

# COMPUTATIONAL SOLID MECHANICS - ASSIGNMENT #1

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## 1 Rate independent models

### 1.1 Non-symmetric & Tension-only models

Implementation of continuum isotropic damage model of non-symmetric case is shown in figure 1. The elastic domain is non-symmetric and the points are chosen in elastic domain. Therefore, damage variable remains zero as time pass. The following figure (figure 2) shows tension-only damage model. As we expect

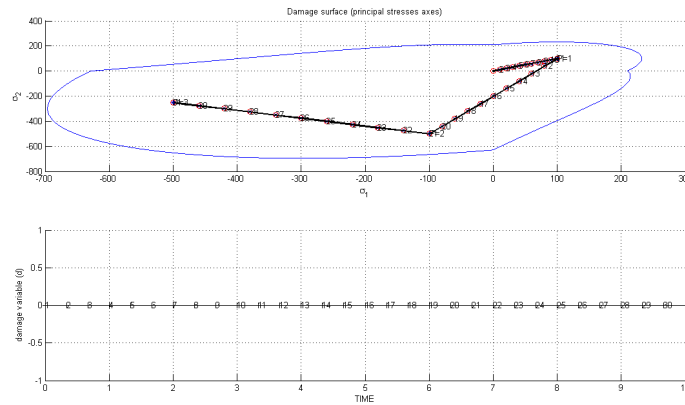


Figure 1: non-symmetric elastic domain & rate of damage variable

the elastic domain includes infinity when  $\sigma_1$  and  $\sigma_2$  goes to  $-\infty$ . Like non-symmetric case, the points are chosen in elastic domain and as a consequent damage variable is zero along the time.

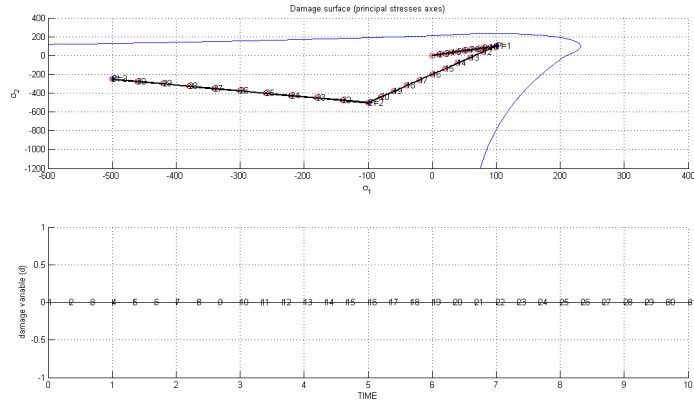


Figure 2: tension-only elastic domain & rate of damage

## 1.2 Linear and exponential hardening and softening

In hardening case ( $H > 0$ ), we implement the code for both linear and exponential cases with the same selected points. In exponential case, we assume  $A > 0$  as a constant value which can obtain by  $A = \frac{Hr_0}{q_\infty - r_0}$  and  $q_\infty = 2.1$  which is greater than  $r_0 = 1.41$ .

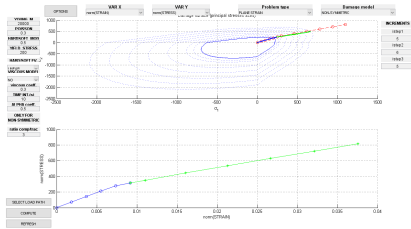


Figure 3: non-symmetric linear hardening ( $H = 0.5$ ), strain-stress norm plot

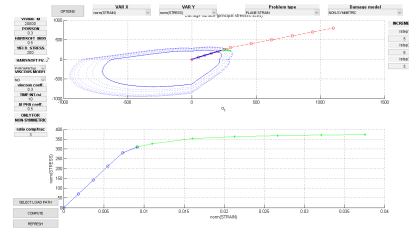


Figure 4: non-symmetric exponential hardening ( $H = 0.5$ ), strain-stress norm plot

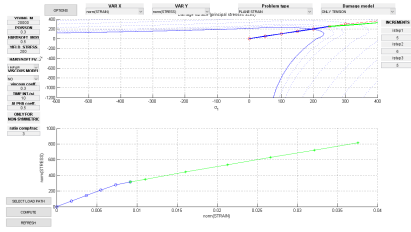


Figure 5: tension-only linear hardening ( $H = 0.5$ ), strain-stress norm diagram

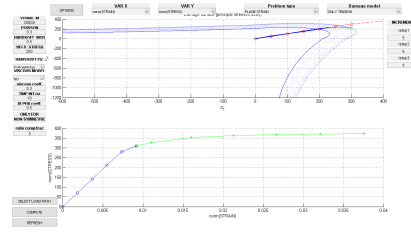


Figure 6: tension-only exponential hardening ( $H = 0.5$ ), strain-stress norm diagram

The first diagrams of figures 3,4,5, and 6 show hardening case since the elastic domain is getting expanded. The yield stress in these diagrams is the point on blue line in the initial elastic domain. In the second diagram of figures 3 and 5, the strain-stress norm plot, the strain marked as a green lines prove that there is a linear hardening while the green line in the second plots in figures 4 and 6 show hardening is in exponential way as it is not straight line.

In the softening case ( $H < 0$ ), we assume  $q_\infty = 0.7$  to hold  $q_\infty < r_0$ . The following figures clarify the difference between exponential and linear cases for both models. In addition, elastic surface compresses in softening case as the first diagram in each figure make it clear.

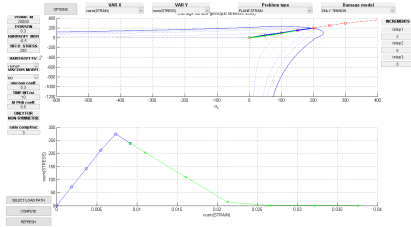


Figure 7: tension-only linear softening ( $H = -0.5$ ), strain-stress norm diagram

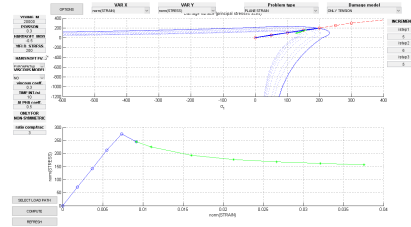


Figure 8: tension-only exponential softening ( $H = -0.5$ ), strain-stress norm diagram

The top point in the diagrams of strain-stress norm is the point which yield stress happens.

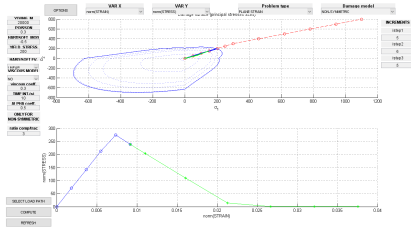


Figure 9: non-symmetric linear softening ( $H = -0.5$ ), strain-stress norm diagram

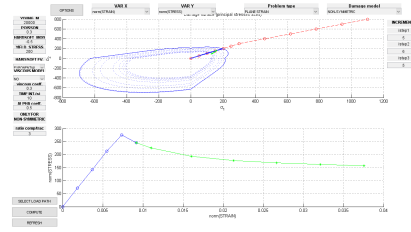


Figure 10: non-symmetric exponential softening ( $H = -0.5$ ), strain-stress norm diagram

### 1.3 Correctness of implementation

In the case 1, we consider uniaxial tensile loading, uniaxial tensile unloading and uniaxial tensile loading for non-symmetric model with positive linear hardening. Therefore,

$$\begin{aligned}\Delta\bar{\sigma}_1^{(1)} &= 300, & \Delta\bar{\sigma}_2^{(1)} &= 0 \\ \Delta\bar{\sigma}_1^{(2)} &= -150, & \Delta\bar{\sigma}_2^{(2)} &= 0 \\ \Delta\bar{\sigma}_1^{(3)} &= 400, & \Delta\bar{\sigma}_2^{(3)} &= 0\end{aligned}$$

According to figure 11, the elastic surface expands so there is tensile loading.

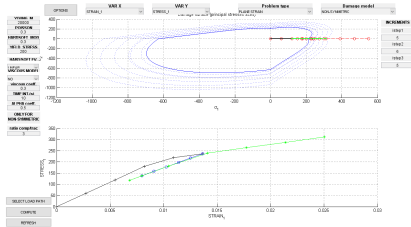


Figure 11: strain-stress space

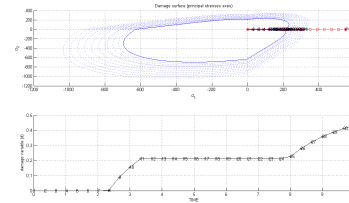


Figure 12: rate of damage variable

In strain-stress space, black line represent first segment, blue line is the second segment, and green line shows the third segment. loading happens when stress is greater than yield stress ( $\sigma_u = 200$ ). In the second segment, stress and strain decrease, so there is unloading. In the third path, as soon as the strain and stress gets greater than previous maximum of them, we face loading. Figure 12 shows variation of damage with respect to time. Each segment path has 10 points. when damage is constant, the points are in elastic domain but it increases when there is a pure loading. Thus, tensile loading happens in the first and third segments (Point 8 and 24) according to figure 12.

Similarly we consider case 1 for tension-only model and we obtain following results:

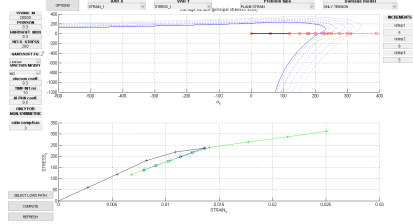


Figure 13: strain-stress space

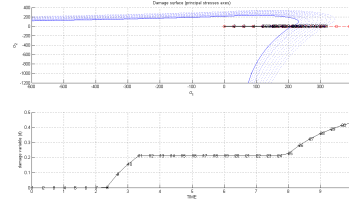


Figure 14: rate of damage variable

In case 2, we consider uniaxial tensile loading, biaxial tensile unloading, and tensile loading in each segment for both damage models.

$$\begin{aligned} \Delta\bar{\sigma}_1^{(1)} &= 300, & \Delta\bar{\sigma}_2^{(1)} &= 0 \\ \Delta\bar{\sigma}_1^{(2)} &= -200, & \Delta\bar{\sigma}_2^{(2)} &= -200 \\ \Delta\bar{\sigma}_1^{(3)} &= 500, & \Delta\bar{\sigma}_2^{(3)} &= 500 \end{aligned}$$

The result for tension-only model is shown in figures 15 and 16. first diagram in

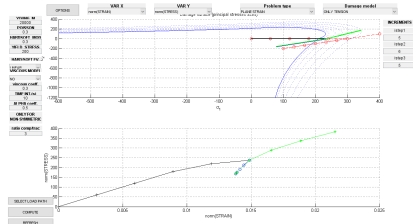


Figure 15: strain-stress norm diagram

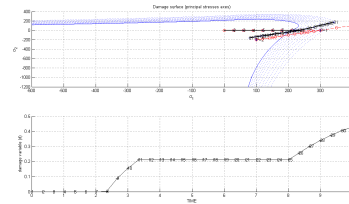


Figure 16: rate of damage variable

figures 15 and 16 shows elastic surface in stress space. According to figure 15, loading happens when the stress is greater than yield stress as it is in the black line when the  $\|\sigma\| \geq \sigma_u = 200$ . then, the blue line represent unloading when the path return to elastic domain and strain and stress decrease. Again, the green line after passing the previous maximum stress and strain, we encounter with another loading. In figure 16, when the rate of damage variable dose not change, we see unloading or staying in elastic state. On the other hand, when it increases, loading starts. Therefore the loading happens in points 8 and 25. Point 11 shows unloading as damage variable does not increase.

The following figures represent the same discussion for non-symmetric model:  
In case 3, we assume

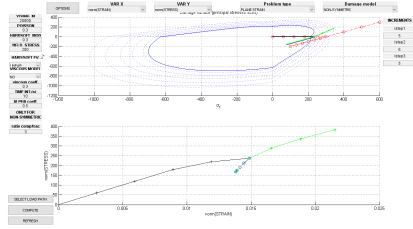


Figure 17: strain-stress norm diagram

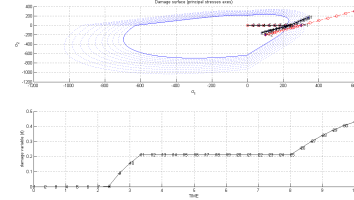


Figure 18: rate of damage variable

$$\begin{aligned} \Delta \bar{\sigma}_1^{(1)} &= 300, & \Delta \bar{\sigma}_2^{(1)} &= 300 \\ \Delta \bar{\sigma}_1^{(2)} &= -100, & \Delta \bar{\sigma}_2^{(2)} &= -100 \\ \Delta \bar{\sigma}_1^{(3)} &= 300, & \Delta \bar{\sigma}_2^{(3)} &= 300 \end{aligned}$$

Following figures shows loading and unloading similar to previous cases but all loading and unloading are in both directions (biaxial).

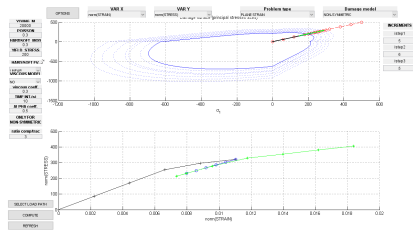


Figure 19: strain-stress norm diagram (non-symmetric model)

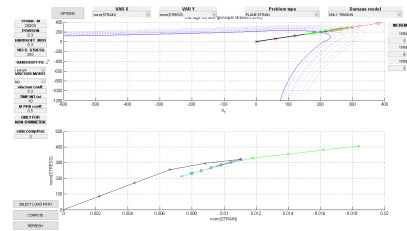


Figure 20: strain-stress norm diagram (tension-only model)

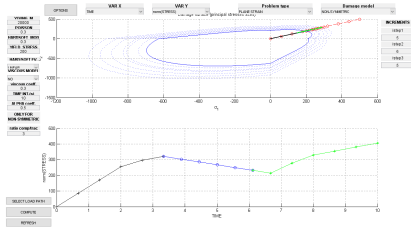


Figure 21: rate of stress norm (non-symmetric model)

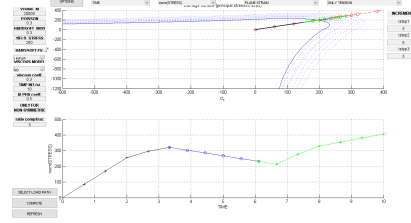


Figure 22: rate of stress norm (tension-only model)

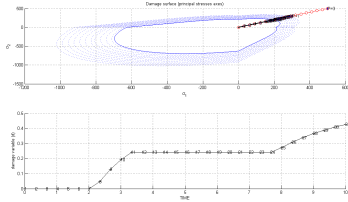


Figure 23: rate of damage variable (non-symmetric model)

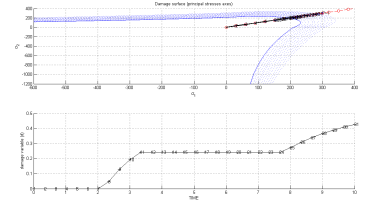


Figure 24: rate of damage variable (tension-only model)

## 2 Rate dependent models

In rate dependent models, we have new parameter of viscosity  $\eta$ . Let  $H = 0.3$  and  $\nu = 0.3$ . Hardening type is linear and  $\sigma_u = 200$ . Moreover, each segment path includes 5 points to better view of results. To obtain different strain rate, we change length of the time interval. It is enough to check correctness of implementation for uniaxial loading, so the Points to make the path in stress space are:

$$P_1 = (300, 0), \quad P_2 = (200, 0), \quad P_3 = (600, 0)$$

First, we fix  $\eta = 0.5$  and choose different length of time interval ( $\Delta T$ ). Whenever  $\Delta T$  gets greater, strain rate decrease. we also let  $\alpha = 0.5$ . The figures 25 and 26 shows the difference between different value of strain rates with fixed  $\eta$ . Comparing point 16 in each figure in stress-strain diagram shows increase of strain rate gives increase on stress.

Figures 27 and 28 represent different  $\eta$  with fixed  $\Delta T$ . Similarly by comparing stress value of point 16 in each graphs, we find that the greater viscosity parameter, the greater stress value.

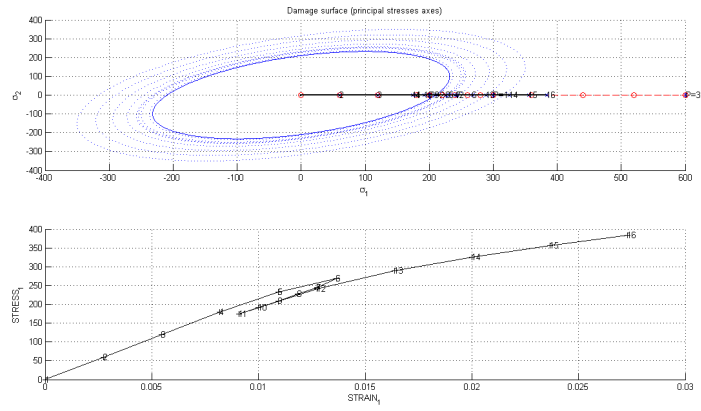


Figure 25: stress-strain plot-  $\Delta T = 10$ ,  $\eta = 0.5$ ,  $\alpha = 0.5$

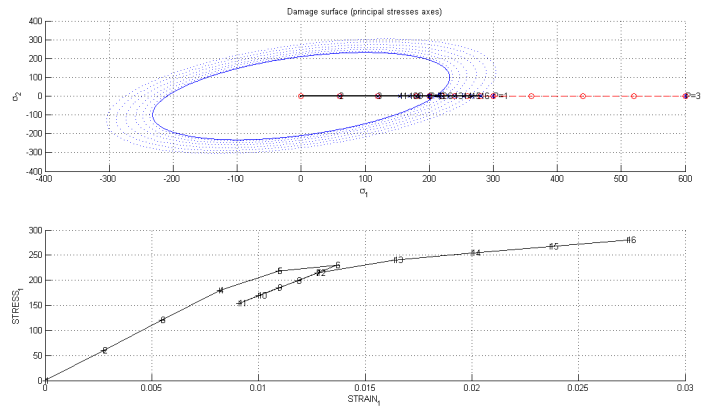


Figure 26: stress-strain plot-  $\Delta T = 100$ ,  $\eta = 0.5$ ,  $\alpha = 0.5$



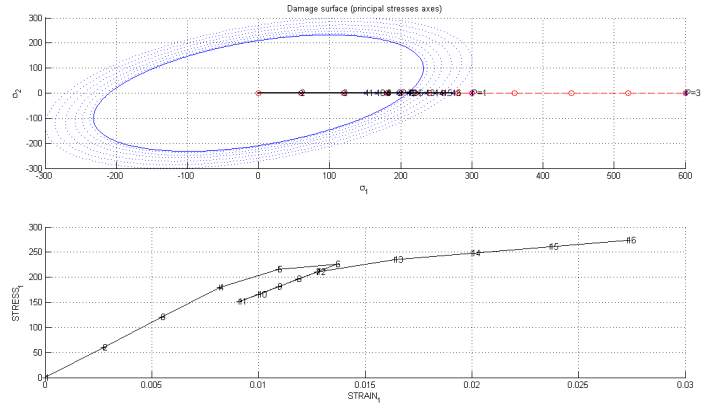


Figure 27: stress-strain plot-  $\Delta T = 100$ ,  $\eta = 0.2$ ,  $\alpha = 0.5$

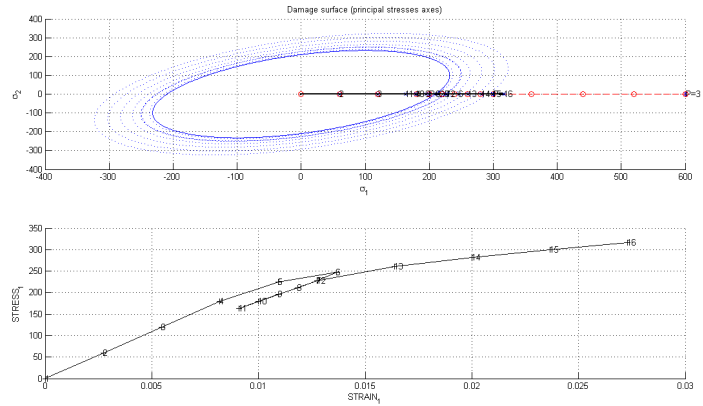


Figure 28: stress-strain plot-  $\Delta T = 100$ ,  $\eta = 2$ ,  $\alpha = 0.5$

Now we fix  $\eta = 0.5$ ,  $\Delta T = 10$  and change the value of  $\alpha$ . The following pictures show stress-strain diagram for different  $\alpha$ 's. For  $\alpha = 0$  (figure 33), the result is not appropriate but it does not show instability we expect. It is the same for  $\alpha = 0.25$ . If we accept from theory that  $\alpha = 0.5$  is the most accurate integration method for our problem, the other cases except  $\alpha = 0$  are acceptable for our case.

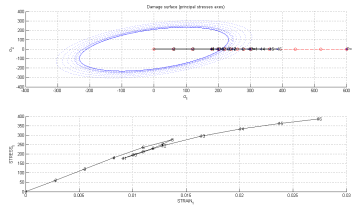


Figure 29:  $\alpha = 0.25$

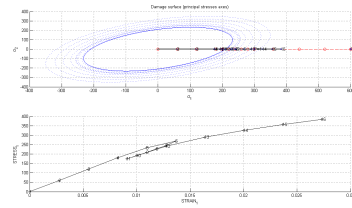


Figure 30:  $\alpha = 0.5$

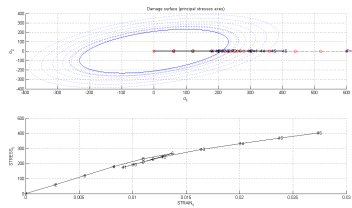


Figure 31:  $\alpha = 0.75$

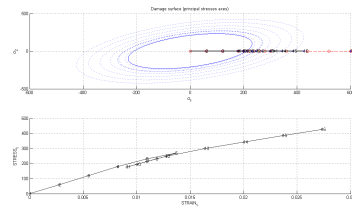


Figure 32:  $\alpha = 1$

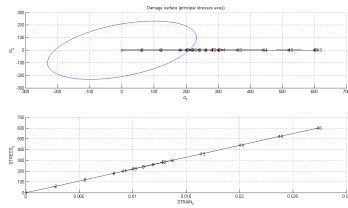


Figure 33:  $\alpha = 0$