COMPUTATIONAL SOLID MECHANICS
ASSIGNMENT-1

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INTRODUCTION:

A continuum damage model has been analysed in this report for two cases: Rate-independent model(Tensile damage and non-symmetric damage case) and Rate-dependent model(only for Symmetric case). The damage model is studied under various Karush/Kuhn/tucker loading and unloading conditions. The model is studied for strain driven case.

So the constitutive equation followed in the code is:

$$g(\epsilon, r) = \tau_\epsilon - r$$

where,

g is the damage function,
τ_ϵ is the norm of the strain,
r is the internal variable.

1. Rate Independent Model (Inviscid case)

a) The implementation of the MATLAB CODE has been included in the appendices for i)tensile damage model and ii) non-symmetric damage model.

b)The code has been implemented for all the three cases for linear and exponential hardening/softening ($H > 0$ and $H < 0$)

For a particular case the plot has been shown below:

Tensile Damage Model

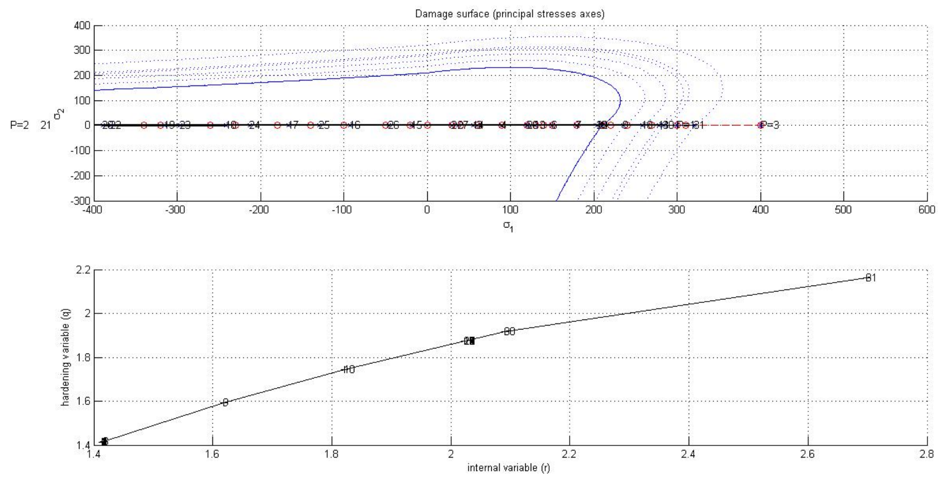

Figure 1: Diagram showing exponential plot for internal variable (r) vs. hardening variable (q) for hardening case

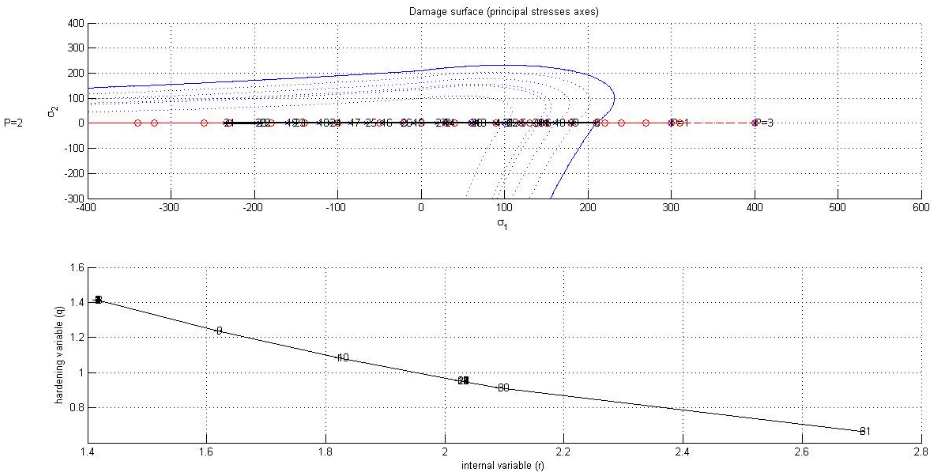

Figure 2: Diagram showing exponential plot for internal variable (r) vs. hardening variable (q)for softening case

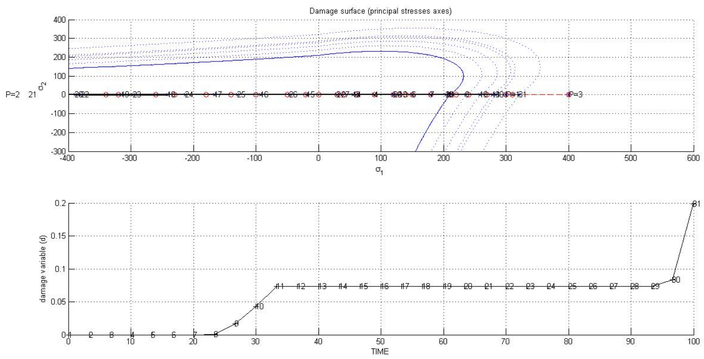

Figure 3: Diagram showing damage variable (d) vs time varying exponentially for hardening case

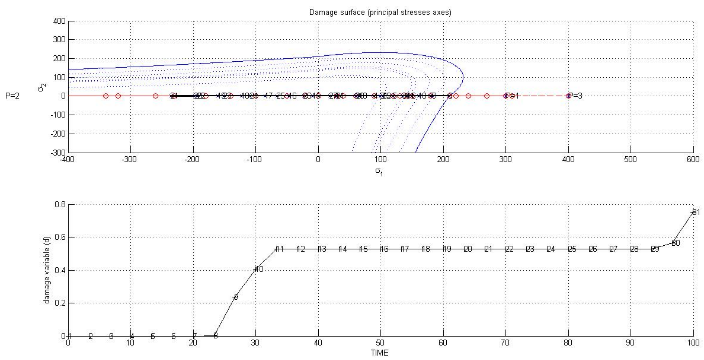

Figure 4: Diagram showing damage variable (d) vs time varying exponentially for softening case

Tension-Compression Damage Model

Case1:

$\Delta\bar{\sigma_1} = 300$; $\Delta\bar{\sigma_1} = 0$(uniaxial tensile loading)
$\Delta\bar{\sigma_1} = -500$; $\Delta\bar{\sigma_1} = 0$(uniaxial tensile unloading/compressive loading)
$\Delta\bar{\sigma_1} = 400$; $\Delta\bar{\sigma_1} = 0$(uniaxial compressive unloading/ tensile loading)

When uniaxial tensile loading is applied at first, the model remains within the elastic region i.e upto its elastic limit. When the load path exceeds the yield stress,deformation takes place. Since,this model is studied for softening case,it gets shrinked and a new damage surface is formed. Now when there is compressive unloading, it is observed that even though the model crosses the yield stress limit it doesn't get damaged and behaves elastically. Now upon further tensile loading, it is observed that the damage surface shrinks until it reaches a new yield stress value.
Below are the figures showing stress space, stress-strain curve and internal variable(r)-time plot.

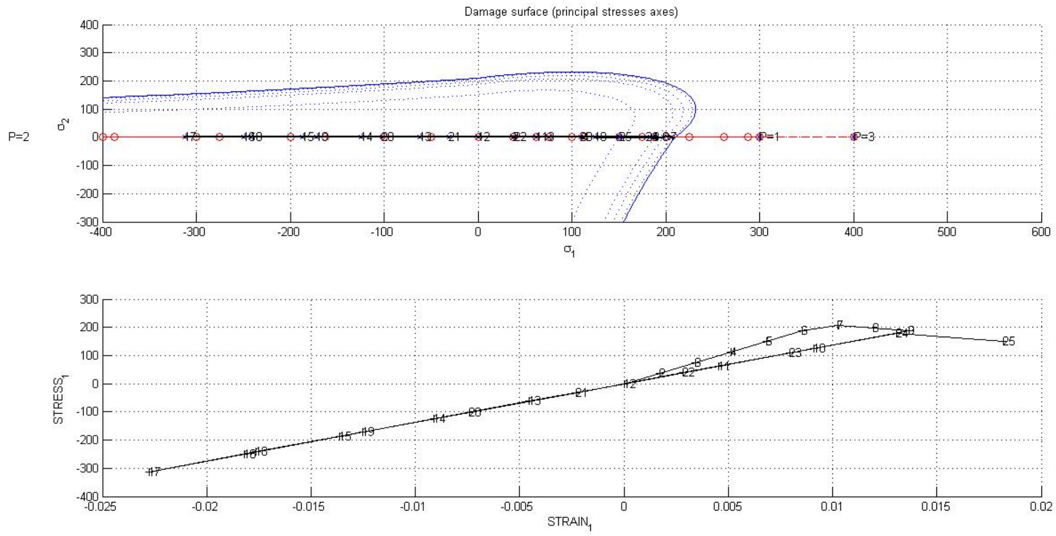

Figure 5: Diagram showing stress space and stress-strain curve for tension-compression damage model

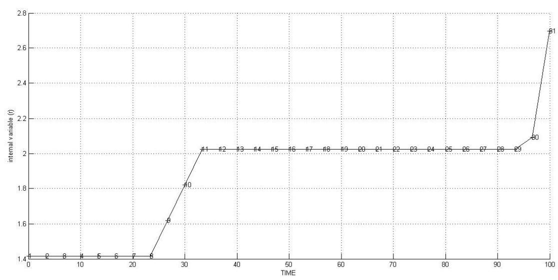

Figure 6: Diagram showing internal variable vs time

From the above internal variable (r) vs. time plot, we can study the model precisely.We observe that when the model is within elastic domain, there is no evolution of r. As soon as the model exceeds yield stress, it starts shrinking(due to softening)and there is evolution in r. Upon compressive unloading, the model is within the new damage surface, hence r remains constant again.When again tensile loading starts, there is evolution of r which means that the point has exceeded the new damage surface. It is a case of **pure loading**$(\dot{r} > 0, \dot{\tau_\epsilon} > 0)$

Case2:

$\Delta\bar{\sigma_1} = 300 \quad ; \quad \Delta\bar{\sigma_1} = 0$(uniaxial tensile loading)
$\Delta\bar{\sigma_1} = -200 \quad ; \quad \Delta\bar{\sigma_1} = -200$(biaxial tensile unloading/compressive loading)
$\Delta\bar{\sigma_1} = 250 \quad ; \quad \Delta\bar{\sigma_1} = 250$(biaxial compressive unloading/ tensile loading)

The case is similar to the first one. Except when it is inelastically loading, the point remains within the new damage surface, which implies there is no evolution of r. It is a case of **unloading**$(\dot{\mathbf{r}} = \mathbf{0}, \dot{\tau_\epsilon} = \mathbf{0})$. Hence there is no further deformation.

It is shown in the figure below.

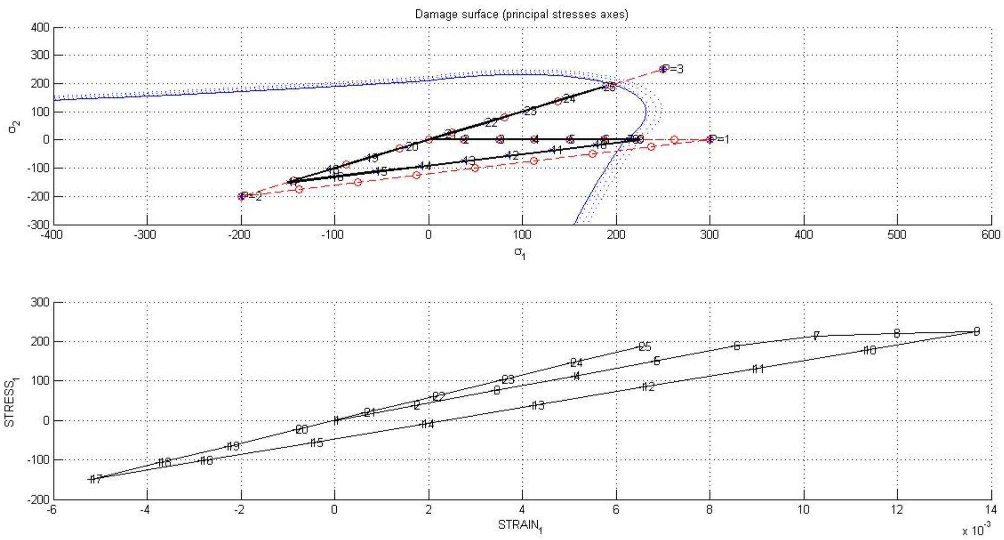

Figure 7: Diagram showing stress space and stress-strain curve for tension-compression damage model

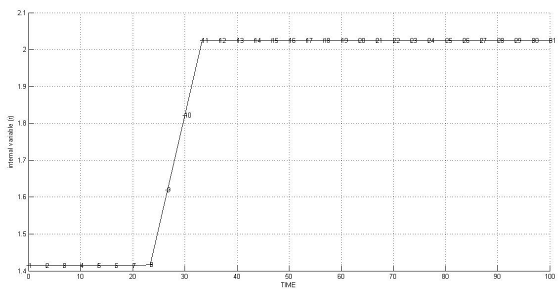

Figure 8: Diagram showing internal variable vs time

CASE 3:

$\Delta\bar{\sigma_1} = 250 \quad ; \quad \Delta\bar{\sigma_1} = 250$(uniaxial tensile loading)
$\Delta\bar{\sigma_1} = -250 \quad ; \quad \Delta\bar{\sigma_1} = -250$(uniaxial tensile unloading/compressive loading)
$\Delta\bar{\sigma_1} = 100 \quad ; \quad \Delta\bar{\sigma_1} = 100$(uniaxial compressive unloading/ tensile loading)

When uniaxial tensile loading is applied at first, the model remains within the elastic region as expected.So r remains constant. When the loading exceeds the yield stress,it starts deforming. Since,this model is studied for hardening case,it gets expanded which can be seen in the figure below.Now, the model has a new damage surface.There is evolution of r. Upon unloading, the point remains within the new surface and so r remains constant as expected.Now, when it is inelastically loading, it is observed that the point approximately stays on the damage surface. There is no evolution of r.It is a case of **neutralloading**$(\dot{\mathbf{r}} = \mathbf{0}, \dot{\tau_\epsilon} = \mathbf{0})$. It is shown in the figure below.

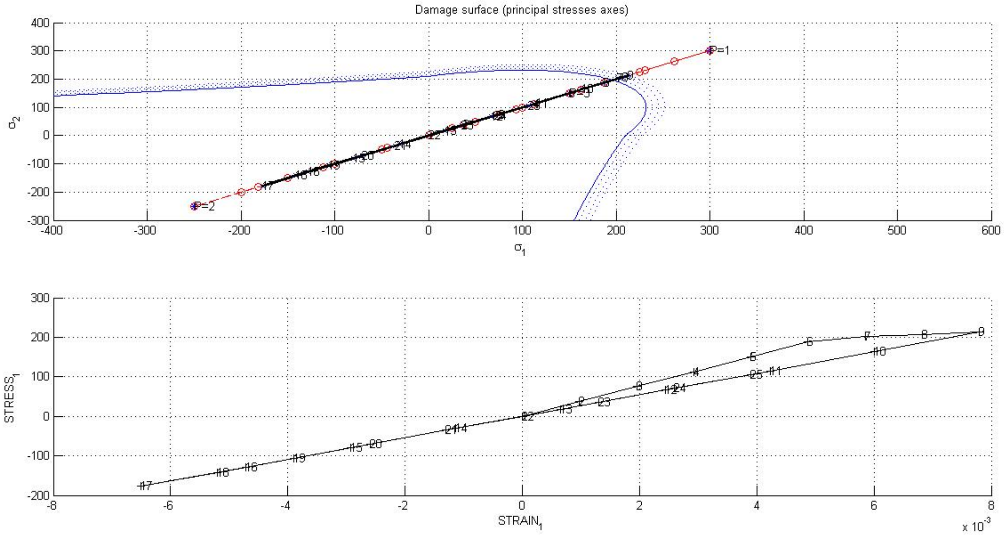

Figure 9: Diagram showing stress space and stress-strain curve for tension-compression damage model

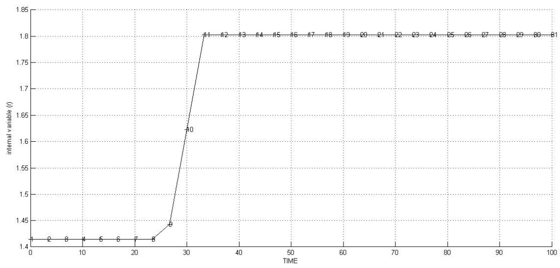

Figure 10: Diagram showing internal variable vs time

<u>Other case studies</u>:

For pure tensile:

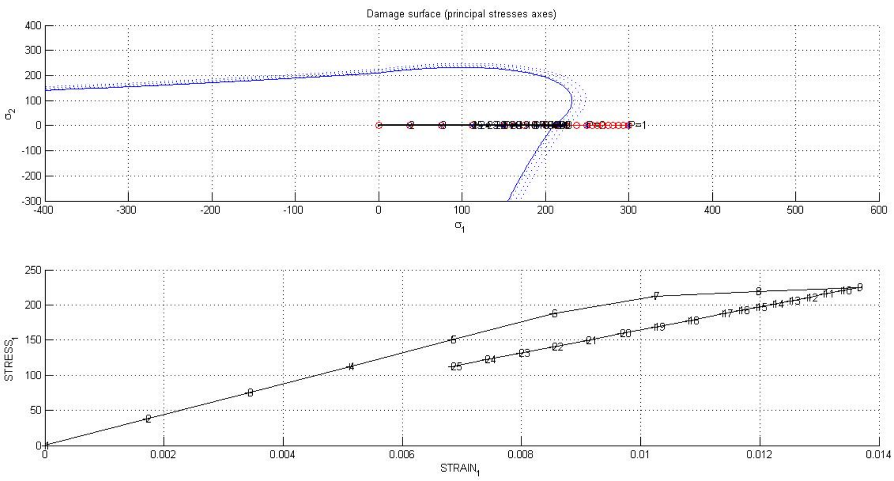

Figure 11: Diagram showing stress space and stress-strain for pure tensile case

For pure compression:

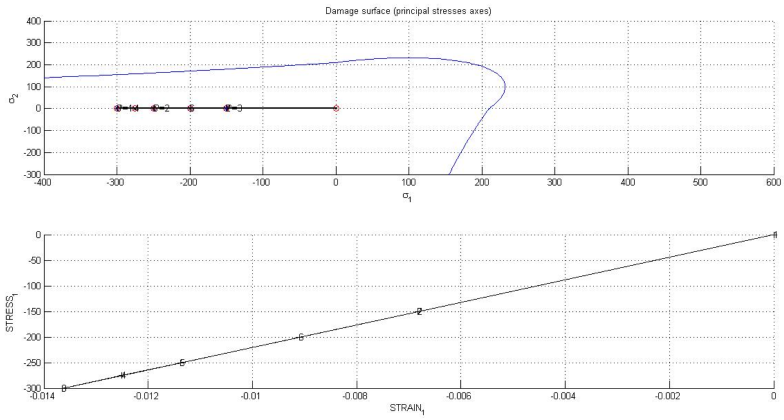

Figure 12: Diagram Diagram showing stress space and stress-strain for pure compression case

From above figure we can observe that, upon compressive loading or unloading, the model is always within the elastic domain.

<u>NON-SYMMETRIC DAMAGE MODEL</u>:

In non-symmetric damage model case we observe that it follows the same trend as tension damage model except in the compression loading cases, the domain enlarges by a term $\frac{1}{n}$ where n is the ratio of compression and tension.

The following three cases have been studied.
CASE1:
$\Delta\bar{\sigma_1} = 300$; $\Delta\bar{\sigma_1} = 0$(uniaxial tensile loading)
$\Delta\bar{\sigma_1} = -500$; $\Delta\bar{\sigma_1} = 0$(uniaxial tensile unloading/compressive loading)
$\Delta\bar{\sigma_1} = 400$; $\Delta\bar{\sigma_1} = 0$(uniaxial compressive unloading/ tensile loading)

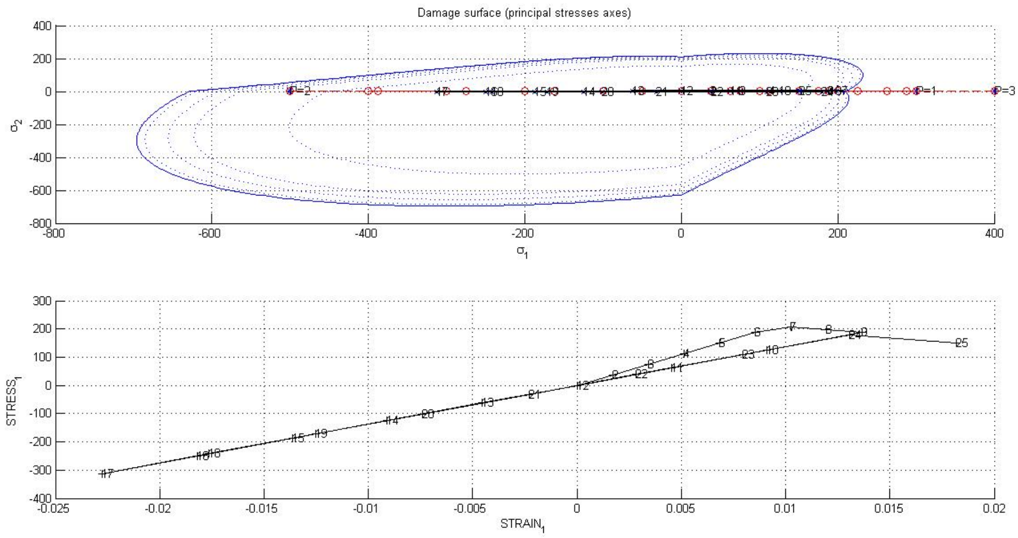

Figure 13: Diagram showing stress space and stress-strain curve for non-symmetric damage model

the stress-strain curve is similar to the tension damage model.We observe that when the model is within elastic domain, there is no evolution of r. As soon as the model exceeds yield stress, it starts shrinking(due to softening)and there is evolution in r. Upon compressive unloading, the model is within the new damage surface, hence r remains constant again.When again tensile loading starts, there is evolution of r which means that the point has exceeded the new damage surface. It is a case of **pure loading**

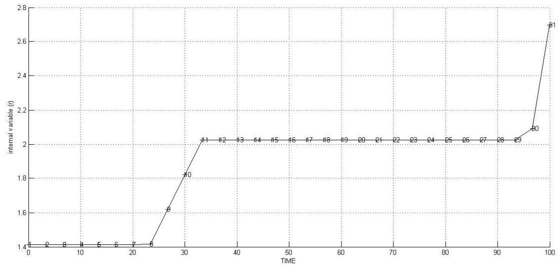

Figure 14: Diagram showing internal variable vs time

CASE 2:

$\Delta\bar{\sigma_1} = 300$; $\Delta\bar{\sigma_1} = 0$(uniaxial tensile loading)
$\Delta\bar{\sigma_1} = -200$; $\Delta\bar{\sigma_1} = -200$(biaxial tensile unloading/compressive loading)
$\Delta\bar{\sigma_1} = 250$; $\Delta\bar{\sigma_1} = 250$(biaxial compressive unloading/ tensile loading)

A case of unloading in the inelastic regime

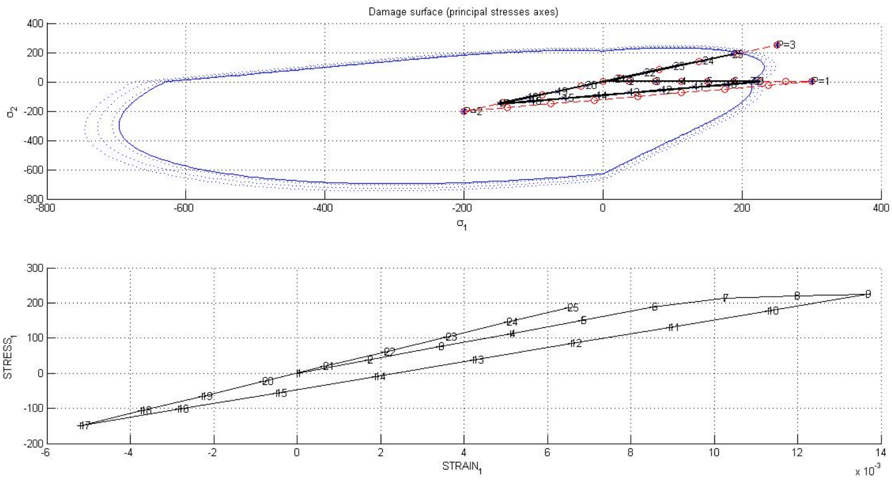

Figure 15: Diagram showing stress space and stress-strain curve for non-symmetric damage model

CASE 3:

$\Delta\bar{\sigma_1} = 300$; $\Delta\bar{\sigma_1} = 0$(uniaxial tensile loading)
$\Delta\bar{\sigma_1} = -200$; $\Delta\bar{\sigma_1} = -200$(biaxial tensile unloading/compressive loading)
$\Delta\bar{\sigma_1} = 250$; $\Delta\bar{\sigma_1} = 250$(biaxial compressive unloading/ tensile loading)

A case of unloading(neutral loading) in the inelastic regime

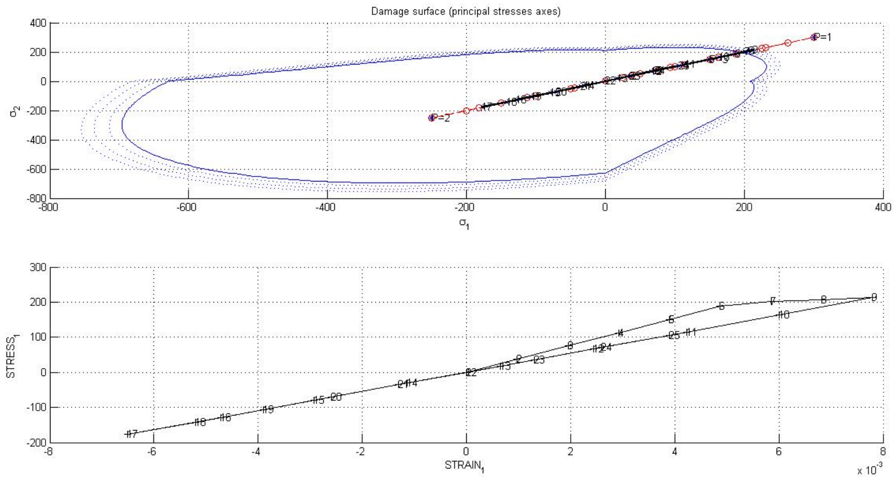

Figure 16: Diagram showing stress space and stress-strain curve for non-symmetric damage model

II. Rate dependent model(viscous case)

Now, the code has been implemented for viscous model and presented in the appendix.

Case is studied for Symmetric model considering hardening.

For, different parameters of alpha(α), eta(η) and at different strain rate conditions the stress-strain curve is studied and shown below.

i. For $\eta = 0, .5, 2, 3$ and 5 Pas^{-1}

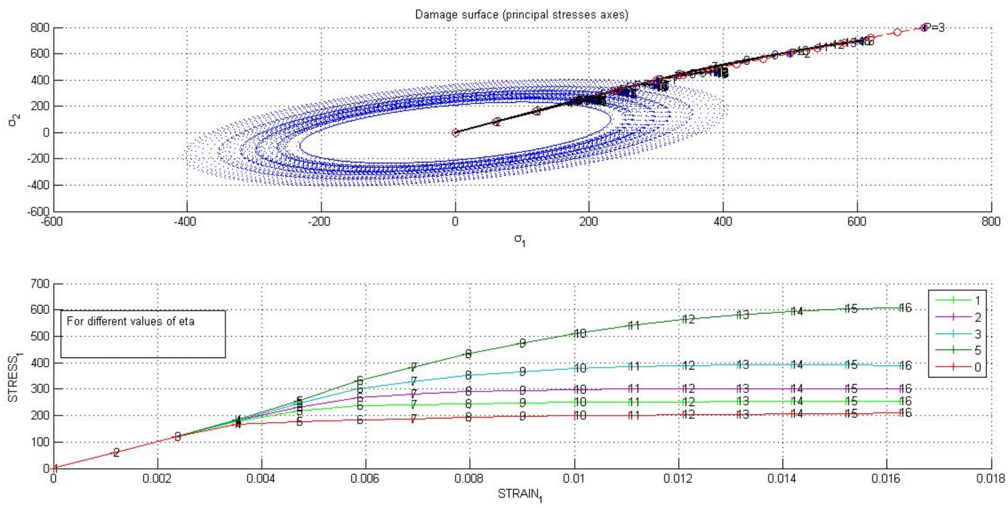

Figure 17: Diagram showing stress-strain curve for different η value

We observe that as η increases the model becomes more viscous.Hence the stress increases and the model behaves elastically until it reaches the ultimate stress. As η decreases the stress decreases and model behaves like inviscid $\eta = 0$.

ii. For different values of strain rate $(\dot{\epsilon})$ i.e For time steps 10,100,500

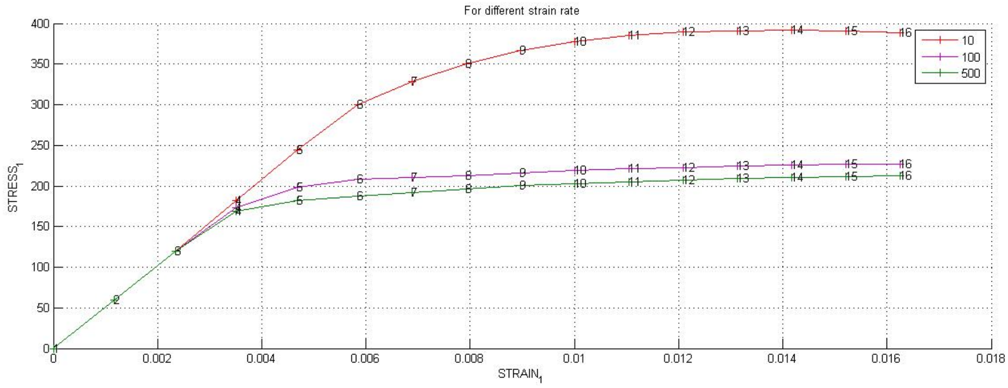

Figure 18: Diagram showing stress-strain curve for different time increment

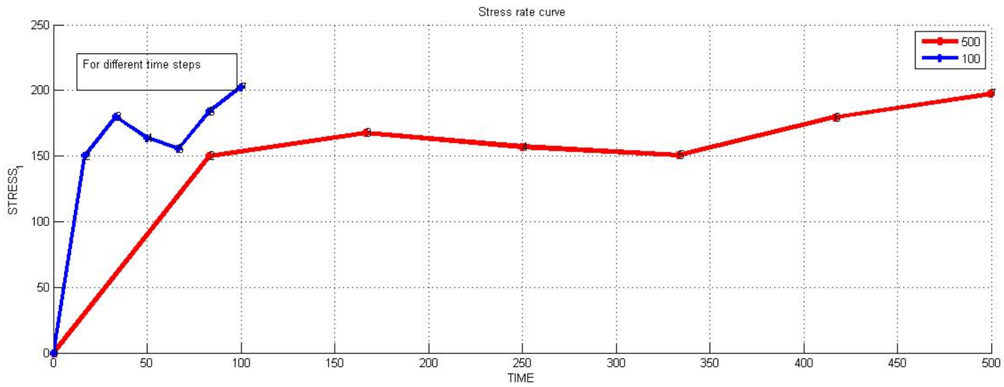

Figure 19: Diagram showing stress vs time plot

The strain rate effect has similar effects on the stress-strain curve like for the viscosity case. The yield stress decreases when time increment increases and

iii. For $\alpha = [1/4, 1/2, 3/4, 1]$

It is observed that when ($\alpha = 0.5$), the stress-strain curve gives better results and is second order accurate. when α is between [0,1] it is first order accurate. From the graph it is seen that, when alpha is [0,1/2] the plot is stable and gives better results than when $\alpha = [0, 0.5]$

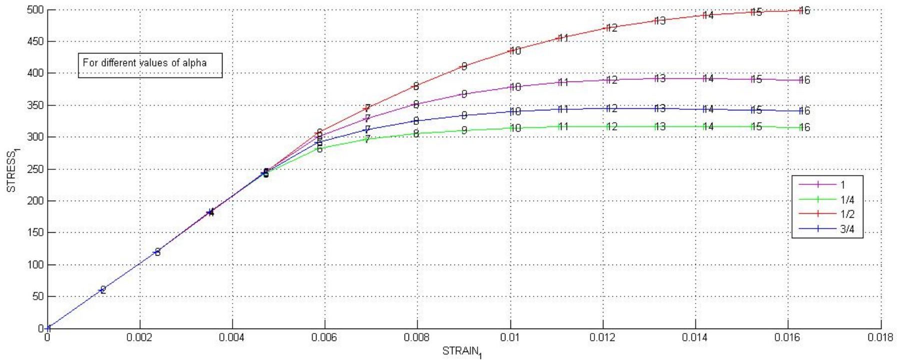

Figure 20: Diagram showing stress-strain curve for different values of α

Modification for Tensile- Compression and Non-symmetric Damage Model in Modelos-de-dano1

```
if (MDtype==1)      %* Symmetric
rtrial= sqrt(eps_n1*ce*eps_n1');

elseif (MDtype==2)  %* Only tension
stress=ce*eps_n1';
stress(stress<0)=0;
rtrial=sqrt(eps_n1*stress);

elseif (MDtype==3)  %*Non-symmetric
    stress=ce*eps_n1';
    stress_plus=stress;
    stress_plus(stress_plus<0)=0;
    num = sum((stress_plus));
    den = sum((abs(stress)));
    theta = num/den;
    rtrial = (theta + (1-theta)/n)* sqrt(eps_n1 * ce*eps_n1');

end
%*************************************************************************
return
```

Modifications in dibujar-criterio-dano1

```
elseif MDtype==2
    tetha=[0:0.01:2*pi];
    %*************************************************************************
    %* RADIUS
    D=size(tetha);                     %*  Range
    m1=cos(tetha);                     %*
    n1=m1;
    n1(n1<0)=0;
    m2=sin(tetha);                     %*
    n2=m2;
    n2(n2<0)=0;
    Contador=D(1,2);                   %*



    radio = zeros(1,Contador) ;
    s1    = zeros(1,Contador) ;
    s2    = zeros(1,Contador) ;

    for i=1:Contador
        radio(i)= q/sqrt([n1(i) n2(i) 0 nu*(n1(i)+n2(i))]*ce_inv*[m1(i) m2(i) 0 ...
            nu*(m1(i)+m2(i))]');

        s1(i)=radio(i)*m1(i);
        s2(i)=radio(i)*m2(i);

    end
    hplot =plot(s1,s2,tipo_linea);
     axis([-400 600 -300 400])
```

```
elseif MDtype==3                        % For NON SYMMETRIC Model
tetha=[0:0.01:2*pi];
    D=size(tetha);                      %*  Range
    m1=cos(tetha);
    m2=sin(tetha);
    Contador=D(1,2);
    radio = zeros(1,Contador) ;
    s1    = zeros(1,Contador) ;
    s2    = zeros(1,Contador) ;
    for i=1:Contador
        numerator=(m1(i)+m2(i)+abs(m1(i))+abs(m2(i)))/2;
        denominator=abs(m1(i))+abs(m2(i));
        ang=numerator/denominator;
        radio(i)=((1)/(ang+((1-ang)/(n))))* (q/sqrt([m1(i) m2(i) 0 nu*(m1(i)+m2(i))]*ce_inv*
        [m1(i) m2(i) 0 ...
        nu*(m1(i)+m2(i))]'));
        s1(i)=radio(i)*m1(i);
        s2(i)=radio(i)*m2(i);
        end
    hplot =plot(s1,s2,tipo_linea);
```

Modification for Exponential Hardening/Softening Law in rmap-dano1

```
else
        % *****************************************************
         Hardening/Softening exponential law'
        if H>0
            q_infi=r0*1.3;
        A = (H*r0)/(q_infi-r0);
        H_new= (A*(q_infi-r0)*exp(A*(1-rtrial/r0)))/r0;
        q_n1= q_n + H_new*delta_r;
        else
             q_infi=r0*0.5;
        A = (H*r0)/(q_infi-r0);
        H_new= (A*(q_infi-r0)*exp(A*(1-rtrial/r0)))/r0;
        q_n1= q_n + H_new*delta_r;
        end
```

Modification for Viscous symmetric only case in Modelos-de-dano1

```
viscpr = Eprop(6);
if Eprop(6)                             % Viscous
    alpha=Eprop(8);
    if (MDtype==1)                      % Symmetric
        Tn=sqrt(eps_n*ce*eps_n');
        Tn1=sqrt(eps_n1*ce*eps_n1');
        T_alpha=(1-alpha)*Tn+alpha*Tn1;
       rtrial=T_alpha;
    end
end
```

Modification in rmap-dano1 for viscous case

```
if(rtrial > r_n)
%*   Loading

fload=1;
delta_r=rtrial-r_n;
r_n1= (((eta-delta_t*(1-alpha))/(eta+alpha*delta_t))*r_n) + ....
    ((delta_t/(eta+alpha*delta_t))*rtrial);
if hard_type == 0
    %  Linear
    q_n1= q_n+ H*delta_r;
else
    % ******************************************************
    %'Hardening/Softening exponential law)
     q_infi=r0*1.3;
     A = (H*r0)/(q_infi-r0);
     H_new= (A*(q_infi-r0)*exp(A*(1-rtrial/r0)))/r0;
     q_n1= q_n + H_new*delta_r;
end
```