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MSc Computational Mechanics

Computational Solid Mechanics

ASSIGNMENT 2

25/05/06

1D: Rate Independent /Rate Dependent Plasticity Model

The cases are solved for material properties, Young's Modulus E= 400 Pa, Yield stress = 200 Pa. The value of cyclic loading is taken as, tensile loading is given as 350, tensile unloading or compressive loading is taken as -350 and tensile loading of 450 is applied again. The ultimate tensile strength in case of non-linear hardening is taken as $\sigma_{inf} = 300$Pa.

Case 1: Perfect Plasticity

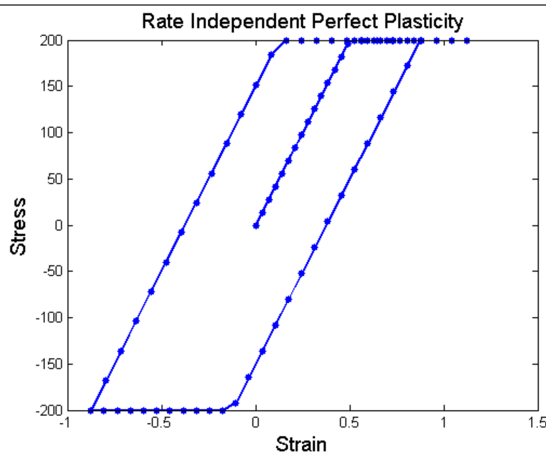

Figure 1: 1D Rate independent Perfect plasticity

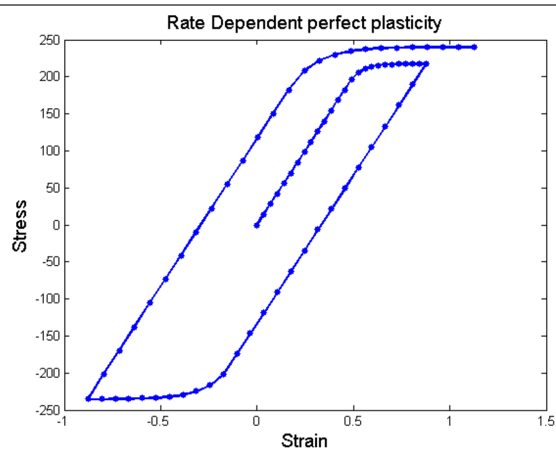

Figure 2: 1D Rate dependent Perfect Plasticity for $\eta = \mathbf{5}$

The above case is solved for uniaxial cyclic plastic loading/elastic unloading. It can be seen from the above figure that for perfect plasticity rate independent case, the tensile loading and unloading doesn't exceed the yield stress limit as it is solved for linear case, and hence deformation does not occur.

But in case of rate dependent case, viscosity is added to the material, hence the material behaves slightly nonlinear and stress exceeds the yield stress value of 200 Pa and hence deformation takes place during tensile loading and unloading. The value of yield stress keeps increasing due to deformation during cyclic loading and unloading.

For rate dependent case, a plot of stress vs time in figure 3 shows that with increase in viscosity parameter, the yield stress value increases at a given time. Figure 4 shows as the load rate increases, the slope of deformation increases. When the load rate is decreased i.e. when δt is increased very high, the material reaches quasi-static condition. **When the viscosity and load rate are very less, the rate independent model is reproduced from rate dependent model as seen in blue curve in figure 3 and red curve in figure 4.**

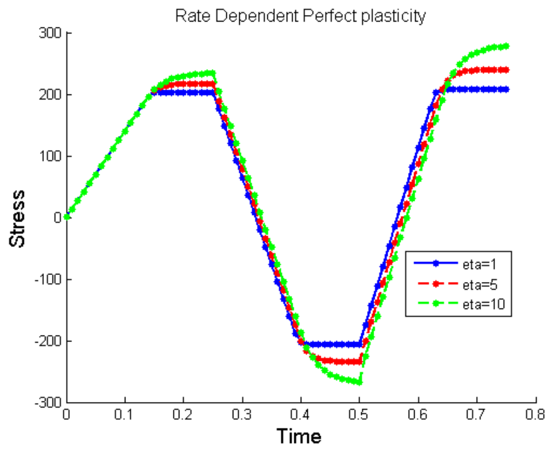

Figure 3: 1D Rate dependent stress vs time for varying viscosity for perfect plasticity

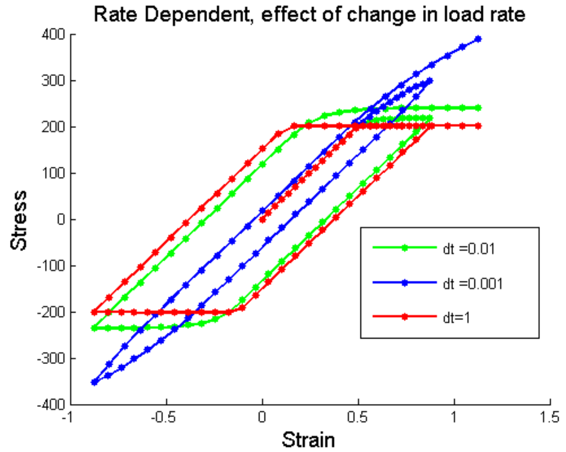

Figure 4: 1D Rate dependent stress vs strain for effect if change in load rate

Case 2: Linear isotropic hardening plasticity

In the linear Isotropic hardening case, It can be observed that with increase in isotropic hardening parameter 'K' the plasticity of the material decreases. At K=0 the material shows perfect plasticity and as K tends to infinity, the slope of plastic deformation increases and the size of yield stress increases during tensile/compressive loading and unloading due to plastic deformation as shown in the figure below. This means that on further cycles of tensile loading and unloading, the material will eventually deform along elastic line of stress strain curve. It happens when K=E. Similar behavior is observed for rate dependent linear isotropic hardening plasticity case. One case is shown for rate dependent case, at K=100 in figure 4, during tensile unloading, the yield stressed is increased upto -300 Pa and is further increased during tensile loading again. Unlike in rate independent case, where the yield stress doesn't go beyond 300 during compressive loading at K=100. Also if material is loaded in tension past yield, and then loaded in compression, it will not yield in compression until it reaches the level past yield that was reached when it was loaded in tension. It is observed that in rate dependent case also the slope of deformation increases as the isotropic hardening parameter increases. Effect of viscosity and load rate is same as discussed in case of perfect plasticity.

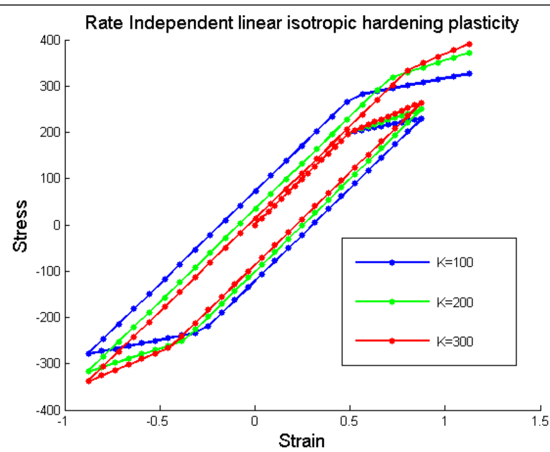

Figure 5: 1D Rate independent linear isotropic hardening plasticity

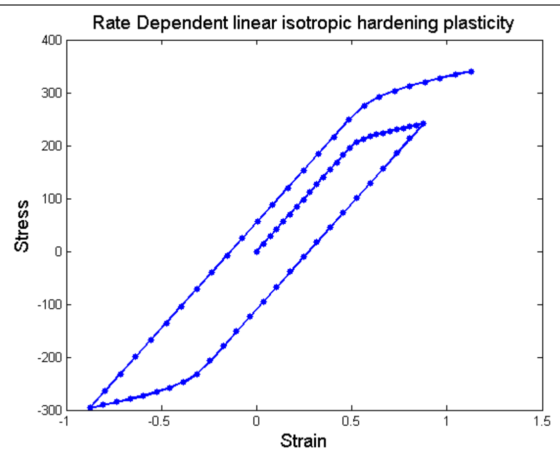

Figure 6: 1D Rate dependent linear isotropic hardening plasticity

Case 3: Linear kinematic hardening plasticity

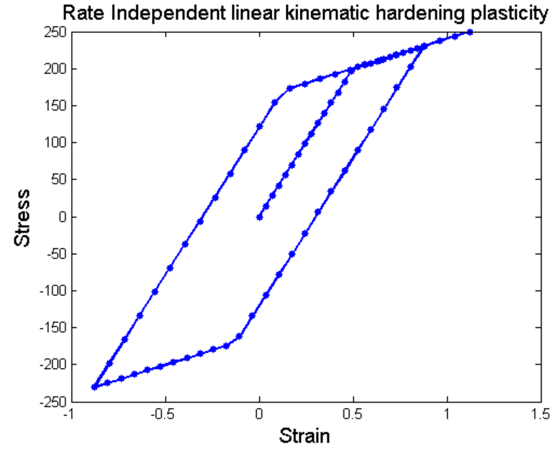

Figure 7: 1D Rate independent linear kinematic hardening plasticity

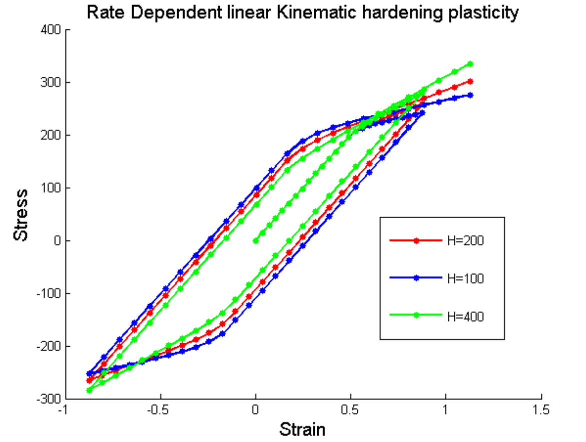

Figure 8: 1D Rate dependent linear kinematic hardening plasticity

The above figure shows the behavior of material under various kinematic hardening parameters. Here the material shows the **Bauschinger effect**. In rate dependent case, the graph is observed by varying the kinematic hardening parameter 'H' as H=100, H=200 and H=400. It can be observed that, as H increases the material softens on compression. On compressive loading, the size of yield stress is observed to decrease and it further decreases with further tensile loading. Hence the material deforms elastically in compressive loading and further tensile loading. Similar effect is observed in rate independent case. After tensile loading, the material deforms and as the yield stress values increases and at the same time the yield stress value during compressive loading decreases and further decreases during compressive unloading.

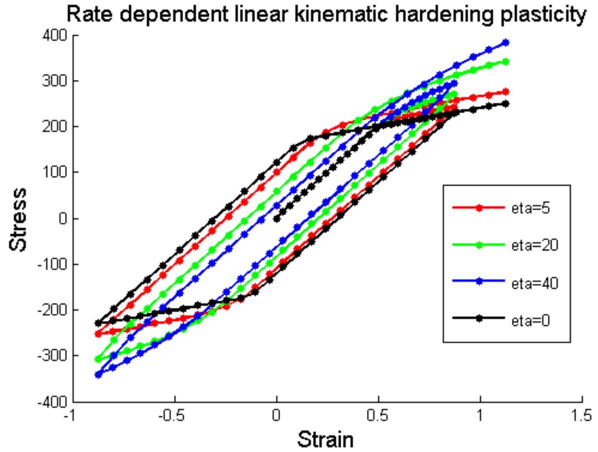

Figure 9: Rate dependent varying viscosity for linear kinematic hardening plasticity

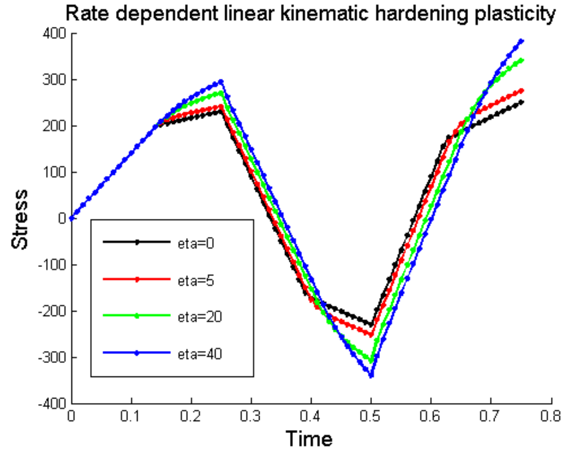

Figure 10: Rate dependent varying viscosity for linear kinematic hardening plasticity stress vs time

The above graph shows the influence of change in viscosity on the material with linear kinematic hardening case. It can be seen in stress vs strain curve that as η (viscosity) increases, the slope of plastic deformation increases. When η decreases, the yield stress for compressive loading decreases. The stress vs time plot shows that increase in η gives higher yield stress for the given time. This effect of viscosity is observed in all rate dependent cases.

<u>Case 4: Nonlinear isotropic hardening plasticity</u>

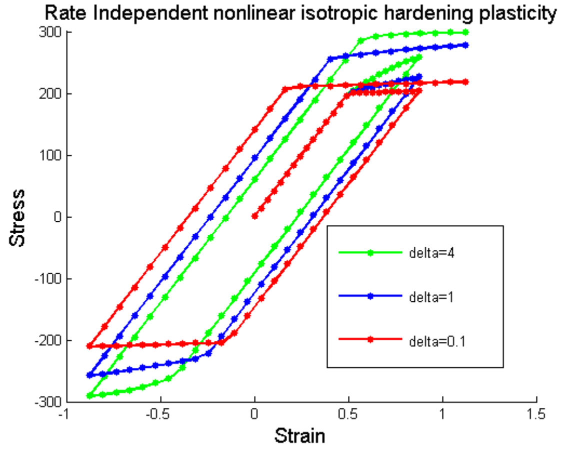

Figure 11: 1D Rate independent nonlinear isotropic hardening plasticity

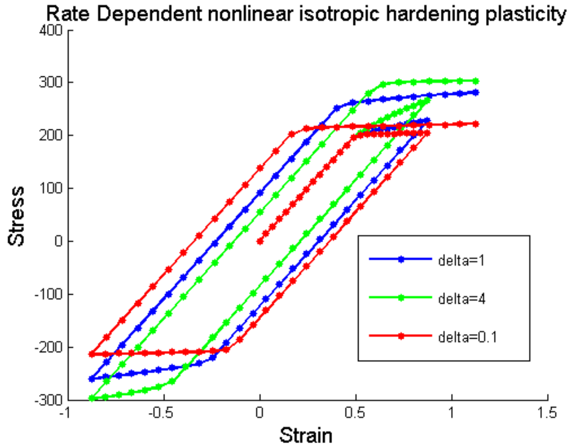

Figure 12: 1D Rate dependent linear isotropic hardening plasticity

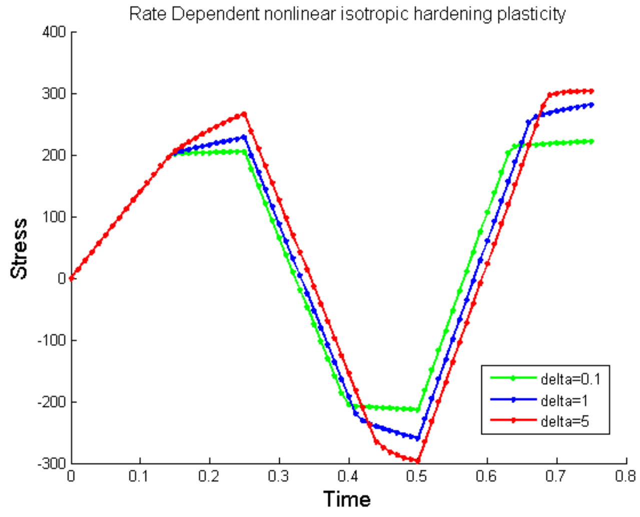

Figure 13: 1D Rate dependent linear isotropic hardening plasticity stress vs time

In the above cases it can be seen that by changing exponential hardening coefficient δ the material behavior changes. The above graphs are plot for $\delta = 0.1$, $\delta = 1$, and $\delta = 4$ for both, rate independent and rate dependent isotropic hardening case. In both cases it is observes that as the exponential hardening coefficient increases, the material tries to deform faster and tries to reach $\sigma_{inf} = 300\ Pa$ (ultimate tensile strength) faster, but ofcourse does not cross it. Rate dependent case shows that it reaches faster to σ_{inf} than rate independent case due to the viscous effect of the material. The slope of plastic deformation is more when the material is made subject to tensile loading, then the slope decreases when it is given further compressive loading. As the value of δ is decreased, the material behaves more and more plastic. At $\delta = 0$, the material shows perfect plasticity. It can be seen from the rate dependent stress vs time plot for varying exponential hardening coefficient that as δ increases, yield stress increases for given time.

Case 5: Nonlinear isotropic and linear kinematic hardening plasticity

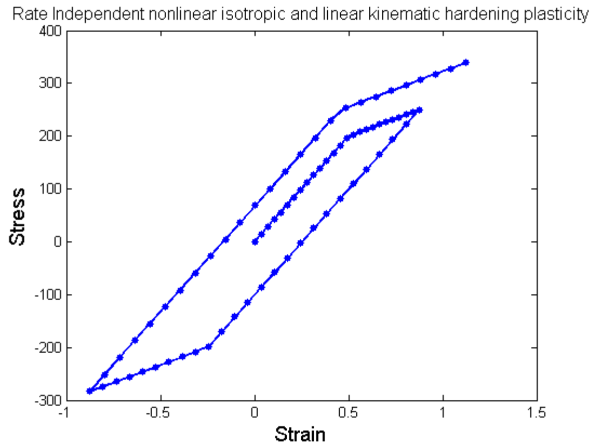

Figure 14: 1D Rate independent Nonlinear isotropic and linear kinematic hardening plasticity

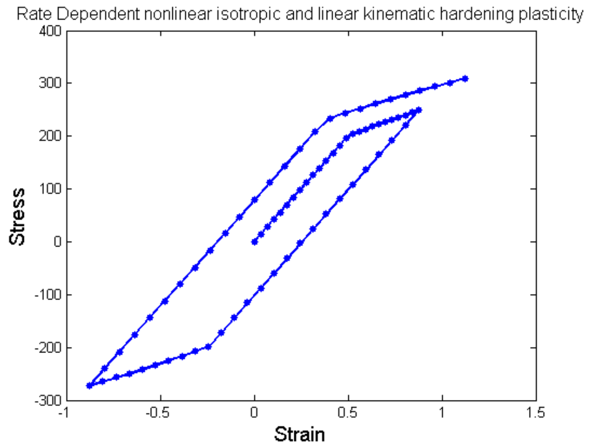

Figure 15: 1D Rate dependent Nonlinear isotropic and linear kinematic hardening plasticity

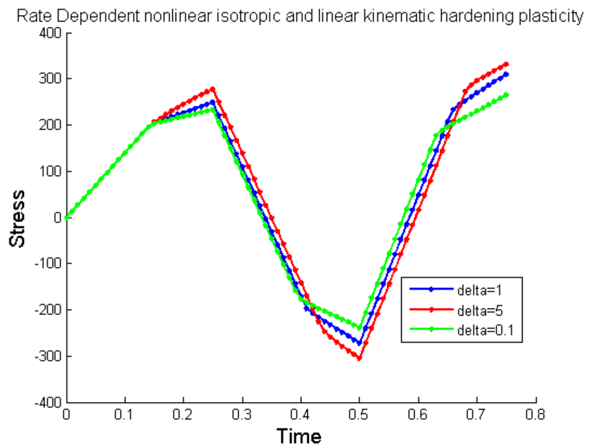

Figure 16: 1D rate dependent Nonlinear isotropic and linear kinematic hardening plasticity stress vs time varying δ

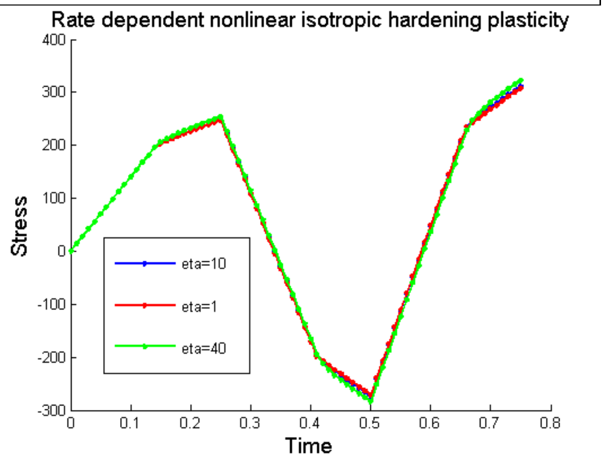

Figure 17: 1D rate dependent Nonlinear isotropic and linear kinematic hardening plasticity stress vs time varying η

In non-linear isotropic and linear kinematic hardening, it can be observed that, change in load rate has no effect on the material behavior during tensile loading and unloading. Change in viscosity has very less effect on the material behavior. As viscosity increases considerably, there is very less increase in yield stress of the material at a given time as seen above. From the figure of stress vs time for varying δ it can be seen that, change in exponential hardening coefficient δ, effects the behavior of material. As the value of δ increases, the size of yield stress increases uniformly at a given time while cyclic tensile loading and unloading. It crosses the value of $\sigma_{inf} = 300\ Pa$. This is due to linear kinematic hardening. Since there is not much effect of viscosity for rate dependent nonlinear isotropic and linear kinematic hardening plasticity, the loading/unloading graph for rate dependednt and rate independent case shows similar behaviour.

3D: Rate Independent /Rate Dependent Plasticity Model

The cases are solved for material properties, Young's Modulus E= 400 Pa, Yield stress = 200 Pa. The ultimate tensile strength in case of non-linear hardening is taken as $\sigma_{inf} = 300\text{Pa}$. Viscosity parameter =5. The value of uniaxial cyclic loading is taken as,

$$\sigma_{loading} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 350 & 0 & 0 & 0 & 0 & 0 \\ -350 & 0 & 0 & 0 & 0 & 0 \\ 450 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Case 1: Perfect Plasticity

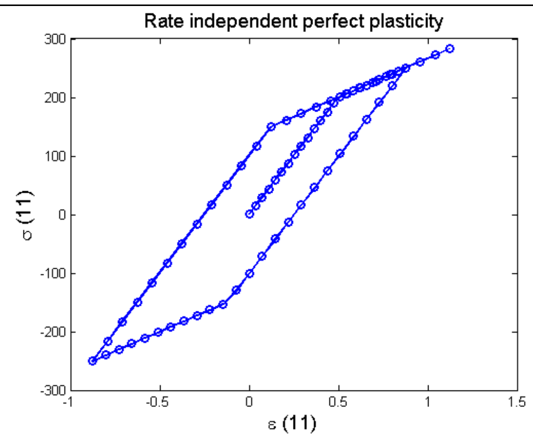

Figure 18: 3D Rate independent Perfect Plasticity

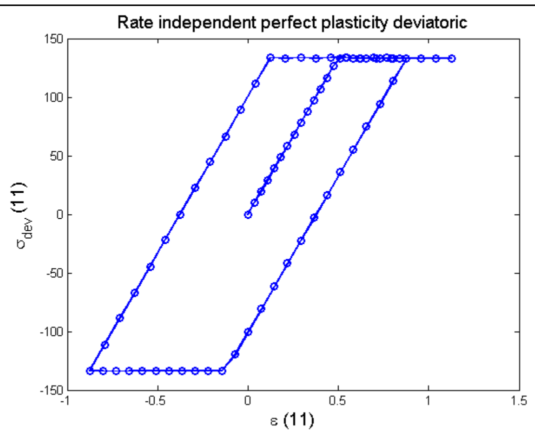

Figure 19: 3D Rate independent Perfect Plasticity

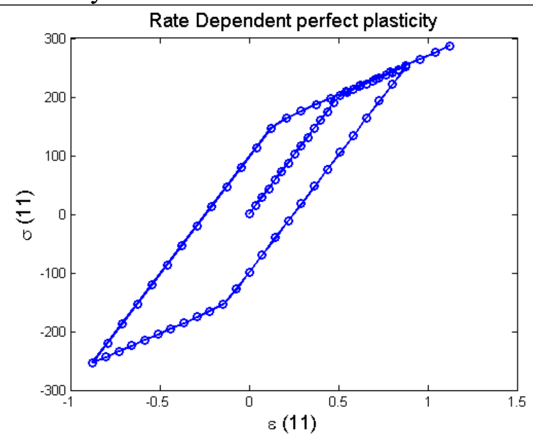

Figure 20: 3D Rate dependent Perfect Plasticity

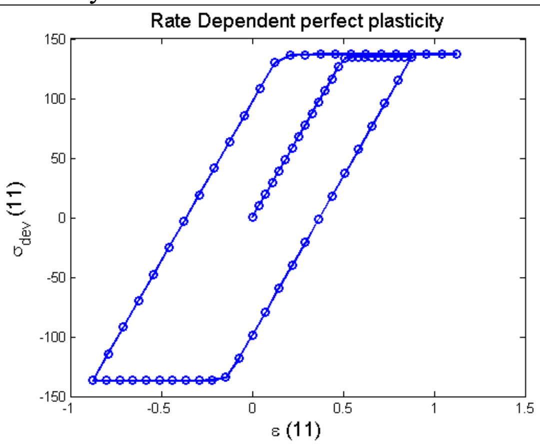

Figure 21: 3D Rate independent Perfect Plasticity

The above figures show the graphs for perfect plasticity of rate independent and rate dependent case. It can be seen that the deviatoric stress saturates as it reaches 2/3rd of yield stress. Hence perfect plasticity curve can be observed. But the plot for normal stress strain curve shows that on tensile loading the deformation occurs and yield surface increases on compressive loading. When stress (11) increases at intersection of yield surface and stress, stress (22) and stress (33) starts increasing which allows stress (11) to go beyond yield surface. The stress (11) vs strain (11) graphs for rate independent and dependent case with viscosity parameter of 5 are shown in the figures above. In the cases below, the material behavior would be discussed using deviatoric stress (11) graphs as it is easier to understand.

Case 2: Linear isotropic hardening plasticity

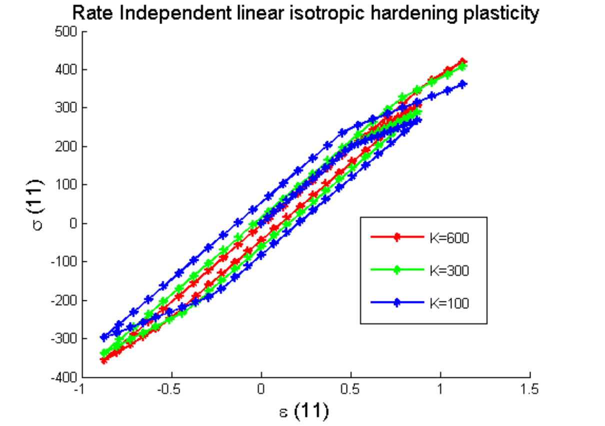

Figure 22: Rate independent Linear isotropic hardening plasticity

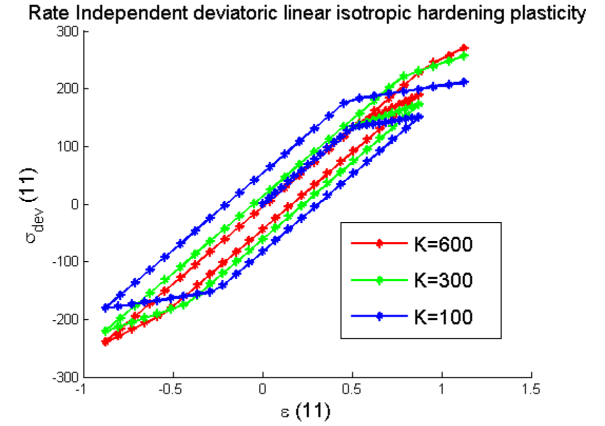

Figure 23: Rate independent deviatoric Linear isotropic hardening plasticity

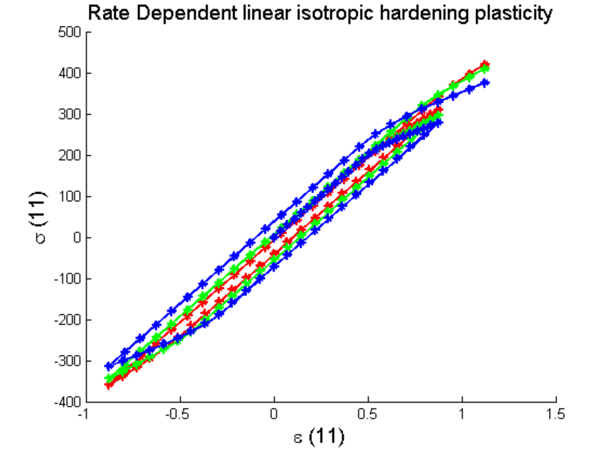

Figure 24: Rate dependent Linear isotropic hardening plasticity

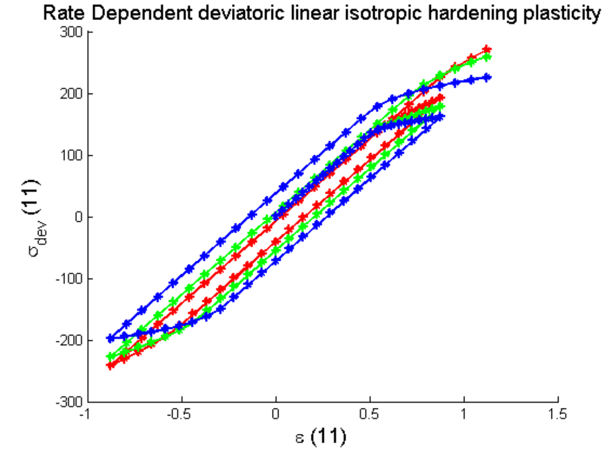

Figure 25: Rate independent deviatoric Linear isotropic hardening plasticity

In the above figure, material is made subject to isotropic hardening parameter K=100, 300 and 600. It can be seen that as isotropic hardening parameter 'K' increases, the slope of plastic deformation of the material increases. As K tends to zero, the material tries to behave perfectly plastic. If material is loaded in tension past yield, and then loaded in compression, it will not yield in compression until it reaches the level past deviatoric yield stress which is 2/3rd of yield stress that was reached when it was loaded in tension. As K tends to infinity, the slope of plastic deformation increases and the size of yield stress increases during uniaxial tensile/compressive loading and unloading due to plastic deformation as shown in the figure below. This means that on further cycles of tensile loading and unloading, the material will eventually deform along elastic line of stress strain curve.

From the figure given for deviatoric stress vs time, it can be observed that with increase in viscosity parameter η the value of deviatoric stress also increases at the given time. A stress vs time graph for varying K is also shown. It is observed that increase in isometric parameter, the yield stress increases.

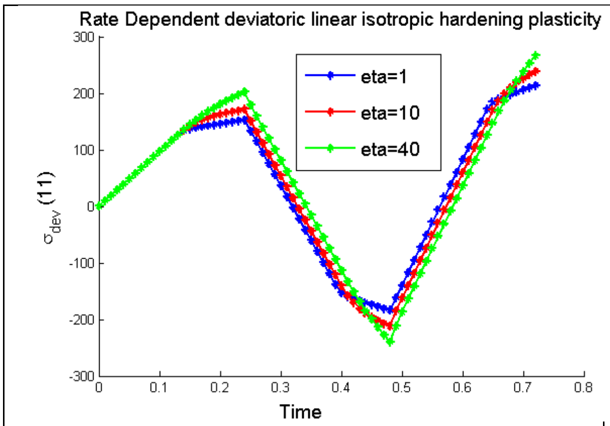

Figure 26: Rate dependent deviatoric Linear isotropic hardening plasticity

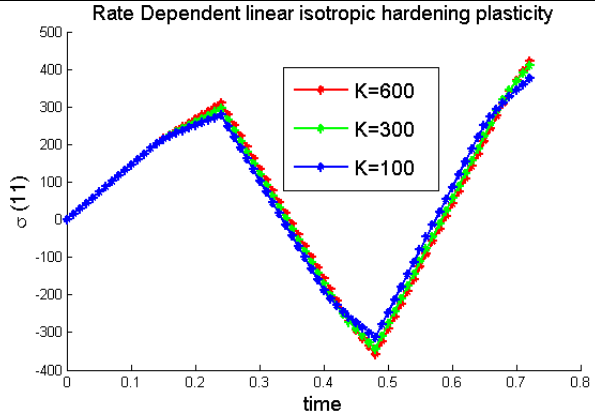

Figure 27: Rate dependent Linear isotropic hardening plasticity

<u>Case 3: Linear kinematic hardening plasticity</u>

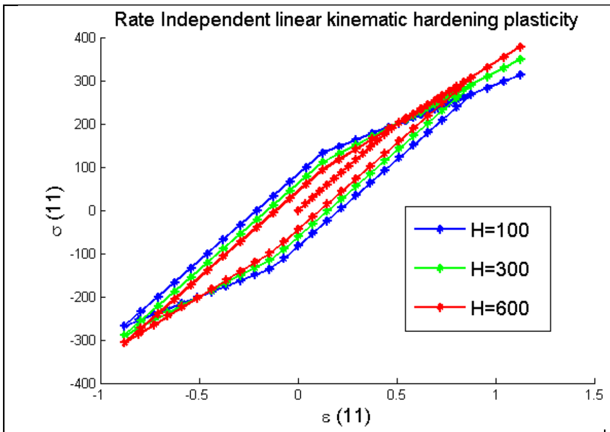

Figure 28: Rate independent Linear kinematic hardening plasticity

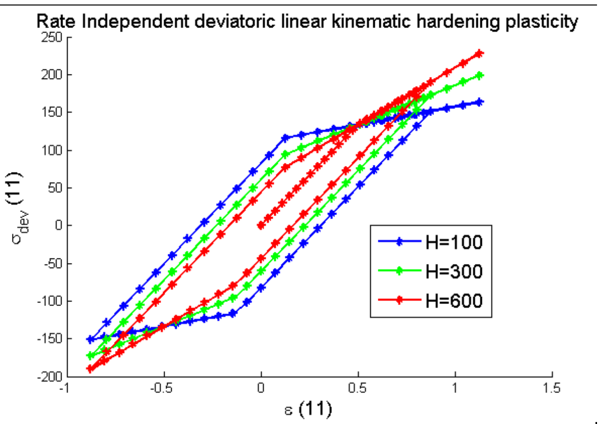

Figure 29: Rate independent dev Linear kinematic hardening plasticity

Here the material shows the **Bauschinger effect**. In rate dependent case, the graph is observed by varying the kinematic hardening parameter ‘H’ as H=100, H=300 and H=600. It can be observed that, as H increases the material softens on compression. On compressive loading, the size of yield stress is observed to decrease and it further decreases with further tensile loading. Hence the material deforms elastically in compressive loading and further tensile loading. Similar effect is observed in rate dependent case. After tensile loading, the material deforms and as the yield stress values increases and at the same time the yield stress value during compressive loading decreases and further decreases during compressive unloading. The variation in viscosity parameter in rate dependent case is gives very less effect on material behavior under kinematic hardening.

In the following figure the behavior of material by change in load rate is shown for rate independent normal stress and deviatoric stress case. It can be observed that, as the load rate is decreased i.e. when δt is increased very high, the material reaches quasi-static condition as observed in deviatoric stress plot.

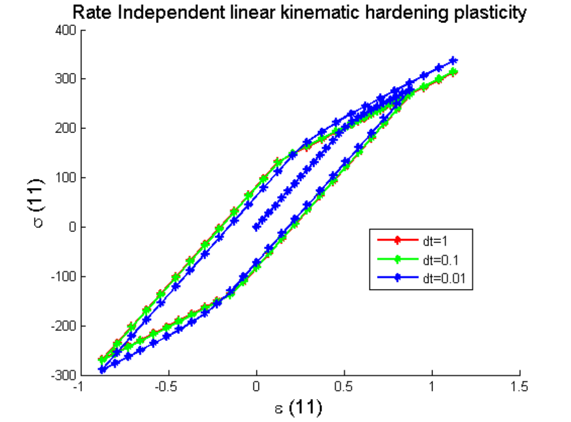

Figure 30: Rate independent Linear kinematic hardening plasticity

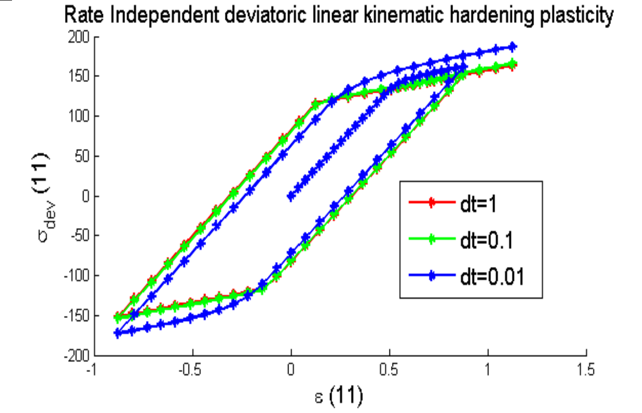

Figure 31: Rate independent dev Linear kinematic hardening plasticity

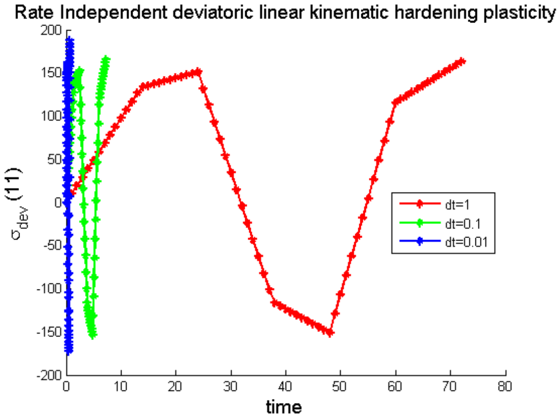

Figure 32: Rate independent dev Linear kinematic hardening plasticity stress vs time

Case 4: Non Linear isotropic hardening plasticity

For non-linear isotropic hardening case, rate dependent model is discussed. It can be seen that by changing exponential hardening coefficient δ the material behavior changes. The below graphs are plot for $\delta = 1$, $\delta = 10$, and $\delta = 30$ for both, rate independent and rate dependent isotropic hardening case. In both cases it is observes that as the exponential hardening coefficient increases, the material tries to deform faster and σ_{dev} tries to reach $2/3^{rd}$ of $\sigma_{inf} = 300\,Pa$ (ultimate tensile strength) faster, but ofcourse does not cross it. Rate dependent case shows that it reaches faster to σ_{inf} than rate independent case due to the viscous effect of the material. The slope of plastic deformation is more when the material is made subject to tensile loading, then the slope decreases when it is given further compressive loading. As the value of δ is decreased, the material behaves more and more plastic. At $\delta = 0$, the material shows perfect plasticity. It can be seen from the rate dependent stress vs time plot for varying exponential hardening coefficient that as δ increases, yield stress increases for given time.

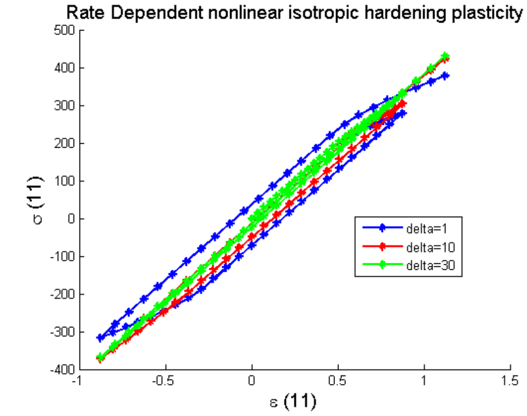

Figure 33: Rate dependent Non Linear isotropic hardening plasticity

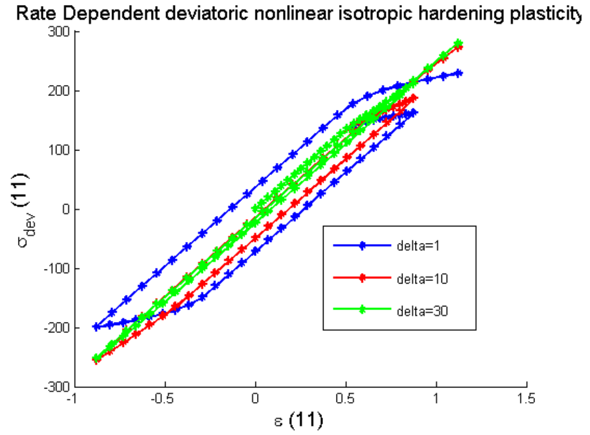

Figure 34: Rate dependent dev Non Linear isotropic hardening plasticity

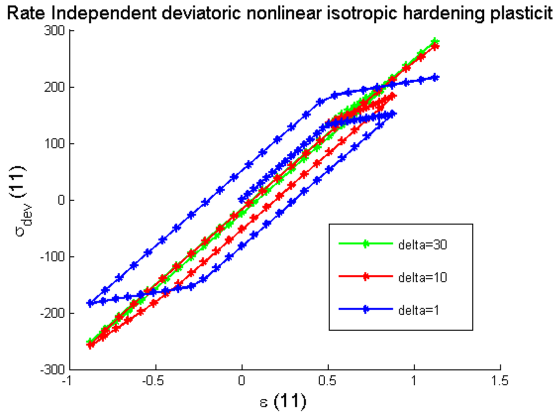

Figure 35: Rate dependent dev Non Linear isotropic hardening plasticity

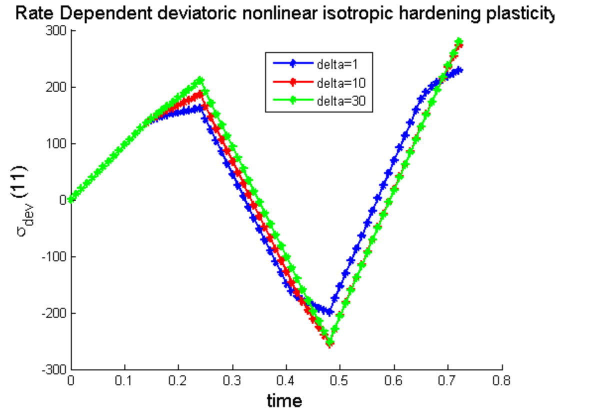

Figure 36: Rate dependent dev Non Linear isotropic hardening plasticity

Case 5: Non Linear isotropic and linear kinematic hardening plasticity

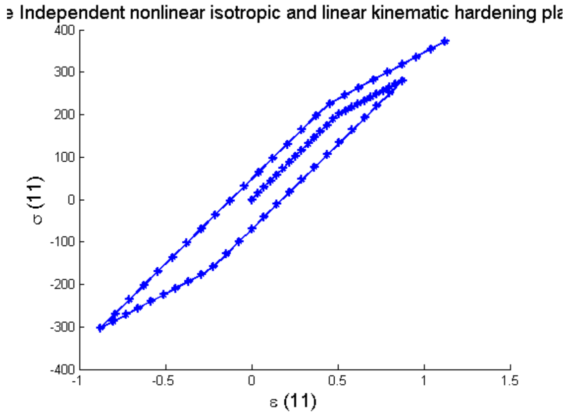

Figure 37: Rate independent Non Linear isotropic and linear kinematic hardening plasticity

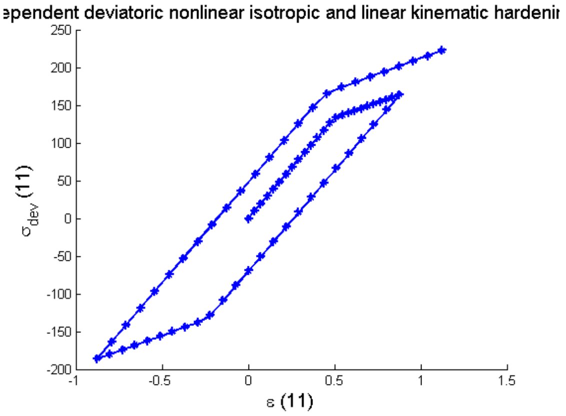

Figure 38: Rate independent dev Non Linear isotropic and linear kinematic hardening plasticity

It was observed that, change in load rate has no effect on the material behavior. Change in viscosity has very less effect on the material behavior. As viscosity increases considerably, there is very less increase in 2/3[rd] of yield stress of the material for deviatoric stress at a given time. From the figure of stress vs time for varying δ it can be seen that, change in exponential hardening coefficient δ, effects the behavior of material. As the value of δ increases, the size of yield stress increases uniformly at a given time while cyclic tensile loading and unloading. It crosses the value of $\sigma_{inf} = 300\ Pa$. This is due to linear kinematic hardening.

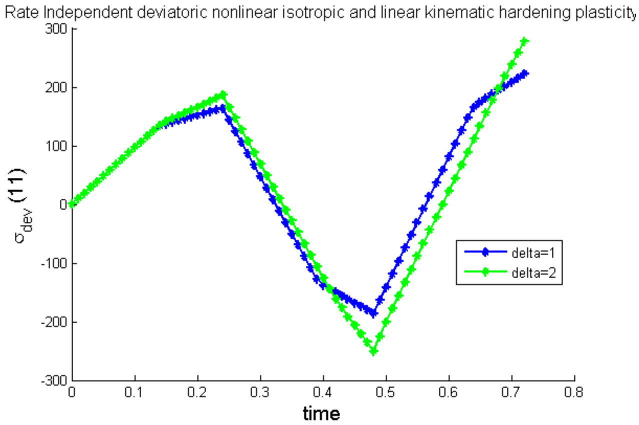

Figure 39: Rate independent Non Linear isotropic and linear kinematic hardening plasticity

Rate Dependent case for producing perfect plasticity as in rate independent model

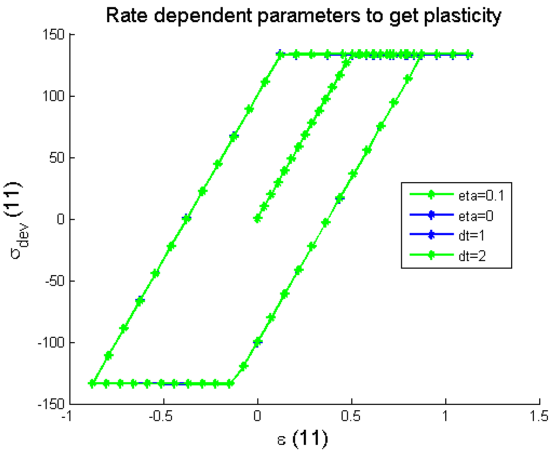

Figure 40: Rate dependent case to produce perfect plasticity by varying eta and load rate

The above case is simulated by varying viscosity parameter once it is taken as 0.01 and once 0.1 and by changing the load rate i.e. by changing the dt=1 and dt=2. Normally the dt was taken as 10e-2. At such high dt the material behaves quasi static, and rate dependent case behaves as perfectly plastic. Thus decreasing the load rate makes the material plastic. Decreasing the viscosity has the similar effect on the material. When viscosity tends to zero, the material becomes fully plastic.

APPENDIX

Matlab code for 1D rate dependent/ independent plasticity model

Code for RI1D.m:

```
clear all
clc
%-----------Material Properties are assumed as follows---------------%
E=400;                  %young's modulus
yield=200;              %yield stress
sigma_infy=300;
delta=1;
dt=5;
eta=5;                  %viscosity
K=100;                   %Isotropic Hardening
H=100;                   %Kinematic Hardening
%--------------------------------------------------------------------%
disp('[1] Rate independent/dependent plasticity Linear case')
disp('[2] Rate independent/dependent plasticity Nonlinear case')
method=input('Choose method to be solved:');

%---------------------Loading----------------------------------------%
load=[350;-350;450];
load_Steps=size(load,1) ;
timeSteps=25*ones(1,load_Steps);
%-------------------------------------------------------------------
strain =getStrain(E,load,timeSteps);

if method==1

trail_sigma = zeros(1 , length(strain));
trial_zeta = zeros(1 , length(strain));
sigma = zeros(1 , length(strain));
trial_f = zeros(1 , length(strain));
qbar = zeros(1 , length(strain));
alpha = zeros(1 , length(strain));
eps = zeros(1 , length(strain));

for n=1 : length(strain)-1
    trail_sigma(n+1) = E * (strain(n+1) - eps(n)) ;
    trial_zeta(n+1) = trail_sigma(n+1) - qbar(n) ;
    trial_f(n+1) = abs(trial_zeta(n+1)) - ( yield + K * alpha(n) ) ;
    if trial_f(n+1) <= 0
        sigma(n+1) = trail_sigma(n+1) ;
        eps(n+1) = eps(n);
        qbar(n+1) = qbar(n);
        alpha(n+1) = alpha(n) ;
    else
        delta_gamma = ramp_fn(trial_f(n+1)) / (E + K + H + eta / dt) ;
        sigma(n+1) = trail_sigma(n+1) - delta_gamma * E *sign...
                                                (trial_zeta(n+1)) ;
        eps(n+1) = eps(n) + delta_gamma * sign(trial_zeta(n+1)) ;
        alpha(n+1) = alpha(n) + delta_gamma ;
        qbar(n+1) = qbar(n) + delta_gamma * H * sign(trial_zeta(n+1)) ;

    end
end

elseif method==2
```

```
[sigma,eps] = RtrnMapViscoExpo(E,H,K,strain,yield,eta,sigma_infy,delta);

end

time=0:dt:(dt*25*load_Steps);
hold on
% figure
plot(strain,sigma,'b .-','LineWidth',2,'MarkerSize',15)
xlabel('Strain','FontSize',14);
ylabel('Stress','FontSize',14);
title('Rate Independent nonlinear isotropic hardening
plasticity','FontSize',14);

figure
plot(time,sigma,'r *-','LineWidth',1.5,'MarkerSize',4)
xlabel('Time','FontSize',14);
ylabel('Stress','FontSize',14);
title('Rate Independent nonlinear isotropic hardening
plasticity','FontSize',14);
```

Code for RtrnMapViscoExpo.m

```
function [sigma,eps] = RtrnMapViscoExpo(E,H,K,strain,yield,eta,...
                                                        sigma_infy,delta)


dt=1e-1;
trail_sigma =          zeros(1 , length(strain)) ;
trial_zeta  =          zeros(1 , length(strain)) ;
sigma       =          zeros(1 , length(strain)) ;
trial_f     =          zeros(1 , length(strain)) ;
qbar        =          zeros(1 , length(strain)) ;
q           =          zeros(1 , length(strain)) ;
alpha       =          zeros(1 , length(strain)) ;
eps         =          zeros(1 , length(strain)) ;
gamma       =          zeros(1 , length(strain)) ;

for n=1 : length(strain)-1
    trail_sigma(n+1)  = E * (strain(n+1) - eps(n)) ;
    trial_zeta(n+1)   = trail_sigma(n+1) - qbar(n) ;
    q(n)              = - pi(alpha(n),delta,sigma_infy,yield) ;
    trial_f(n+1)      = abs(trial_zeta(n+1)) - yield + q(n) ;
    if trial_f(n+1) <= 0
        sigma(n+1)    = trail_sigma(n+1) ;
        eps(n+1)      = eps(n);
        qbar(n+1)     = qbar(n);
        alpha(n+1)    = alpha(n) ;
    else

        gamma(n+1)    = NRMethod(trial_f(n+1),dt,E,H,eta,alpha(n),delta,...
                                                        sigma_infy,yield);
        sigma(n+1)    = trail_sigma(n+1) - gamma(n+1)*dt * E *sign...
                                                        (trial_zeta(n+1));
        eps(n+1)      = eps(n) + gamma(n+1)*dt * sign(trial_zeta(n+1));
        alpha(n+1)    = alpha(n) + gamma(n+1)*dt;
        qbar(n+1)     = qbar(n) + gamma(n+1)*dt * H * sign(trial_zeta(n+1));

    end
end
```

Code for NRMethod.m :

```
function gamma2=NRMethod(sigma_infy,gamma, eta ,delta,dt, yield,H ,alpha ,
E)
         gamma1=0;
         rela_Error=1;

while (rela_Error>1e-13)

    gamma2    =     gamma1-g_fn(gamma1,trial_f,dt,E,H,eta,alpha,delta,...
                    sigma_infy,yield)/Dg_fn(gamma1,dt,E,H,eta,alpha,delta,...
                    sigma_infy,yield);

    rela_Error =    abs(gamma2-gamma1);

    gamma1    =     gamma2;

end
end
```

Code for getStrain.m:

```
function strain=getStrain(E,stress,istep)

stress=[0;stress];
strain_step = zeros(size(stress,1),1);

for   I = 1:size(stress,1)-1

    sigma_0 =stress(I+1,1);
    strain1=sigma_0/E;
    strain_step(I+1,1)=strain1;
end

[strain] = calstrain_IN(istep,strain_step);
end
```

Code for pi.m:

```
function value=pi(,delta, yield ,sigma_infy, alpha)

value = (sigma_infy - yield)*(1 - exp(-delta*alpha)) ;

end
```

Code for calstrain_IN.m:

```
function [strain]=calstrain_IN(istep,STRAIN)

mstrain = size(STRAIN,2) ;
strain = zeros(sum(istep)+1,mstrain) ;
acum = 0 ;
PNT = STRAIN(1,:) ;
for iloc = 1:length(istep)
```

```
    INCSTRAIN = STRAIN(iloc+1,:)-STRAIN(iloc,:);
    for i = 1:istep(iloc)
        acum = acum + 1;
               PNT = PNT + INCSTRAIN/istep(iloc);
        strain(acum+1,:) = PNT ;
    end

end
end
```

Code for g_fn.m:

```
function value =g_fn(sigma_infy,gamma, eta ,delta,dt, yield,H ,alpha , E,f)

value = trial_f - gamma * dt * (E+H+eta/dt) -...
            (pi(alpha + gamma * dt,delta,sigma_infy,yield) -...
            pi(alpha,delta,sigma_infy,yield));
end
```

Code for Dg.m:

```
function value = Dg_fn(sigma_infy,gamma, eta ,delta,dt, yield,H ,alpha , E)

value =  - dt * (E + (sigma_infy - yield) * delta *...
            exp(- delta * (alpha + gamma * dt)) + H + eta / dt);

end
```

Matlab code for 3D rate dependent/ independent plasticity model

Code for 3D.m:

```
%clear all
clc
young=400;
nu=0.3;
yield=200;

mu = young / ( 2 * (1 + nu) ) ;

sigma_infy=300;
delta=10;
eta=0.01;
K=00;
H=00;
disp('[1] Linear case')
disp('[2] Nonlinear case')
method=input('Choose method:');


load=[0 0 0 0 0 0;350 0 0 0 0 0;-350 0 0 0 0 0;450 0 0 0 0 0];
load_Steps=size(load,1)-1 ;
timeSteps = 25*ones(1,load_Steps);

dt            =   1 ;

strain =getStrain(young,nu,load,timeSteps);


[ce] = elastic_tensor(young,nu);

if method==1


trail_sigma =   zeros(6 , size(strain,2)) ;
trial_zeta  =   zeros(6 , size(strain,2)) ;
sigma       =   zeros(6 , size(strain,2)) ;
trial_f     =   zeros(1,  size(strain,2)) ;
qbar        =   zeros(6 , size(strain,2)) ;
q           =   zeros(1,  size(strain,2)) ;
eps         =   zeros(6 , size(strain,2)) ;

for n=1 : size(strain,2)-1

    SSS = ce * (strain(:,n+1) - eps(:,n)) ;
    trail_sigma(:,n+1) =  SSS ;
    ZZZ = dev_sigma(trail_sigma(:,n+1)) ;
    trial_zeta(:,n+1) = ZZZ - qbar(:,n);
    trial_f(n+1) = norm(trial_zeta(:,n+1)) - sqrt(2/3)*( yield - q(n) ) ;

    if trial_f(:,n+1) <= 0
        eps(:,n+1) = eps(:,n);
        qbar(:,n+1) = qbar(:,n);
        q(n+1) = q(n) ;
        sigma(:,n+1) = trail_sigma(:,n+1);
```

```
    else
        delta_gamma = trial_f(:,n+1) / (2*mu + 2/3*K + 2/3*H + eta/dt) ;
        sigma(:,n+1) = trail_sigma(:,n+1) - delta_gamma * 2*mu *
unit_vector(trial_zeta(:,n+1));
        eps(:,n+1) = eps(:,n) + delta_gamma *
unit_vector(trial_zeta(:,n+1)) ;
        qbar(:,n+1) = qbar(:,n) + delta_gamma * 2/3*H *
unit_vector(trial_zeta(:,n+1)) ;
        q(n+1) = q(n) - delta_gamma * sqrt(2/3)*K ;
    end
end



elseif method==2

sigma = RtrnMapViscoExpo(ce,H,K,strain,yield,mu,eta,sigma_infy,delta);

end

for I=1:size(strain,2)

    devstress(:,I) = dev_sigma(sigma(:,I)) ;
end

time=0:dt:(dt*24*load_Steps);
figure (1)
hold on
plot(time(1,:),devstress(1,:),'g *-','Linewidth',2)
title('Rate Independent deviatoric nonlinear isotropic and linear kinematic
hardening plasticity','FontSize',14)
xlabel('time','FontSize',14) % x-axis label
ylabel('\sigma_{dev} (11)','FontSize',14) % y-axis label

figure (2)
hold on
plot(time(1,:),sigma(1,:),'g *-','LineWidth',2)
title('Rate Independent nonlinear isotropic and linear kinematic hardening
plasticity','FontSize',14)
xlabel('time','FontSize',14) % x-axis label
ylabel('\sigma (11)','FontSize',14) % y-axis label

figure(3)
hold on
plot(strain(1,:),devstress(1,:),'g *-','Linewidth',2)
title('Rate Independent deviatoric nonlinear isotropic and linear kinematic
hardening plasticity','FontSize',14)
xlabel('\epsilon (11)','FontSize',14) % x-axis label
ylabel('\sigma_{dev} (11)','FontSize',14) % y-axis label

figure (4)
hold on
plot(strain(1,:),sigma(1,:),'g *-','LineWidth',2)
title('Rate Independent nonlinear isotropic and linear kinematic hardening
plasticity','FontSize',14)
xlabel('\epsilon (11)','FontSize',14) % x-axis label
ylabel('\sigma (11)','FontSize',14) % y-axis label
```

Code for unit_vector.m

```
function [value] = unit_vector(vector)

value = vector / norm(vector) ;

return
```

Code for NRM.m

```
function gamma2=NRM(trial_f,dt,mu,H,eta,alpha,delta,sigma_infy,yield)
gamma1=0;
relErr=1;

while (relErr>1e-13)

    gamma2=gamma1-
g_fn(gamma1,trial_f,dt,mu,H,eta,alpha,delta,sigma_infy,yield)...
        /Dg_fn(gamma1,dt,mu,H,eta,alpha,delta,sigma_infy,yield);
    relErr=abs(gamma2-gamma1);
    gamma1=gamma2;

end

end
```

Code for chk .m

```
clc
clear all
sigma=[0 0 0 0 0 0;
       30 10 -8 0 0 0;
       0 0 8 0 0 0];
   bigst=[];
n=4;
for i=1:size(sigma,1)-1
    tstart=sigma(i,:);
    tend=sigma(i+1,:);
    strain=[];
for j=1:6
    strain=[strain,linspace(tstart(j),tend(j),n)'];
end
strain=strain(1:end-1,:);
bigst=[bigst;strain];
end
bigst(end+1,:)=sigma(end,:);
```

Code for calstrain_IN .m

```
 function [strain]=calstrain_IN(istep,STRAIN)

mstrain = size(STRAIN,2) ;
strain = zeros(sum(istep)+1,mstrain) ;
acum = 0 ;
PNT = STRAIN(1,:) ;
for iloc = 1:length(istep)
    INCSTRAIN = STRAIN(iloc+1,:)-STRAIN(iloc,:);
    for i = 1:istep(iloc)
        acum = acum + 1;
               PNT = PNT + INCSTRAIN/istep(iloc);
        strain(acum+1,:) = PNT ;
```

```
    end

end
end
```

Code for dev_sigma .m

```
function [value]=dev_sigma(stress)

trace = (stress(1) + stress(2) + stress(3)) / 3 ;

value(1) = stress(1) - trace ;
value(2) = stress(2) - trace ;
value(3) = stress(3) - trace ;
value(4) = stress(4) ;
value(5) = stress(5) ;
value(6) = stress(6) ;

value = value' ;

return
```

Code for Dg_fn .m

```
function value = Dg_fn(gamma,dt,mu,H,eta,alpha,delta,sigma_infy,yield)

value =  - dt * (2*mu + 2/3*(sigma_infy - yield) * delta *...
            exp(- delta * (alpha + gamma * dt * sqrt(2/3))) + 2/3*H + eta /
dt);

end
```

Code for elastic_tensor .m

```
function [ce] = elastic_tensor (E,nu)
%*************************************************************************
***********
%*        Elastic constitutive tensor
%*
%*************************************************************************
***********


%*************************************************************************
***********
%
%*                   mu --------->  Shear modulus
%*
%*
%*
mu = E / ( 2 * (1 + nu) ) ;
lamda = E * nu /( (1 + nu) * (1 - 2 * nu) ) ;

        ce     = zeros(6,6);            % Init.
        C1 = lamda + 2 * mu;
```

```
        ce(1,1) =   C1;
        ce(2,2) =   C1;
        ce(3,3) =   C1;
        ce(1,2) =   lamda;
        ce(1,3) =   lamda;
        ce(2,3) =   lamda;
        ce(4,4) =   mu;
        ce(5,5) =   mu;
        ce(6,6) =   mu;
        ce(2,1) =   lamda;
        ce(3,1) =   lamda;
        ce(3,2) =   lamda;

return
```

Code for g_fn .m

```
function value =
g_fn(gamma,trial_f,dt,mu,H,eta,alpha,delta,sigma_infy,yield)

value = trial_f - gamma * dt * (2*mu + 2/3*H + eta/dt) -...
           sqrt(2/3)*(phi(alpha + gamma * dt *
sqrt(2/3),delta,sigma_infy,yield) -...
            phi(alpha,delta,sigma_infy,yield));
end
```

Code for getStrain .m

```
function strain=getStrain(E,nu,sigma,istep)

ce=elastic_tensor(E,nu);
bigst=[];
n=4;
for i=1:size(sigma,1)-1
    tstart=sigma(i,:);
    tend=sigma(i+1,:);
    s=[];
for j=1:6
    s=[s,linspace(tstart(j),tend(j),istep(i))'];
end
s=s(1:end-1,:);
bigst=[bigst;s];
end
bigst(end+1,:)=sigma(end,:);
strain=ce\bigst';
end
```