Computational Solid Mechanics

Assignment 1

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Master in Numerical Methods
for Engineering

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Rate independent models

1.

In this section the computations have been set in order to see the differences in the stress/strain curve between the linear and the exponential hardening/softening law for the non-symetric and tension only model. Each model shows a hardening example and a softening example, with one of them following the linear law and the other the exponential. The loading/unloading path is uniaxial for all the examples.

a) Non-symmetric model. The cases chosen in order to illustrate the behaviour of this model are: linear hardening and exponential softening.

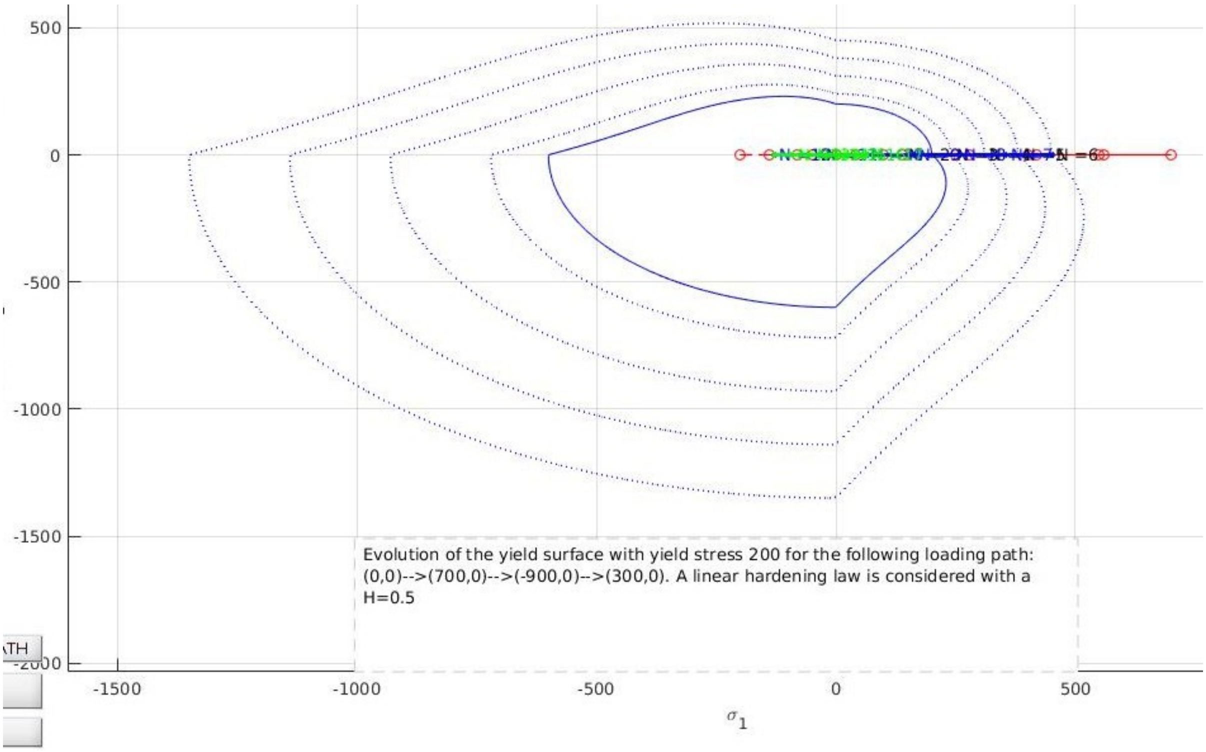

Figure 1. Evolution of the yield surface for the non-symmetric model. The loading path is uniaxial.

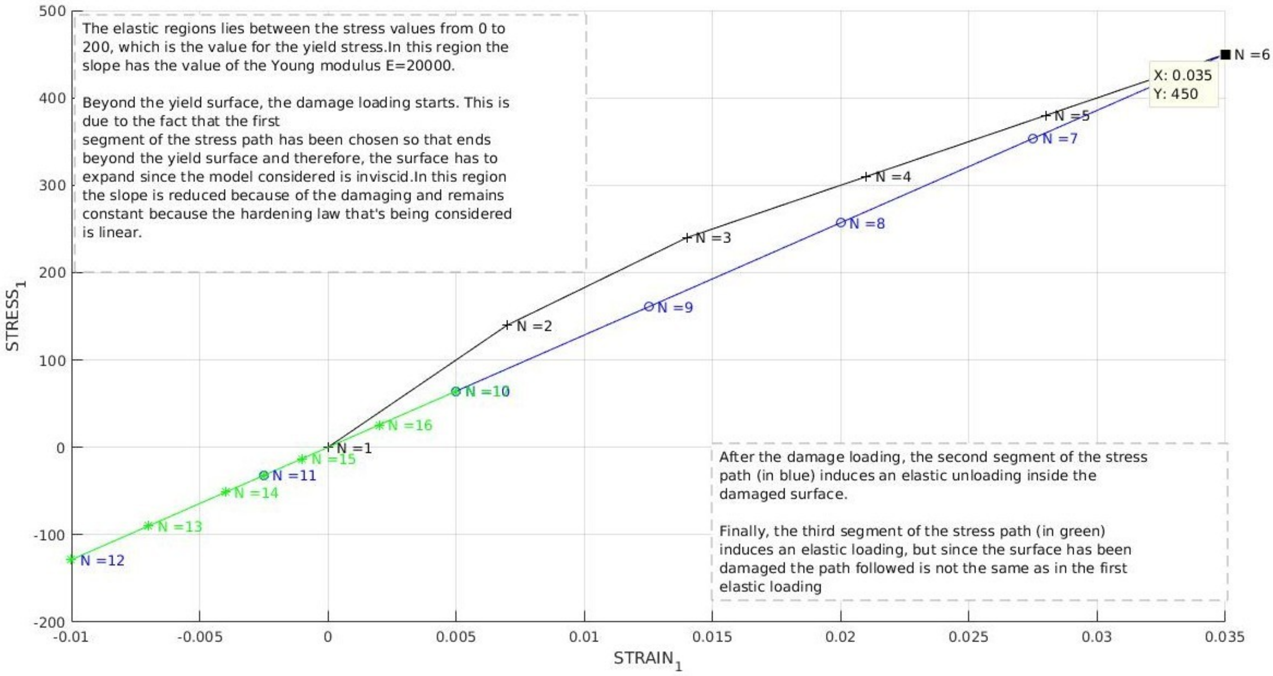

Figure 2. Stress/Strain curve for the path loaded in Figure 1.

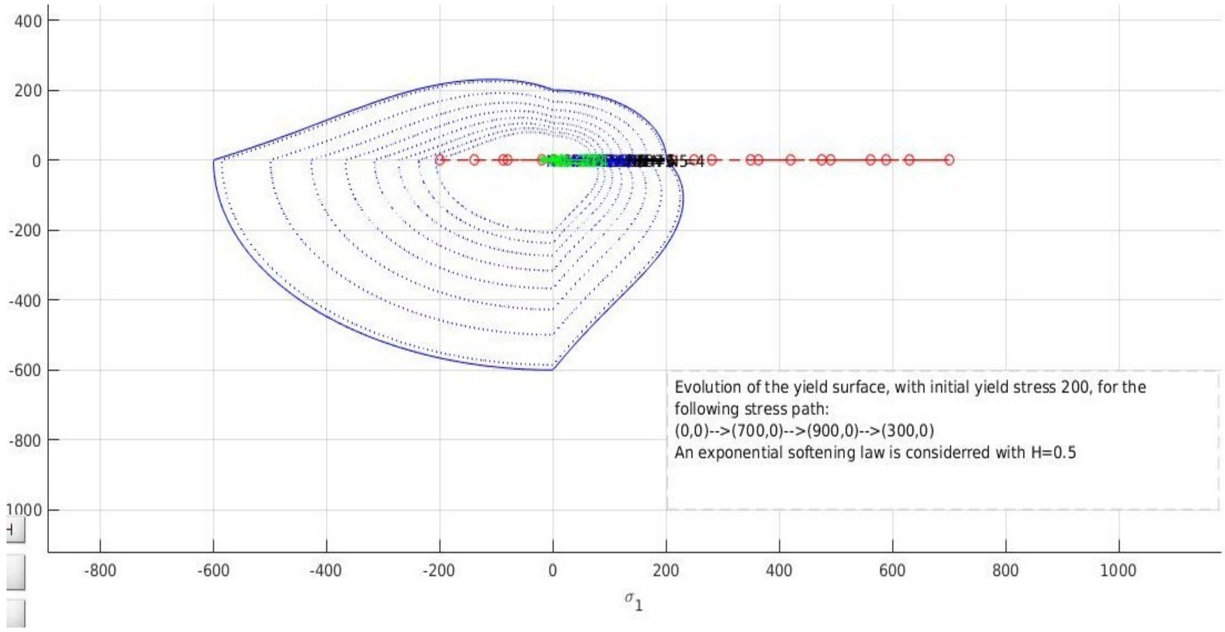

Figure 3. Evolution of the yield surface for the non-symmetric model. The loading path applied is uniaxial

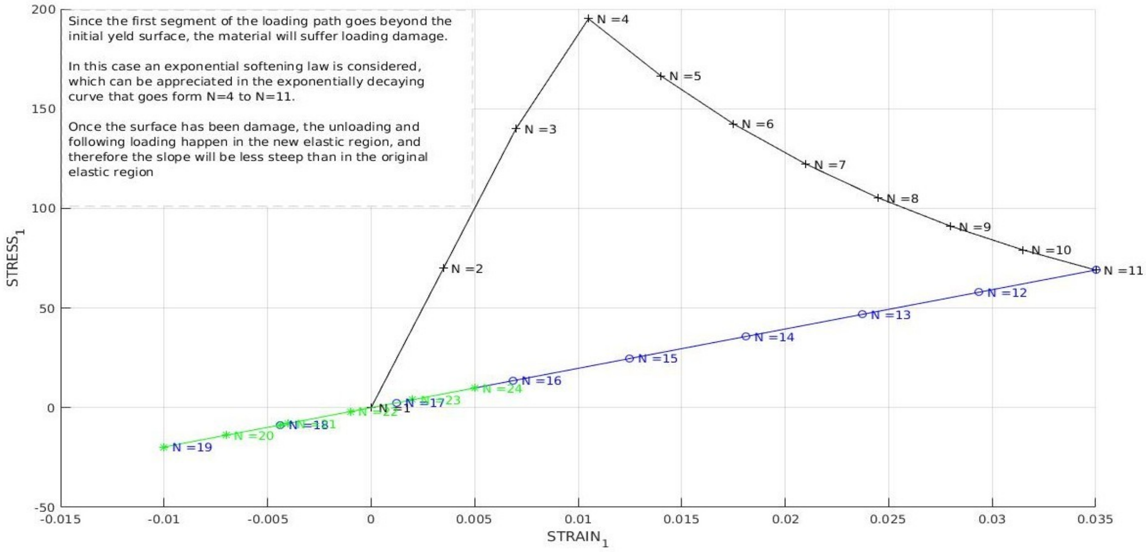

Figure 4. Stress/Strain curve for the loading path applied in Figure 3.

b) Tension-only model. The cases chosen in order to illustrate the behaviour of this model are: linear hardening and exponential softening.

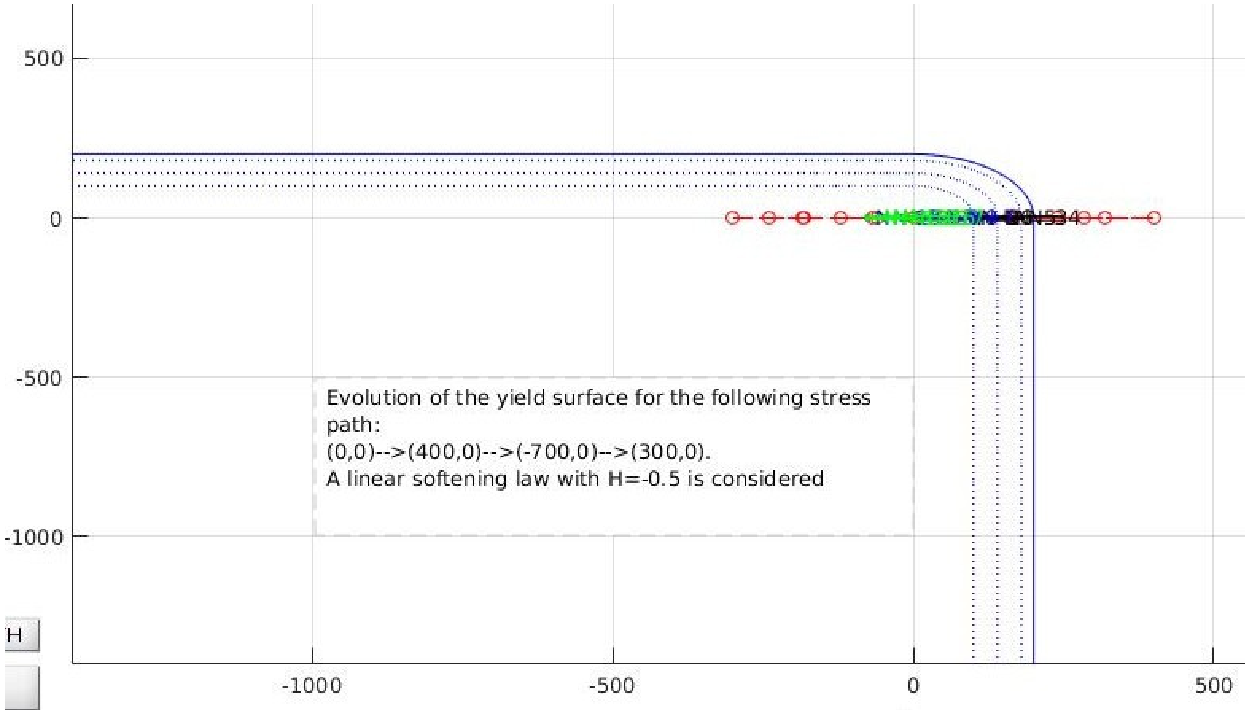

Figure 5. Evolution of the yield surface for the tension-only model

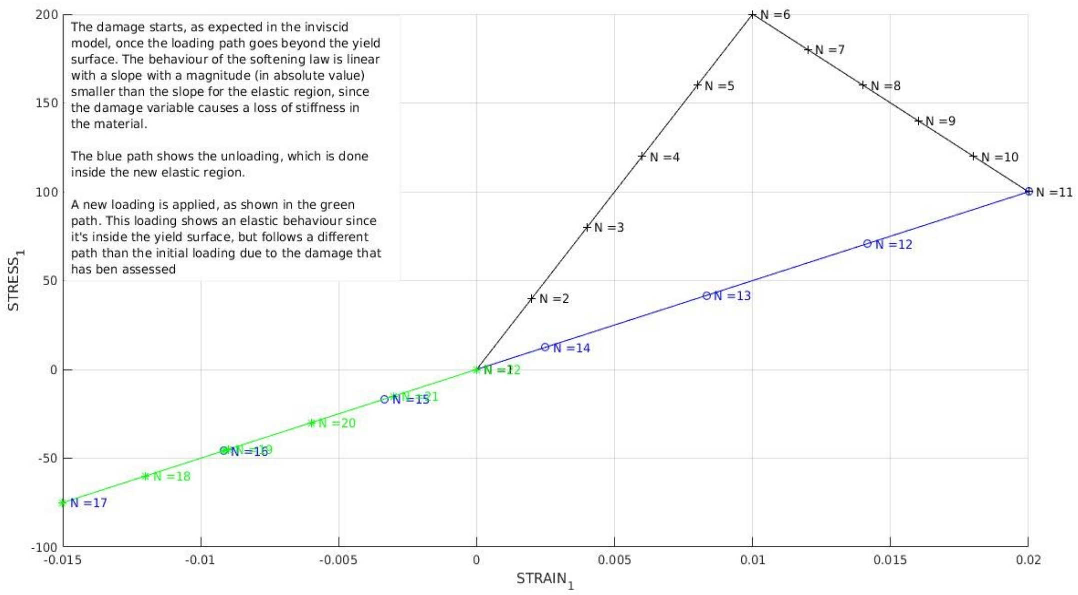

Figure 6. Stress/Strain curve for the path applied in Figure 5.

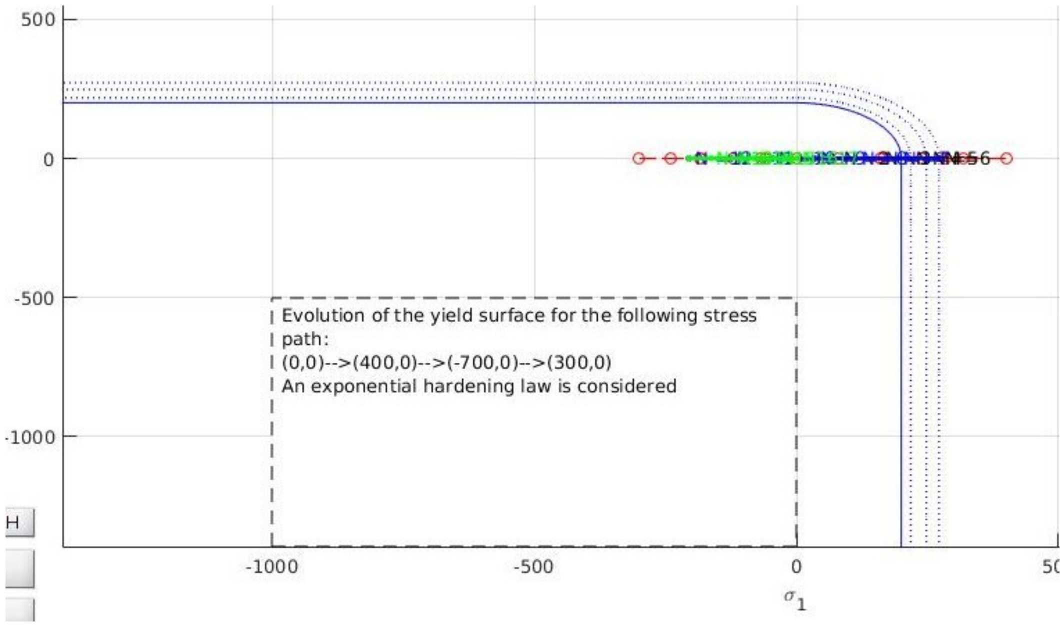

Figure 7. Evolution of the yield surface for the tension-only model. The loading path is uniaxial.

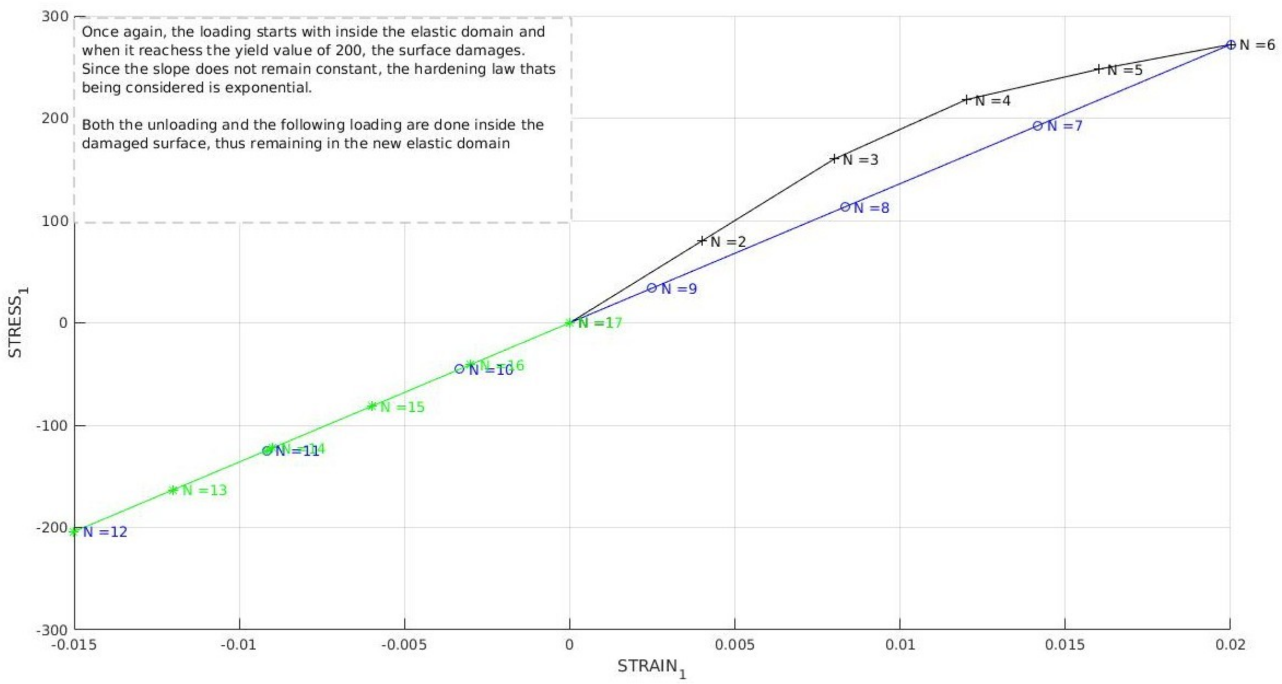

Figure 8. Stress/Strain curve for the path loaded in Figure 7.

2.

In this section, the computations have been carried out in order to show the differences between the non-symmetric and the tension only models.

a) Non-symmetric model.

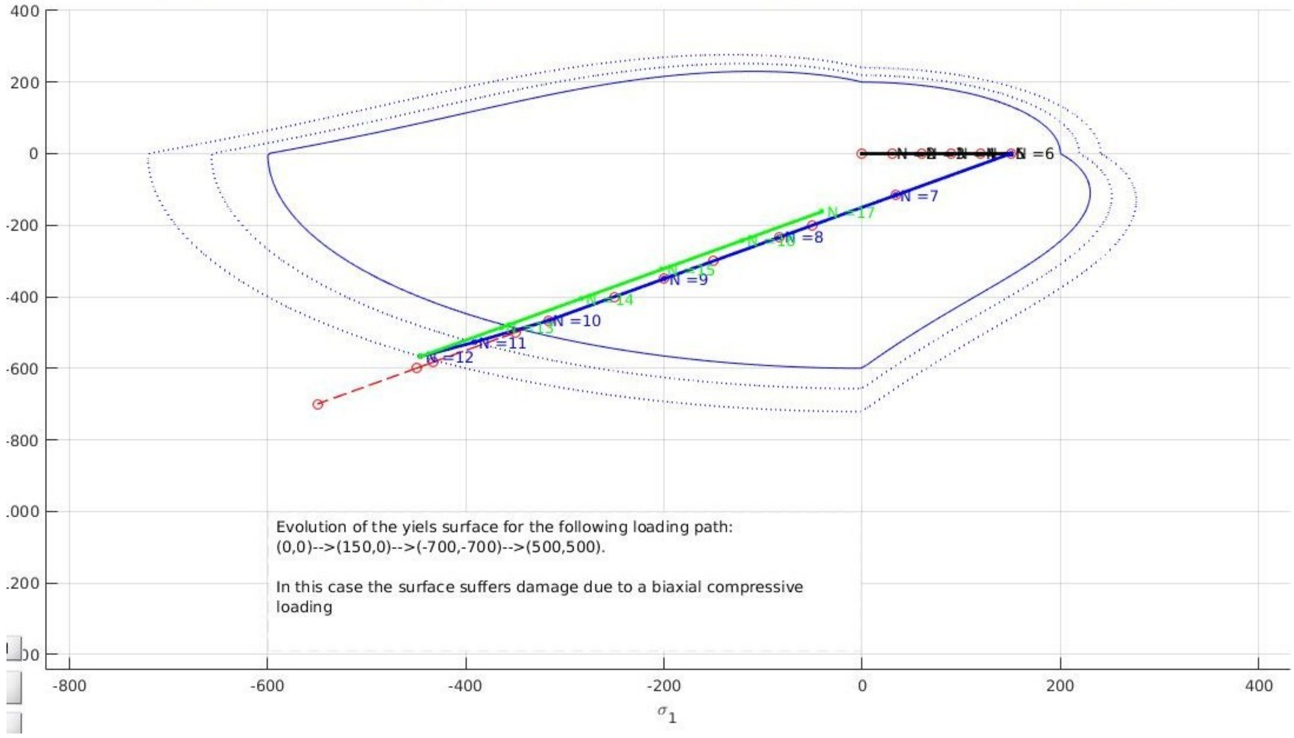

Figure 9. Evolution of the yield surface for the non-symmetric model. The first segment of the loading path is uniaxial and the remaining segments are biaxial

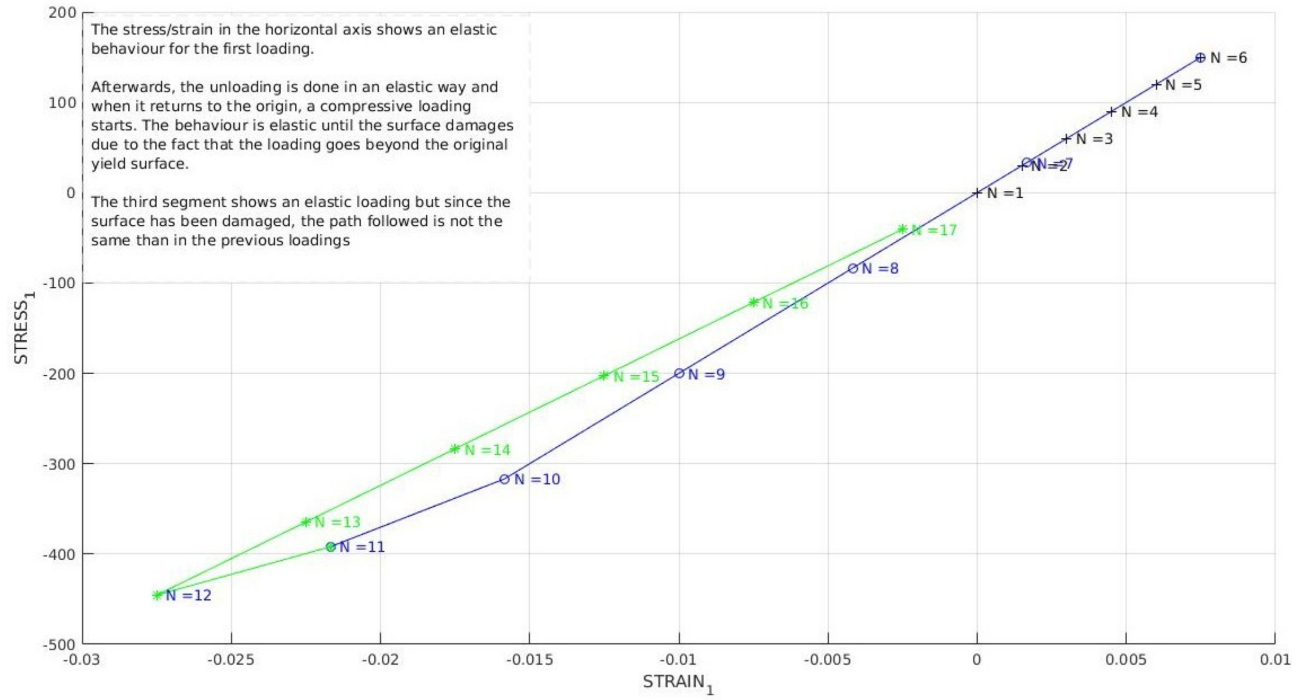

Figure 10. Stress/Strain curve in the horizontal axis for the loading path in Figure 9.

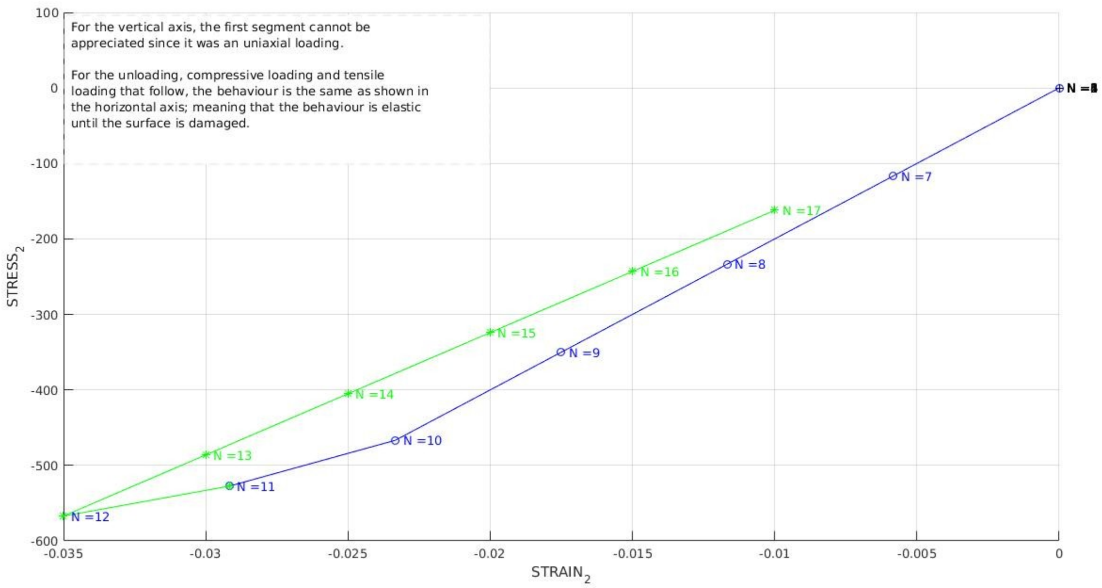

Figure 11. Stress/Strain curve in the vertical axis for the loading path in Figure 9.

b) Tension only model.

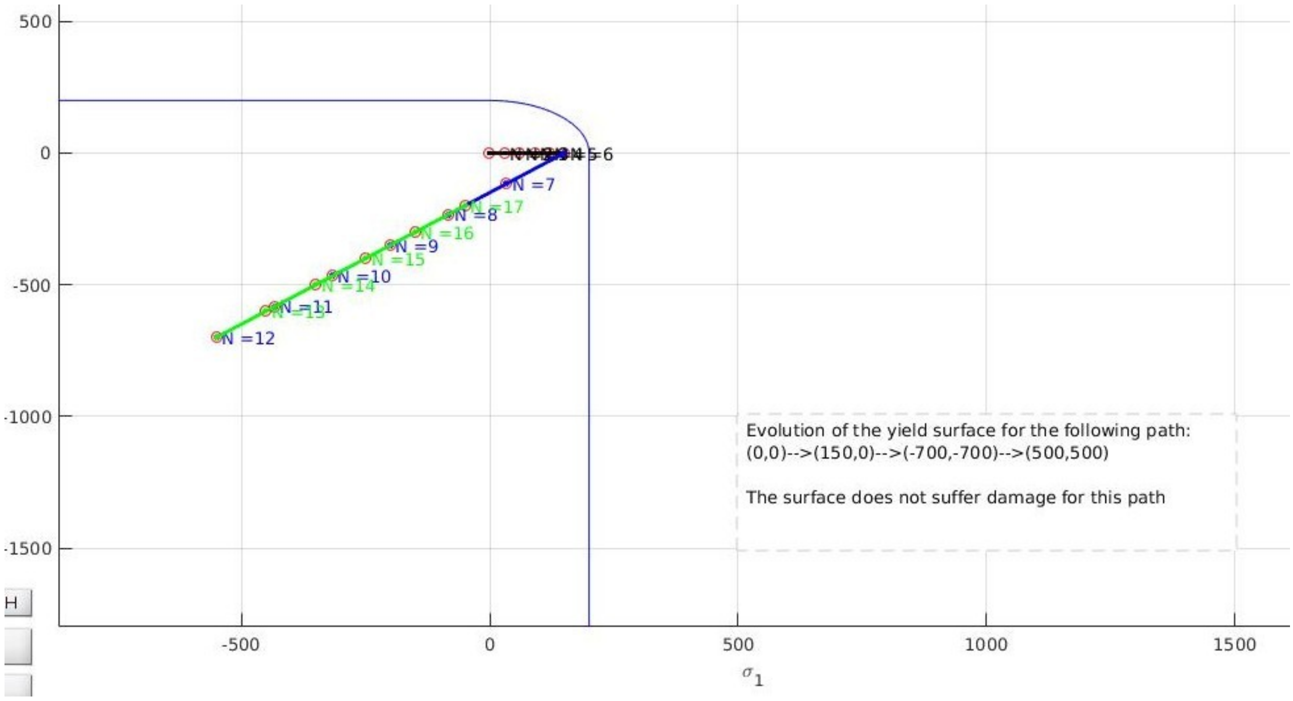

Figure 12. Evolution of the yield surface for the tension only model. The first segment of the loading path is uniaxial and the remaining segments are biaxial

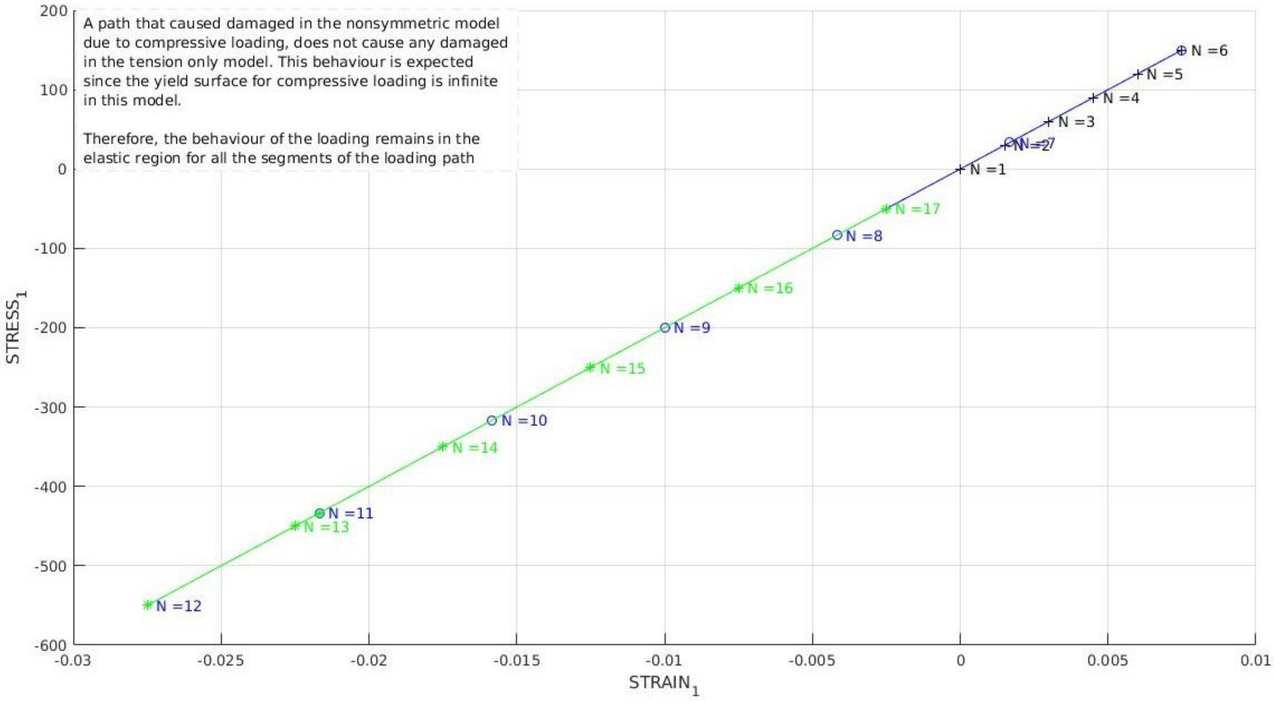

Figure 13. Stress/Strain curve in the horizontal axis for the loading path in Figure 12.

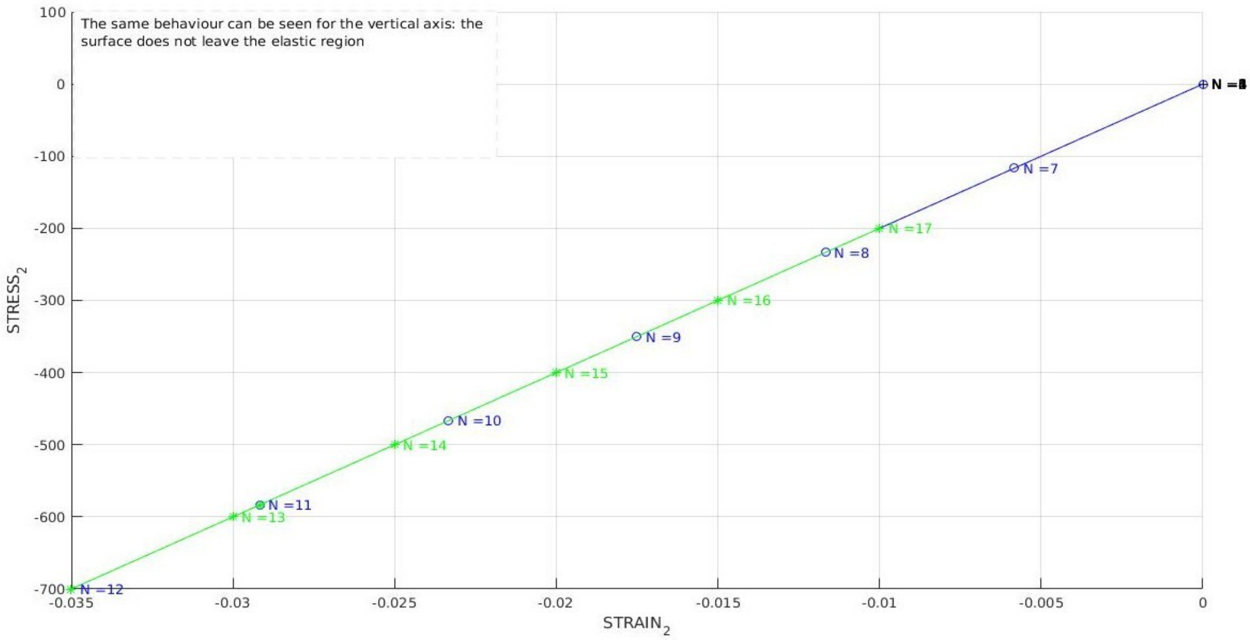

Figure 14. Stress/Strain curve in the vertical axis for the loading path in Figure 12.

3.
Finally, since in the previous section it was shown how compressive loading does not damage a surface that belongs to the tension only model, in this section the goal was to showcase the full stress/strain curve for the non-symmetric model.

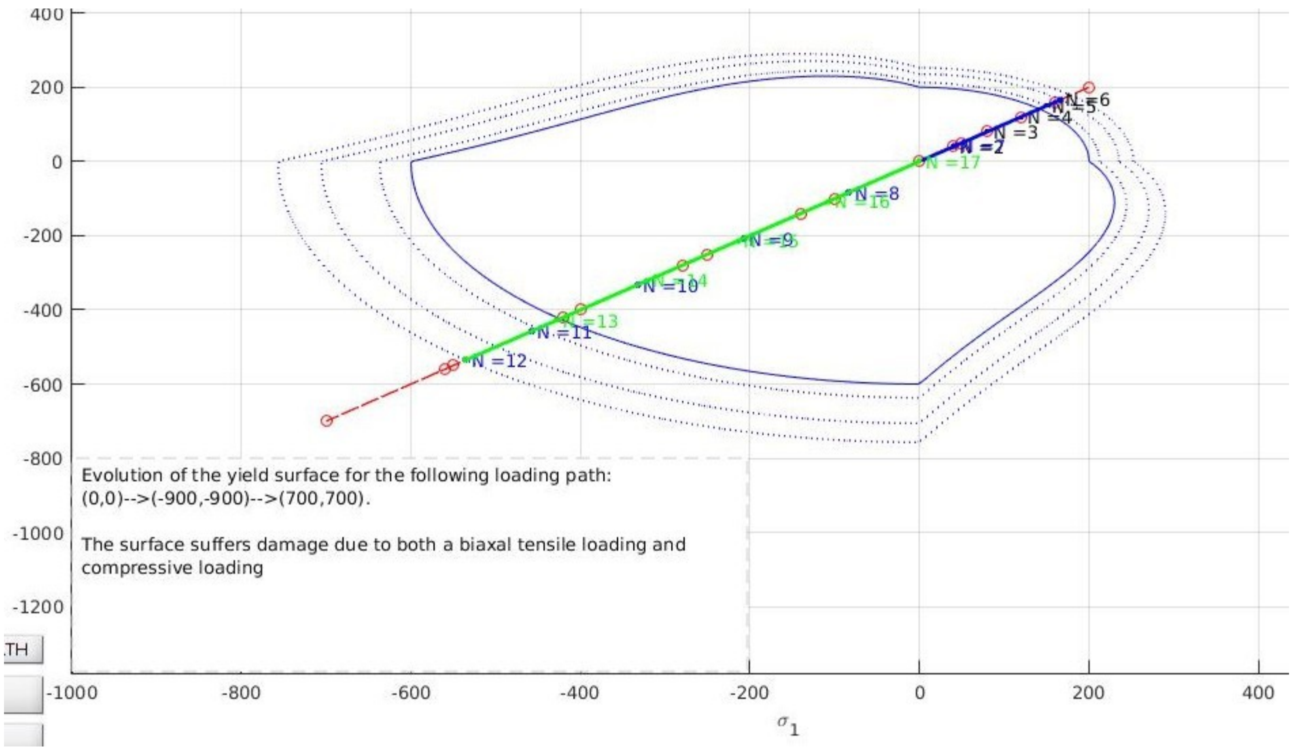

Figure 15. Evolution of the yield surface for the non-symmetric model. The loading path applied is biaxial.

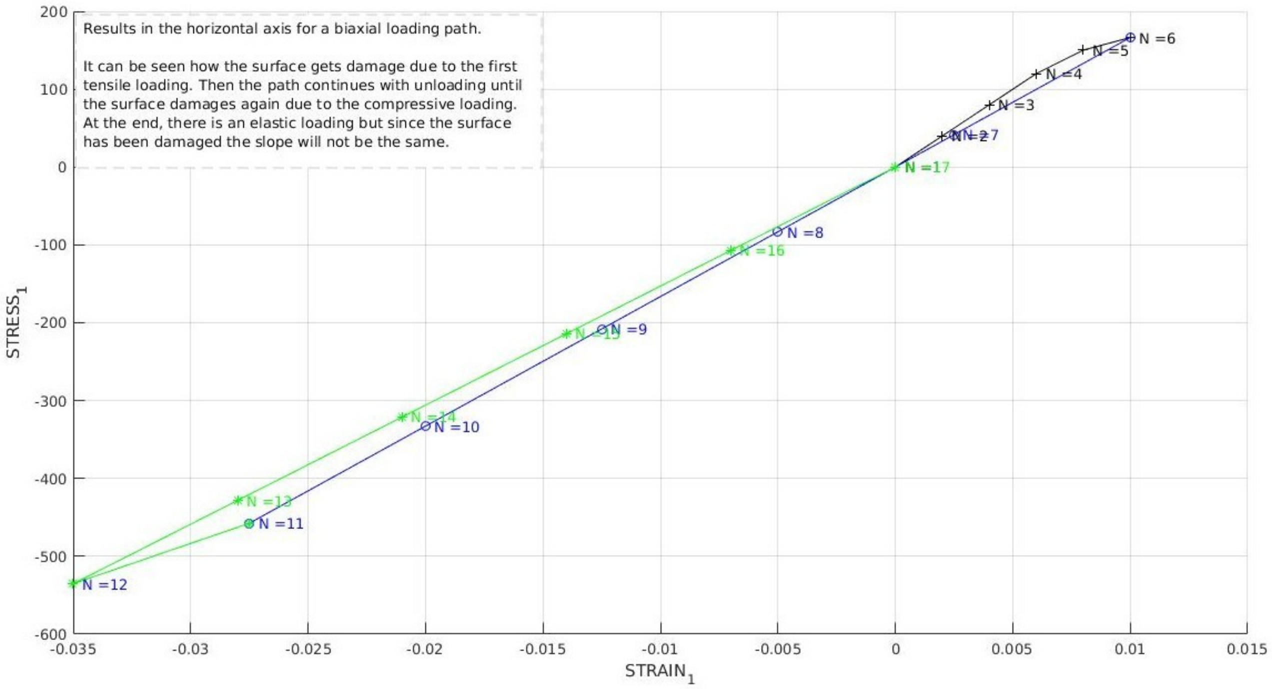

Figure 16. Stress/Strain curve in the horizontal axis for the loading path in Figure 15.

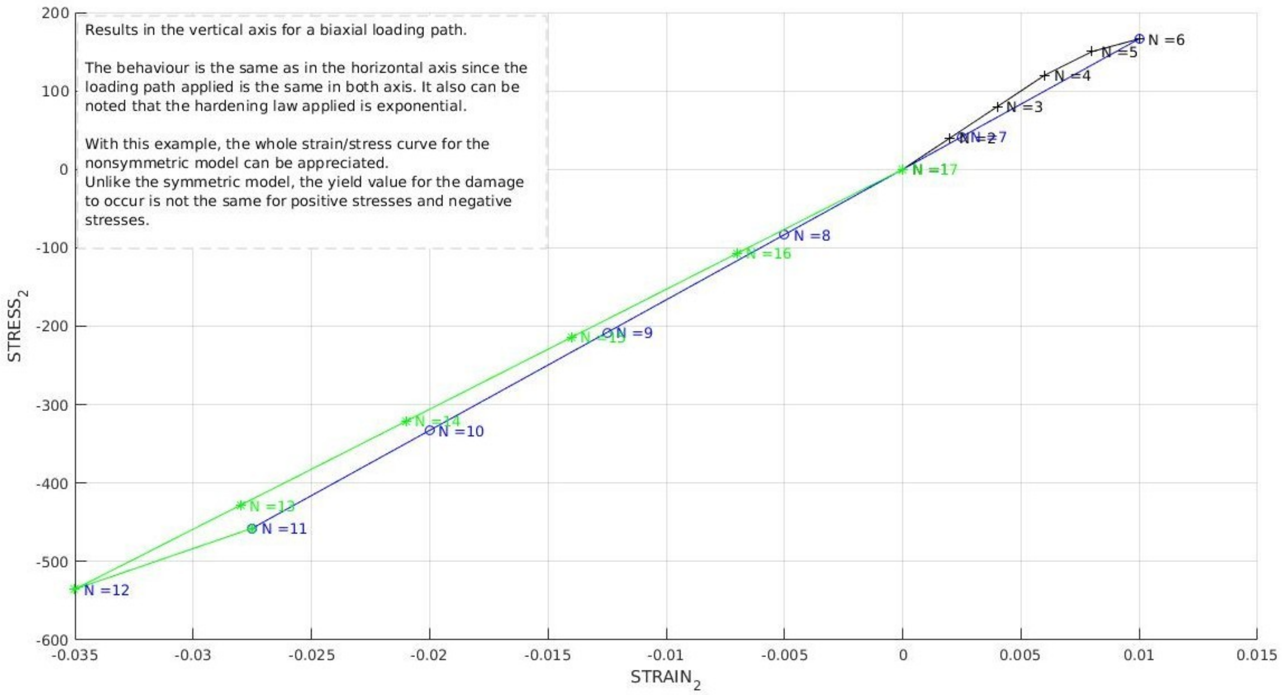

Figure 17. Stress/Strain curve in the vertical axis for the loading path in Figure 15.

Rate independent models

a) Effects of using different viscosity parameters and strain rate values

The following figures show the effects of the viscosity parameter in the symmetric model for the following load path:

(0,0)→(700,0)→(-700,0)→ (0,0)

With α=1/2 and Δt=10 being kept constant in both examples.

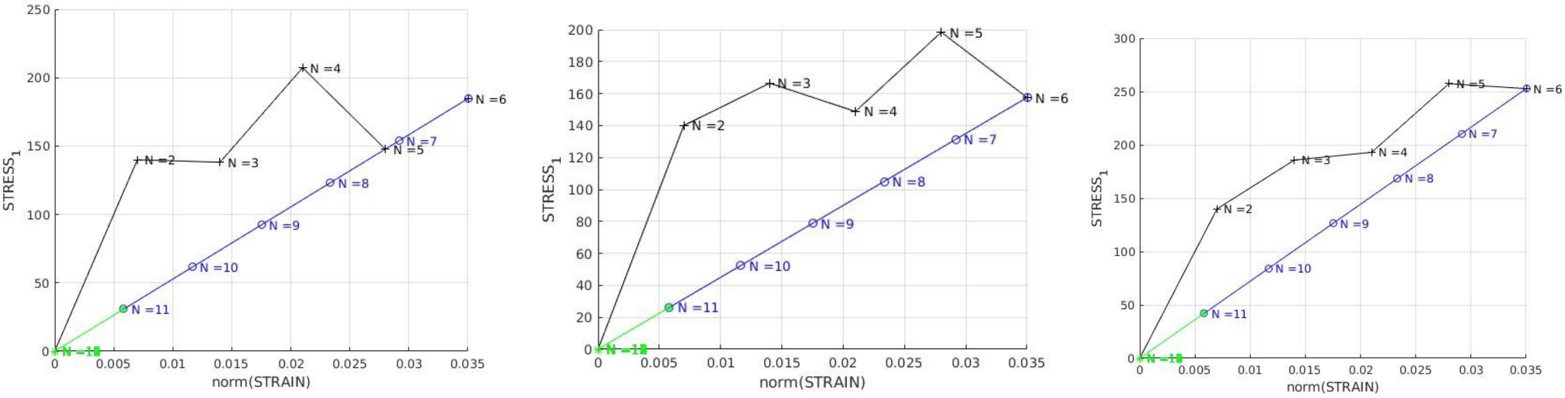

Figure 18. Effects of the viscosity parameter in the Stress/Strain curve. The first graph is for η= 0.3, the second is for η= 0.6 and the thir one for η= 0.9.

The viscous effects that appear when a loading damages the surface show how the surface instead of expanding in a continuous way as in the inviscid case, the rate of expansion changes with time. Therefore the strain/stress curve presents a “saw-like” pattern. When the viscosity increases, the peaks become less pronounced.
Viscous effects do not appear when unloading for this particular model.

In the following examples, the effects of the strain rate change over the stress/strain curve are studied. For these examples the viscosity is kept at 0.3.

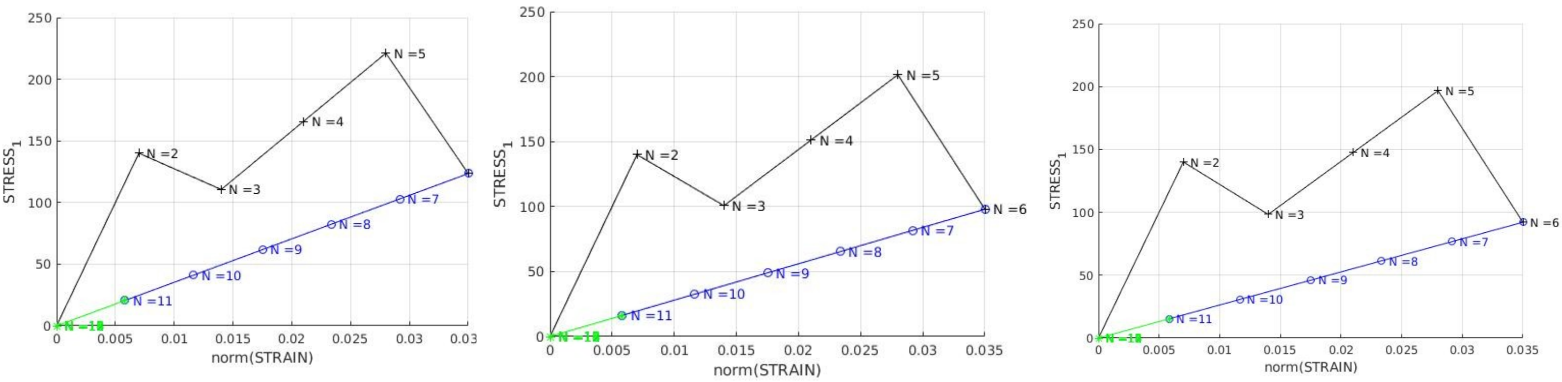

Figure 19. Effects of the strain rate in the Stress/Strain curve. The first graph is for Δt= 30 the second is for Δt= 70 and the thir one for Δt= 100.

For strain rate changes, the pattern of the peaks does not change. Still, it can be seen how the strain/stress curve moves downards the y-axis and reaches lower stress values as the total time of integration increases. Taking a look at point N=5 it can be seen how it moves from having a stress value above 200 to a value below 200.

b) Effects of different α values on the C_{11} component of the tangent and algorithmic constitutive operators

So far, the computations for the viscous model have been done using $\alpha=1/2$, which means that the time integration is done using the Crank-Nicolson scheme. When changing the α values to 0, 0.25, 0.75 and 1, the results obtained showed a really bizarre pattern had no physical meaning. This must be due to the fact that at some point the code implementation has not been done properly. Which means that even though the results in section (a) seemed to behave properly, they are not correct.

The expected results should show how for $\alpha<1/2$,there could appear inestabilities in the solution due to the fact that the integration scheme becomes conditionally stable.

Still, when the viscosity is set to 0 and $\alpha=1$, the implemention recovers the inviscid behaviour:

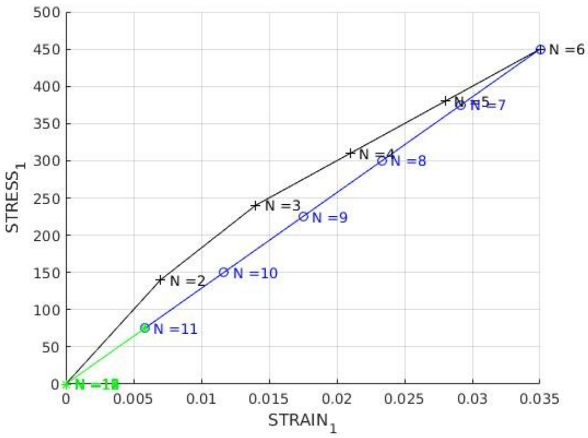

Figure 20. Recovery of the inviscid model setting the proper parameters in the viscous model

Annex 1: Code implementation

1. Nonsymmetric and tension only surfaces

The following lines have been added in the function *dibujar_criterio_dano1.m*

```
elseif MDtype==2
     tetha=[(-pi/2)*0.9999:0.01:pi*0.9999];
    %*************************************************
    %* RADIUS
    D=size(tetha);                      %*  Range
    m1=cos(tetha);                      %*
    m2=sin(tetha);                      %*
    Contador=D(1,2);                    %*


    radio = zeros(1,Contador) ;
    s1    = zeros(1,Contador) ;
    s2    = zeros(1,Contador) ;

    for i=1:Contador
        sigma= [m1(i) m2(i) 0 nu*(m1(i)+m2(i))];
        sigmapos=sigma.*(sigma>0);
        radio(i)= q/sqrt(sigmapos*ce_inv*sigma');

        s1(i)=radio(i)*m1(i);
        s2(i)=radio(i)*m2(i);

    end
    hplot =plot(s1,s2,tipo_linea);

    elseif MDtype==3
        tetha=[0:0.01:2*pi];
        %******************************************************************
        %* RADIUS
        D=size(tetha);                      %*  Range
        m1=cos(tetha);                      %*
        m2=sin(tetha);                      %*
        Contador=D(1,2);                    %*


        radio = zeros(1,Contador) ;
        s1    = zeros(1,Contador) ;
        s2    = zeros(1,Contador) ;


        for i=1:Contador

            sigma=[m1(i) m2(i) 0 nu*(m1(i)+m2(i))];
            sigmapos=sigma.*(sigma>0);
            theta=sum(sigmapos)/sum(abs(sigma));

            radio(i)= (q/sqrt(sigma*ce_inv*sigma'))/(theta+(1-theta)/n);

            s1(i)=radio(i)*m1(i);
            s2(i)=radio(i)*m2(i);

        end
        hplot =plot(s1,s2,tipo_linea);

    end
```

The function Modelos_de_dano1.m was also modified :

```
elseif (MDtype==2)  %* Only tension

    sigma=(eps_n1*ce);
    sigmapos=sigma.*(sigmab>0);

    rtrial=sqrt(sigmapos*eps_n1');

elseif (MDtype==3)  %*Non-symmetric

    sigma=(eps_n1*ce);
    sigmapos=sigma.*(sigma>0);
    theta=(sigmapos(1)+sigmapos(2))/(abs(sigma(1))+abs(sigma(2)));
    rtrial=(tita+(1-tita)/n)*sqrt(eps_n1*ce*eps_n1');
```

2. Exponential hardening law

The following lines have been added in the function *rmap_dano1.m*

```
else
    %Exponential
    q_inf=r0+(r0-zero_q);
    q_n1=q_n+((H*(q_inf-r0)/r0)*exp(H*(1-rtrial/r0)))*delta_r;
end
```

3. Viscous model

The following lines have been added in the function *rmap_dano1.m*

```
%*************************************************************************************
%*       Damage surface       %*
[rtrial_n] = Modelos_de_dano1 (MDtype,ce,eps_n0,n) % TauEps_n (Viscous model)
[rtrial] = Modelos_de_dano1 (MDtype,ce,eps_n1,n) % TauEps_n+1
[rtrial_nalpha]=(1-ALPHA_COEFF)*rtrial_n+ALPHA_COEFF*rtrial ; % TauEps_n+alpha (Viscous model)
%*************************************************************************************


%*************************************************************************************
%*   Ver el Estado de Carga                                                          %*
%*   --------->     fload=0 : elastic unload                                         %*
%*   --------->     fload=1 : damage (compute algorithmic constitutive tensor)       %*
fload=0;

if viscpr == 1
    if (rtrial_nalpha > r_n)
        %*   Loading
        fload=1;
        delta_r=rtrial_nalpha-r_n;
        r_n1=((eta-delta_t*(1-ALPHA_COEFF))/(eta-ALPHA_COEFF*delta_t))*r_n+...
            (delta_t*rtrial_nalpha)/(eta+ALPHA_COEFF*delta_t);

        %  Linear
        q_n1= q_n+ H*delta_r;

        if(q_n1<zero_q)
            q_n1=zero_q;
        end

    else
        %*     Elastic load/unload
        fload=0;
        r_n1= r_n  ;
        q_n1= q_n  ;

    end
```