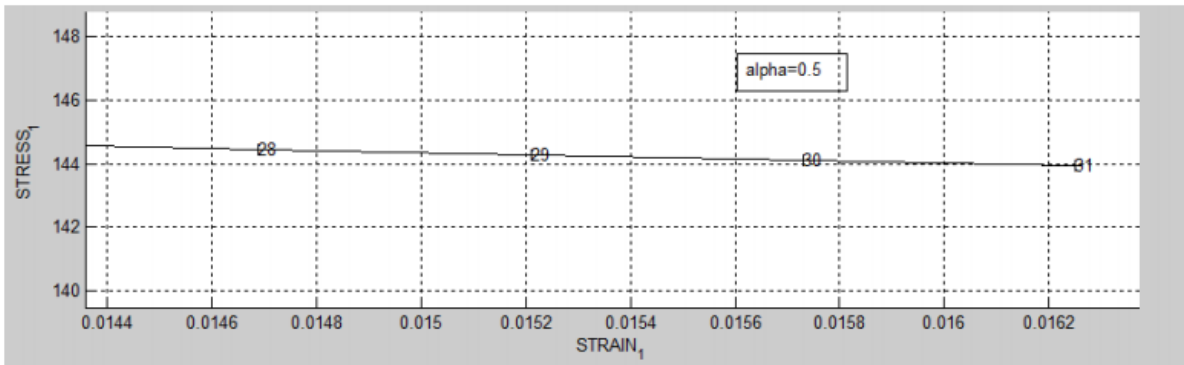


Figure 20: Variation of stress-strain curve for viscous case for $\alpha = 0.5$ (zoomed view)

α is the important parameter in deciding the time integration method. In our case it is behaving as first order accurate that is, it is behaving in the stable manner for α values ranging from 0 to 1 and it complete stability for $\alpha = 0.5$. In both the cases, that is in inviscid and viscid the damage parameter (d) increases and remains constant but never decreases, which can be seen from the Figure 22 below.

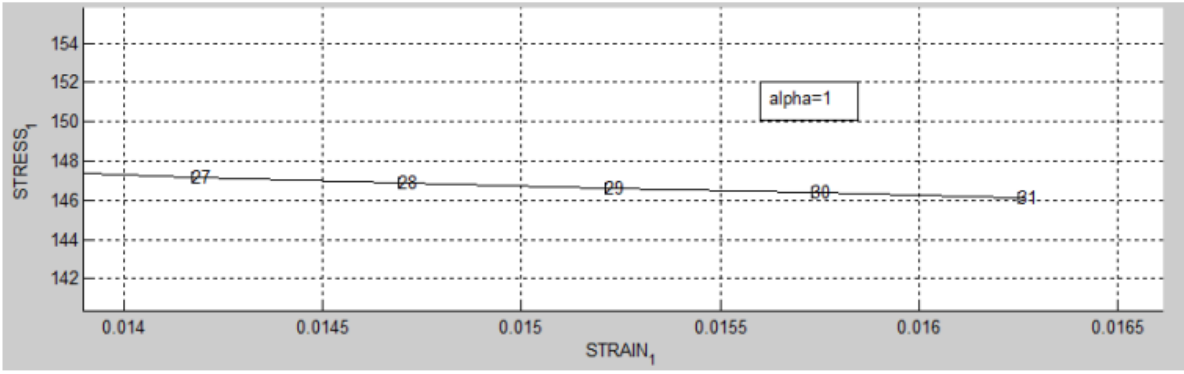


Figure 21: Variation of stress-strain curve for viscous case for $\alpha = 1$ (zoomed view)

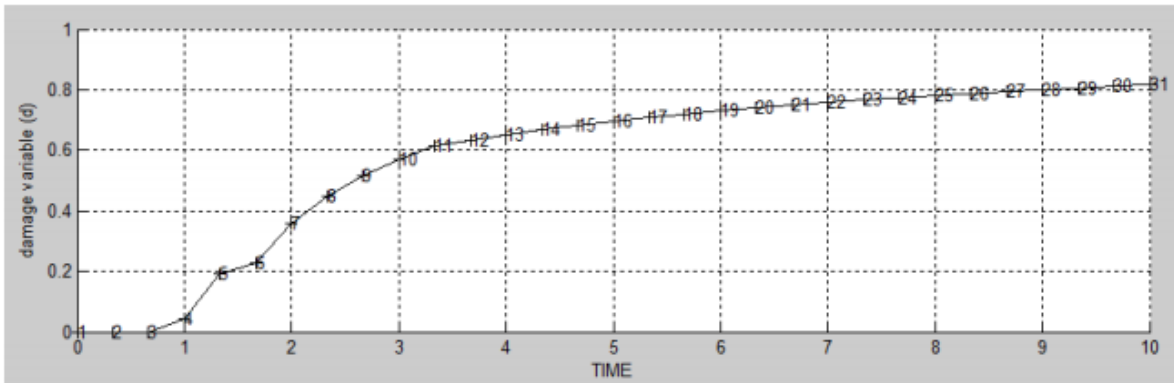


Figure 22: Variation of damage parameter (d) for inviscid and viscous case with respect to time.

3.4 The effects of the α values, on the evolution along time of the C_{11} component of the tangent and algorithmic constitutive operators :

The algorithmic tangent operator is compared with the analytical operator, for varying alpha values. It has been observed that the value of the C_{11} components of the elastic constitutive tensor C are exactly similar at the value of $\alpha = 0$ as observed in figure 23. It has also been observed that the analytical and algorithmic operators match when the Δt value is set as 0. Therefore, the algorithmic tangent operator and analytical operator slightly behaves the same with some slight variation which can be seen in the plots below. And the value of considered $\Delta T = 0.01$.

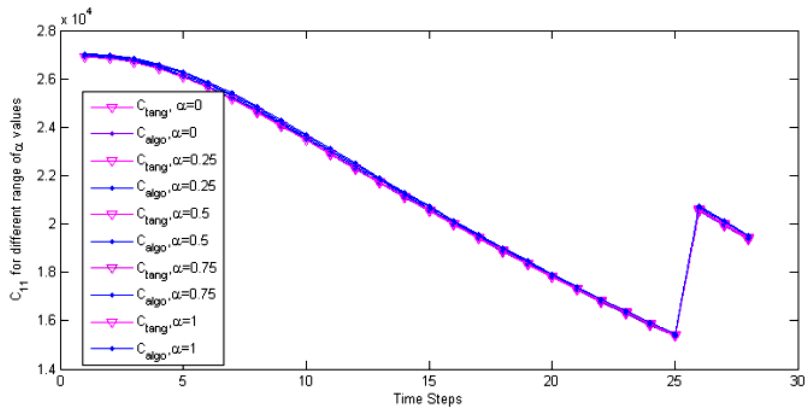


Figure 23: Plot of C_{11} component with different α values along with evolution of time.

Appendices

Appendix 1: MATLAB code modified routines for rate independent models

Tension and Non-Symmetric models:

In *Modelos_de_dano1*

```
if (MDtype==1)      %* Symmetric
rtrial= sqrt(eps_n1*ce*eps_n1') ;          % Computation of strain norm

elseif (MDtype==2) %* Only tension
sigma_ca= ce*eps_n1';                      % Computes the value of Sigma
sigma_plus= (abs(sigma_ca)+sigma_ca)/2;    % Computes the value of Positive Counterpart
                                           % of the stress tensor
rtrial= sqrt(eps_n1*sigma_plus);          % Computation of strain norm

elseif (MDtype==3) %*Non-symmetric
sigma_ca= ce*eps_n1';                      % Computes the value of sigma
sigma_plus= (abs(sigma_ca)+sigma_ca)/2;    % Computes the value of Positive Counterpart
                                           % the stress tensor
theta=sum(sigma_plus)/sum(abs(sigma_plus)); % Computes the value of theta
rtrial = (theta+(1-theta)/n)*sqrt(eps_n1*ce*eps_n1'); % Computes the value of Strain norm
end
```

In *dibujar_criterio_dano1*

```
elseif MDtype==2          % For TENSION ONLY Model
tetha=[0:0.01:2*pi];
D=size(tetha);           %* Range
m1=cos(tetha);
m2=sin(tetha);
Contador=D(1,2);
radio = zeros(1,Contador) ;
s1 = zeros(1,Contador) ;
s2 = zeros(1,Contador) ;
for i=1:Contador
sub=( [m1(i) m2(i) 0 nu*(m1(i)+m2(i))] );
radio(i)= q/sqrt(((sub+abs(sub))/2)*ce_inv*[m1(i) m2(i) 0 ...
nu*(m1(i)+m2(i))]');
s1(i)=radio(i)*m1(i);
s2(i)=radio(i)*m2(i);
end
hplot =plot(s1,s2,tipo_linea);
axis([-1000 500 -1000 500]);
```

```

elseif MDtype==3                                % For NON SYMMETRIC Model
tetha=[0:0.01:2*pi];
D=size(tetha);                                  %* Range
m1=cos(tetha);
m2=sin(tetha);
Contador=D(1,2);
radio = zeros(1,Contador) ;
s1     = zeros(1,Contador) ;
s2     = zeros(1,Contador) ;
for i=1:Contador
    numerator=(m1(i)+m2(i)+abs(m1(i))+abs(m2(i)))/2;
    denominator=abs(m1(i))+abs(m2(i));
    ang=numerator/denominator;
    radio(i)=((1)/(ang+((1-ang)/(n))))* (q/sqrt([m1(i) m2(i) 0 nu*(m1(i)+m2(i))]*ce_inv*
[m1(i) m2(i) 0 ...
nu*(m1(i)+m2(i))]'));
    s1(i)=radio(i)*m1(i);
    s2(i)=radio(i)*m2(i);
end
hplot =plot(s1,s2,tipo_linea);

```

Appendix 2: MATLAB code modified routines for Linear and exponential hardening/ softening

In *rmap_dano1*

```

if hard_type == 0
    if (q_inf-r0)*H<0
        q_inf=zero_q ;                                % Linear Hardening/softening
    end
    r1=r0+(q_inf-r0)/H;
    if r_n1<r1
        q_n1=r0+H*(r_n1-r_n);
    else
        q_n1=q_inf;
    end
else
    q_n1=(q_inf)-((q_inf-r0))*exp( A *( 1 - (r_n1/r0))); %Exponential Hardening/softening
end
if(q_n1<zero_q)                                     %The value of q_inf and A has been
    q_n1=zero_q;                                     %initiated in main_nointeractive.m as
end                                                    % Eprop(9) and Eprop(10).

```

Appendix 3: MATLAB code modified routines for rate dependent models

In *damage_main*

```

if i>1
    eps_n = strain(i-1,:);
else
    eps_n = 0;
end

```

In *rmap_dano1*

```
fload=1;
if Eprop(6)           % Viscous case
    r_n1=((eta-delta_t*(1-alpha))*r_n+(delta_t*rtrial))/(eta+alpha*delta_t)
else
    r_n1= rtrial;    % Inviscid case
end
```

In *Modelos_de_dano1*

```
viscpr = Eprop(6);
if Eprop(6)           % Viscous
    alpha=Eprop(8);
    if (MDtype==1)   % Symmetric
        Tn=sqrt(eps_n*ce*eps_n');
        Tn1=sqrt(eps_n1*ce*eps_n1');
        T_alpha=(1-alpha)*Tn+alpha*Tn1;
        rtrial=T_alpha;
    end
end
```

Appendix 4: MATLAB code modified routines for C_{11} component

In *rmap_dano1*

```
alpha=Eprop(8); eta=Eprop(7);
C_t=(1.d0-dano_n1)*ce;           %Computes the value of Tangent constitutive operators
Tn1=sqrt(eps_n1*ce*eps_n1');
sigma=ce*eps_n1';
C_Alg=C_t+(alpha*delta_t)/(eta+alpha*delta_t)*... % Computes the value of Algorithmic
    aux_var(3)/Tn1*(sigma*sigma');           % constitutive operator.
```