Computational Solid Mechanics Assignment1

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Introduction

Continuum damage models have been widely accepted for simulating the behavior of materials whose mechanical properties are degrading because of the presence of small cracks which propagate during the loading process.

This report shows how to implement a Matlab program about the algorithm of continuum damage constitutive models.Before the implementation starts, some important concepts,such as damage surface,effective stress,elastic domain,Kuhn-Tucker loading/unloading condition, etc,have been reviewed as a background.

Implementation of the Matlab Program

Firstly,we follow the guidance from slides to complete the Matlab program. The program can be totally categorized into seven parts,including executable files and input data, strain history input,plotting damage surface,integration algorithm,return mapping algorithm,viscous case, graphically output representation.

1. Executable File and Input Data
   We have 2 executable files,**main_nointeractive.m** with no user interface and main.m with user interface. Here we choose the former to input the the relevant parameters such as Young's modulus,Possion's coefficient,hardening/ softening modulus,yield stress,type of analysis(plain stress,**plane strain** or 3D),and type of damage surface(symmetric model, **tensile-damage-only model** and **non-symmetric model**).

   Since the type of symmetric damage surface has been implemented, here we just show the other two model.

   (a) Tensile-damage-only model
   Similarly to symmetric model,where we compute the equivalent strain(energy

norm of the strain tensor), here we should use the positive counterpart of a stress tensor using MacAuley bracket. Thus,

$$\tau_{\varepsilon}^{+} = \sqrt{\bar{\sigma}^{+} : \varepsilon}, \quad \bar{\sigma}^{+} = \langle \bar{\sigma} \rangle$$

The code in $Models\_de\_dano1.m$ is as follows:

$$\begin{array}{l} elseif(MDtype == 2) \\ sigma = ce * eps\_n1'; \\ sigma\_plus(sigma\_plus < 0) = 0; \\ rtrial = sqrt(eps_n1 * sigma\_plus); \end{array}$$

Next, $rtrial$ is sent to $rmap\_dano1.m$ and compared with r_n which is the previous equivalent strain, if $rtrial$ is larger than r_n, this is a loading process,and if not, it could be elastic load or unload process.

(b) Non-symmetric model

The only difference between non-symmetric model and the symmetric model is here

$$\tau_{\sigma} = [\theta + \frac{1-\theta}{n}]\sqrt{\sigma : C^{-1} : \sigma} \quad \theta = \frac{\sum \langle \bar{\sigma}_i \rangle}{\sum |\bar{\sigma}_i|}$$

The corresponding code in $Models\_de\_dano1.m$ is

$$\begin{array}{l} elseif(MDtype == 3) \\ sigma = eps\_n1 * ce; \\ denominat = sum(abs(sigma)); \\ numerat = sum(sigma(sigma > 0)); \\ theta = numerat/denominat; \\ coe = theta + ((1 - theta)/n); \\ rtrial = coe * sqrt(eps_n1 * ce * eps\_n1'); \\ end \end{array}$$

Besides, we also have to choose the type of hardening/softening law and finish the exponential case according to $lecture4, page13$.

$$q(r) = q_{\infty} - (q_{\infty} - r_0)e^{A(1-\frac{r}{r_0})} \quad A > 0$$

Here has the same sign to the hardening/softening modulus The corresponding code for this case in $rmap\_dano1.m$ is as follows:

$$\begin{array}{l} A = H; \\ q\_inf = zero\_q; \\ q\_n1 = q\_inf - (q\_inf - r0) * exp(A * (1 - (rtrial)/(r0))); \end{array}$$

2. Plotting Strain History and Damage Surface

The file plotting initial damage has almost been finished from the given program,here we just modify the function $dibujar\_criterio\_dano1.m$ so that we are able to plot the damage surface corresponding to the tensile-only-damage case, as well as non-symmetric case. Since here the we know that the damage surface is defined using polar coordinates

$$\sigma_1 = r(\theta)cos\theta \quad \sigma_2 = r(\theta)sin\theta$$

where

$$r = \frac{q}{\sqrt{\sigma_\theta^T C^{-1}\sigma_\theta}}$$

Here we just show the idea how to implement the code because the idea here is almost the same when we compute $rtrial$ in $Models\_de\_dano1.m$:

(a) Tensile-damage-only model
The difference between the tensile-damage-only model and symmetric model is whether there exists an MacAuley bracket. So

$$r = \frac{q}{\langle\sqrt{\sigma_\theta^T}\rangle C^{-1}\sigma_\theta}$$

(b) Non-symmetric model
Here what we have to do is to add a coefficient as we did before since the damage surface here is defined as

$$r = \frac{q}{[\theta + \frac{1-\theta}{n}]\sqrt{\sigma_\theta^T C^{-1}\sigma_\theta}}$$

It is easy to program the damage surface so here we don't present and explain it in detail.Readers can see the attachment corresponding to the report.

3. Viscous Case
According to Lecture5,pages 15 and 16,in the viscous case,the internal variable r should be updated with α and η, here we do not talk in detail about the formula about the change for r in this case.The full code given in $rmap\_dano1.m$.
Here some key point about the implementation can be noted:

(a) Time interval should be sent to function $rmap\_dano1.m$. So its new argument is like $rmap\_dano1(eps\_n1, hvar\_n, Eprop, ce, MDtype, n, \textbf{delta\_t})$, where **delta_t** is the time interval.

(b) Instead of rtrial in the inviscid case,**r_n1** equals to

$$((\eta - delta\_t*(1-\alpha))*r\_n)/(\eta+\alpha*delta\_t)+((rtrial*delta\_t)/(\eta+\alpha*delta\_t))$$

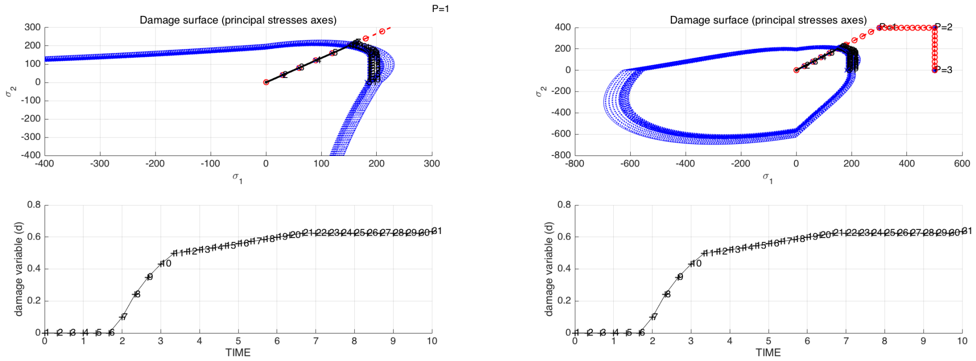
which should be changed in $rmap\_dano1.m$.

Assessment and Discussion

After completing the code according the theory basis, now we can start the assignment.

1. **Rate Independent Models**

(a) In this part, the first two ask us to implement to code for the "non-symmetric tension-compression damage" and "tension-only" damage model,linear and exponential hardening/softening law. All these work has been done previously in this report.As in figure 1 we can plot the result with exponential hardening/softening law if we run $main\_nointeractive.m$.



(a) Tension-only damage model with exponential hardening law

(b) Non-symmetric tension-compression damage model with exponential hardening law

Figure 1: Damage model and the damage parameter under a given loading

(b) In order to assess the correctness of the implementation: for each of the models we implemented before,we compute the path at the stress space and the stress-strain curve,corresponding to appropriate loading paths starting at the point $\sigma_1^{(0)} = 0; \sigma_2^{(0)} = 0$ and described by three-segment paths in the strain space $\Delta\varepsilon^{(1)} \to \Delta\varepsilon^{(2)} \to \Delta\varepsilon^{(3)}$. They are defined, in terms of their corresponding effective stress increments $\Delta\bar{\sigma}^{(1)} = C : \Delta\varepsilon^{(1)} \to \Delta\bar{\sigma}^{(2)} = C : \Delta\varepsilon^{(2)} \to \Delta\bar{\sigma}^{(3)} = C : \Delta\varepsilon^{(3)}$

- We firstly assess the tension-only damage model. This model does not take into account failure by compression,*i.e.* the material can only fail by tension.So if we can verify from our result that stress-strain curve is always a straight line, this means that our model works well.

Case 1:
The loading the as follows:
Uniaxial tensile loading:

$$\Delta\bar{\sigma}_1^{(1)} = 200, \quad \Delta\bar{\sigma}_2^{(1)} = 0$$

Uniaxial tensile unloading/compressive loading:

$$\Delta\bar{\sigma}_1^{(2)} = -300, \quad \Delta\bar{\sigma}_2^{(2)} = 0$$

Uniaxial compressive unloading/tensile loading:

$$\Delta\bar{\sigma}_1^{(3)} = 400, \quad \Delta\bar{\sigma}_2^{(3)} = 0$$

We set all the other parameters as default, and in order to input the strain history,we just press the button located in the lower left-hand part of the UI-figure and identified by the label “SELECT LOAD PATH”.We can input accurate stress by choose“BASE”.The result is obtained in figure 2.In the top part of figure 2, we can see the path of stress in alway zero for $\bar{\sigma}_2$, which is due to the uniaxial tensile/compressive loading/unloading condition prescribed before.

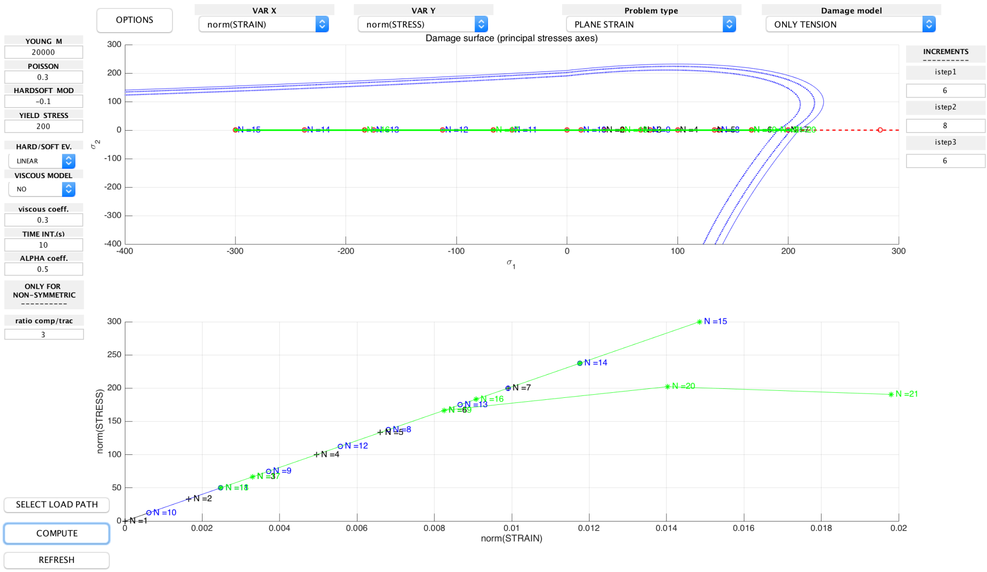


Figure 2: Tension-only damage model under uniaxial tensile/compressive loading/unloading

In the bottom part of figure 2, we can see clearly the behavior of this model:after applied a tensile loading,the stress rises proportionally to the strain in the elastic domain(under 200),from point label 1 to label 7. Secondly, an uniaxial tensile unloading/compressive loading is applied to the model,so the stress decrease in the elastic domain and then goes up to a compressive loading at 300(label 15). *We have to note that even the compressive stress exceeds the yield stress, it is still proportional to strain.* This means that this is a tension-only model, which fail by tension not compression. Finally, the loading is tensile uniaxial, so the stress path moves from point 15 goes down to point 18 and then. Since the loading stress exceeds the yield stress gradually, the model is damage, which explains why stress-strain curve at point 20 and 21 doesn't show significant change.

Case 2:

The loading the as follows:

Uniaxial tensile loading:

$$\Delta\bar{\sigma}_1^{(1)} = 200, \quad \Delta\bar{\sigma}_2^{(1)} = 0$$

Uniaxial tensile unloading/compressive loading:

$$\Delta\bar{\sigma}_1^{(2)} = -300, \quad \Delta\bar{\sigma}_2^{(2)} = -300$$

Uniaxial compressive unloading/tensile loading:

$$\Delta\bar{\sigma}_1^{(3)} = 400, \quad \Delta\bar{\sigma}_2^{(3)} = 400$$

We can follow the label number on the figure 3.At the bottom of the figure,because the first step the tensile loading is up to 200(yield stress),the

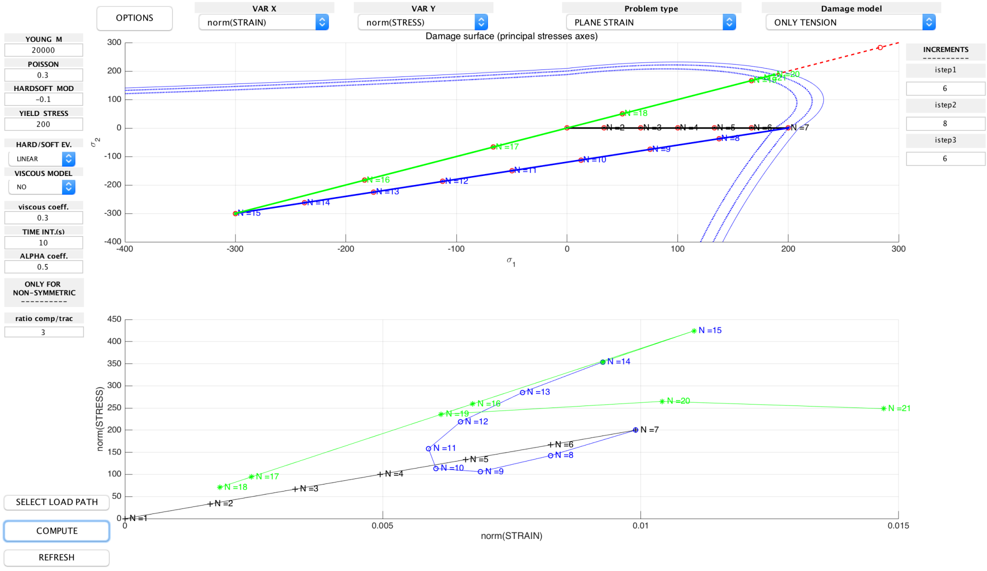


Figure 3: Tension-only damage model from uniaxial tensile loading to biaxial unloading

stress goes up proportionally to the strain where the we can see for point N=1 to 7. Then the stress changes the sign to negative 300 and now it becomes first a biaxial tensile unloading then a compressive loading press.That's what we can see for the blue part of stress-strain curve. The same to the result showed in case 1,*we have also to note that even the compressive stress exceeds the yield stress, it is still proportional to strain.* Finally,from previous state $\Delta\bar{\sigma}_1^{(2)} = -300, \quad \Delta\bar{\sigma}_2^{(2)} = -300$ to $\Delta\bar{\sigma}_1^{(3)} = 400, \quad \Delta\bar{\sigma}_2^{(3)} = 400$, the model experienced compressive unloading to tensile loading, and results is the green part of stress-strain curve.We can conclude that the model verifies very well and describes correctly the real loading/unloading process.

Case 3:

The loading is as follows:

Biaxial tensile loading:

$$\Delta\bar{\sigma}_1^{(1)} = 200, \quad \Delta\bar{\sigma}_2^{(1)} = 200$$

Uniaxial tensile unloading/compressive loading:

$$\Delta\bar{\sigma}_1^{(2)} = -300, \quad \Delta\bar{\sigma}_2^{(2)} = -300$$

Uniaxial compressive unloading/tensile loading:

$$\Delta\bar{\sigma}_1^{(3)} = 400, \quad \Delta\bar{\sigma}_2^{(3)} = 400$$

Similar assessment can be done here for case 3. We can follow the label number on the figure 4.At the bottom of the figure,because the first step the biaxial tensile loading is up to 200(yield stress),the stress goes up proportionally to the strain where the we can see for point N=1 to 7. Then the stress path moves towards to negative 300 and now it becomes first a biaxial tensile unloading then a compressive loading press.That's what we can

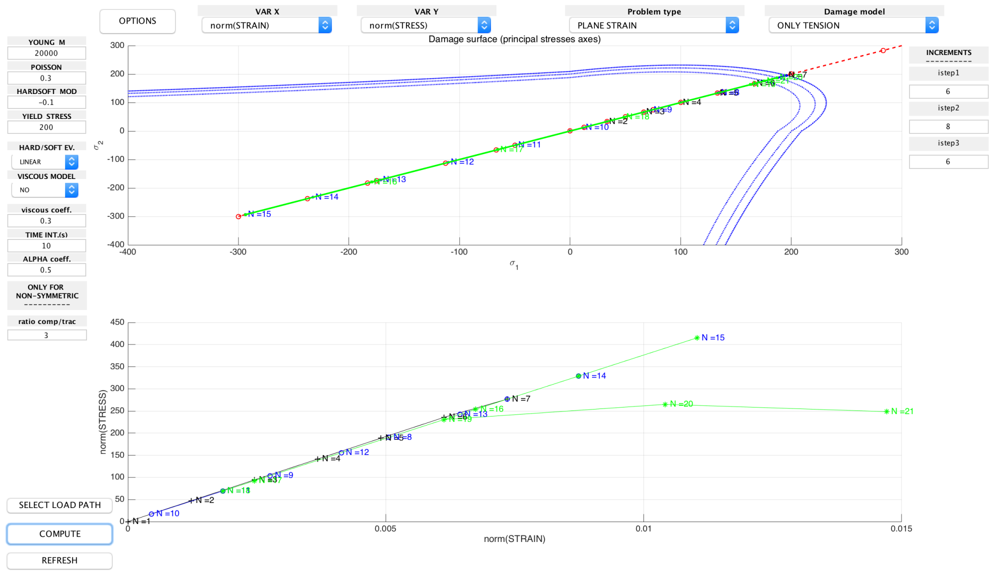


Figure 4: Tension-only damage model from uniaxial tensile loading to biaxial unloading

see for the blue part of stress-strain curve,decrease then increase. We see that during the compression process, the stress-strain curve always holds as a straight line. Finally,from previous state $\Delta\bar{\sigma}_1^{(2)} = -300, \quad \Delta\bar{\sigma}_2^{(2)} = -300$ to $\Delta\bar{\sigma}_1^{(3)} = 400, \quad \Delta\bar{\sigma}_2^{(3)} = 400$, the model experienced compressive unloading to tensile loading, and results is the green part of stress-strain curve.The failure comes out when the tension exceeds the yield stress. We again find that the path changing in stress space describe the real process and the model works well for tension-only damage model.

- We secondly assess the non-symmetrical damage model for materials such as concerete,whose tension domain differs with respect to compression.(Set n=2)

Case 1:
The loading is as follows:
Uniaxial tensile loading:

$$\Delta\bar{\sigma}_1^{(1)} = 250, \quad \Delta\bar{\sigma}_2^{(1)} = 0$$

Uniaxial tensile unloading/compressive loading:

$$\Delta\bar{\sigma}_1^{(2)} = -500, \quad \Delta\bar{\sigma}_2^{(2)} = 0$$

Uniaxial compressive unloading/tensile loading:

$$\Delta\bar{\sigma}_1^{(3)} = 400, \quad \Delta\bar{\sigma}_2^{(3)} = 0$$

As we can see from figure 5, the strain path in stress space is a straight line along σ_1-axis.The model in fact experienced three process.(1) a loading until the $\Delta\varepsilon^{(1)}$,this is the black line from point 1 to point 6.(2)An uniaxial tensile unloading then becoming compressive loading, this is the green line from

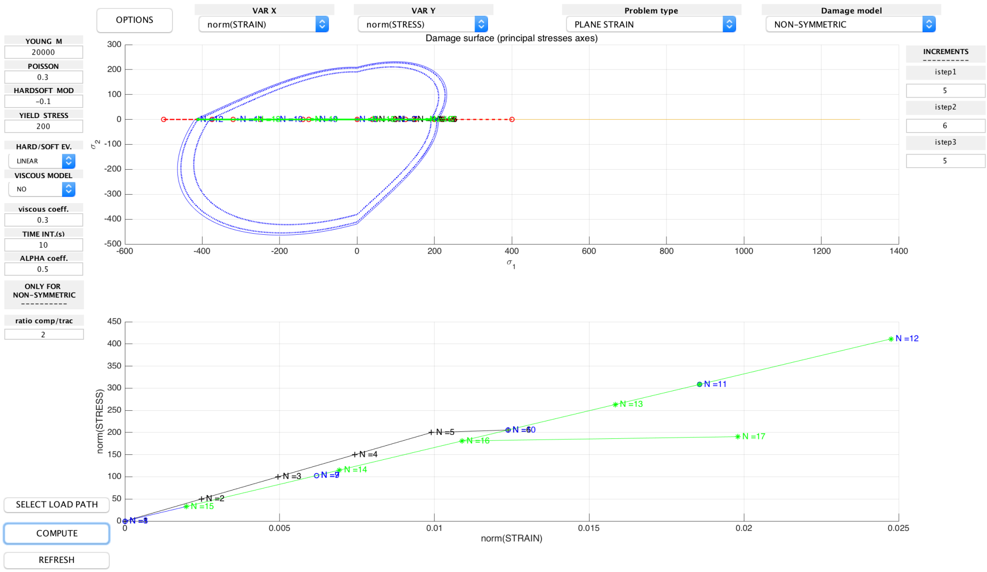


Figure 5: Non-symmetric damage model under uniaxial tensile/compressive loading/unloading to uniaxial loading/unloading process

point 6 to point 12.We have to note that here from point 11 to point 12 the compressive loading exceeds the yield stress(400,n=2),this is the key point for this non-symmetric damage model.So if we apply another two loading cases:

Case 2: Uniaxial tensile loading:

$$\Delta\bar{\sigma}_1^{(1)} = 250, \quad \Delta\bar{\sigma}_2^{(1)} = 0$$

Biaxial tensile unloading/compressive loading:

$$\Delta\bar{\sigma}_1^{(2)} = -500, \quad \Delta\bar{\sigma}_2^{(2)} = -500$$

Biaxial compressive unloading/tensile loading:

$$\Delta\bar{\sigma}_1^{(3)} = 400, \quad \Delta\bar{\sigma}_2^{(3)} = 400$$

Case 3: Biaxial tensile loading:

$$\Delta\bar{\sigma}_1^{(1)} = 250, \quad \Delta\bar{\sigma}_2^{(1)} = 0$$

Biaxial tensile unloading/compressive loading:

$$\Delta\bar{\sigma}_1^{(2)} = -500, \quad \Delta\bar{\sigma}_2^{(2)} = -500$$

Biaxial compressive unloading/tensile loading:

$$\Delta\bar{\sigma}_1^{(3)} = 400, \quad \Delta\bar{\sigma}_2^{(3)} = 400$$

The stress-strain corresponding to case 2 and case 3 are given is figure 6 and figure 7

We can again find that for non-symmetric damage model, the tension domain differs with respect to compression.The parameter n($=\frac{\sigma_Y^c}{\sigma_Y^t}$) is defined in order to describe this difference,

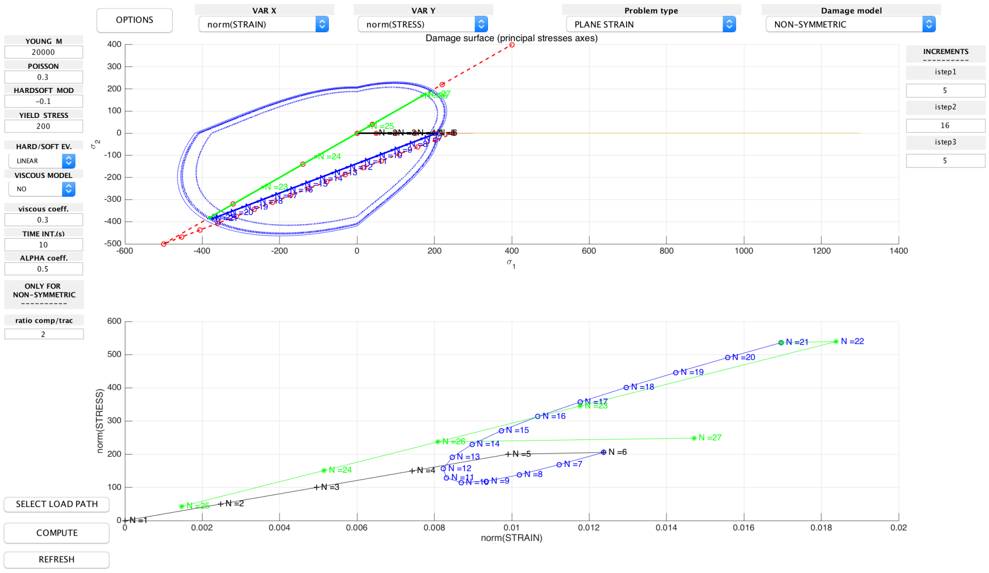


Figure 6: Non-symmetric damage model from uniaxial tensile loading to biaxial unloading

2. **Rate Dependent Models**

   (a) Firstly the code is implemented according to Lecture5,page 15-17. The formula for update internal variable with respect to the viscous coefficient and α. We can analyze the the effects of viscosity parameters η, strain rate $\dot{\varepsilon}$ and α

   (b) The effects of viscosity parameters η is shown in figure 8a, where we set three different viscosity parameters with total time 20s and $\alpha = 0.5$. Here we obtain a consistent result as given in Lecture 5,page 17. With a larger viscosity, the model tends to behavior like viscous fluid, where the stress is determined by the strain rate according to law of Newtonian fluids,

$$\sigma = \eta \frac{d\varepsilon}{dt}$$

   Next, continued result can be obtained if we change the strain rate to see if we can get the result obey this law.

   (c) The effects of strain rate is obtained by changing the total time and maintaining the final strain a constant.Result is shown is figure 8b. The elastic domain become larger than the yield stress if the strain rate become faster.For example, here our actual yield stress is set to 200, but the real yield stress is at about 270 if the total time is 1s.This is the point for changing strain rate.So in the real case, we have to apply the strain slowly so that we are able to obtain the real stress for a material.

   (d) The effects of α on the stress-strain curve is shown is figure 8c.If $\alpha = 0$, the method is explicit Euler method, and in the figure 8c we find that the stress-strain curve is a straight line, which is impossible due to the yield stress.Actually,as suggested in Lecture 5,page 8, the value α should be between $\frac{1}{2}$ and 1 so as to keep the result stable during the numerical integration process.

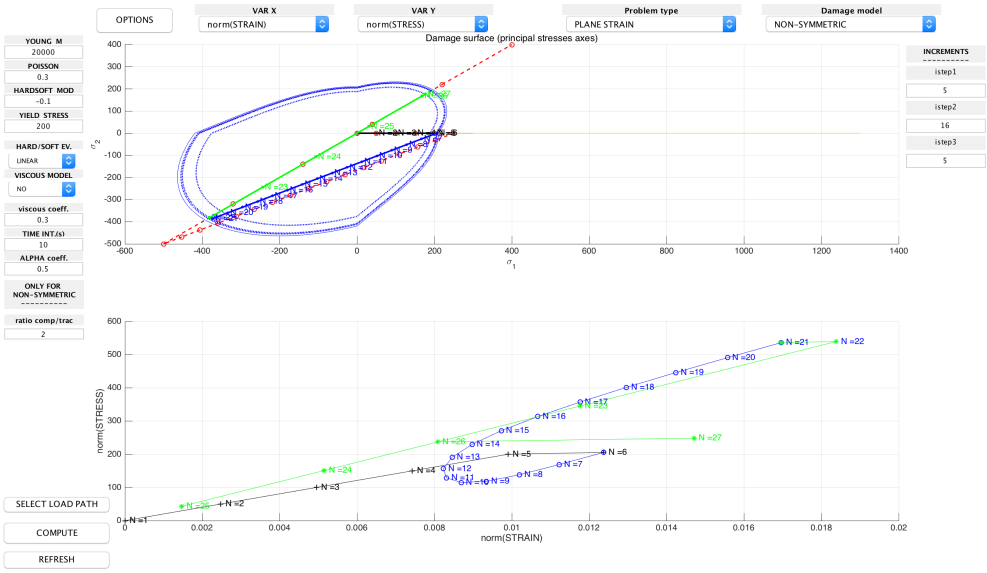


Figure 7: Non-symmetric damage model from uniaxial tensile loading to biaxial unloading

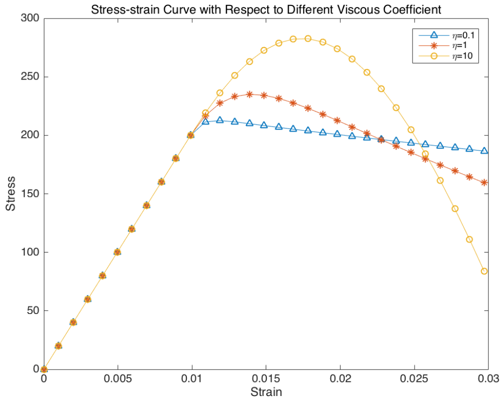
(e) Corresponding to the effects of α on the stress-strain curve shown before, its effects on the evolution along time of C_{11} component of the tangent and algorithmic constitutive operators is plotted in figure 8d.We find that when $\alpha = 0$, C_{11} remains a constant this is a consistent result from figure 8c. Because in this case, it is considered to be alway elastic, the damage variable keeps zero during the whole process.According to algorithm in Lecture 5,page14, 15:

$$d_{n+1} = d_n \quad C^{vd}_{alg,n+1} = (1 - d_{n+1})C$$

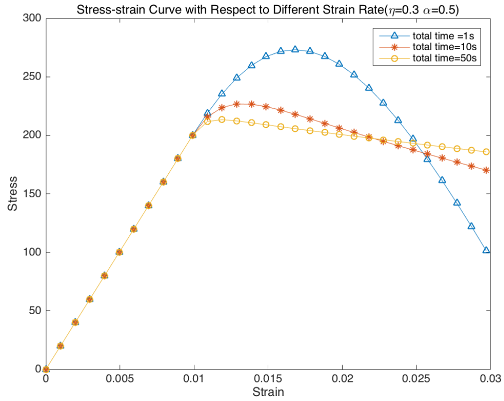
This formula explains why we obtain a constant line when $\alpha = 0$.
During the increasing process of α, we recovered the normal process for strain-stress curve.So in elastic domain, C_{11} decrease linearly due to the updated damage variable each step.Then the material failure comes out as

$$C^{vd}_{alg,n+1} = (1 - d_{n+1})C + \frac{\alpha \Delta t}{\eta + \alpha \Delta t} \frac{1}{\tau_{\varepsilon_{n+1}}} \frac{H_{n+1} r_{n+1} - q(r_{n+1})}{r^2_{n+1}} (\bar{\sigma}_{n+1} \otimes \bar{\sigma}_{n+1})$$

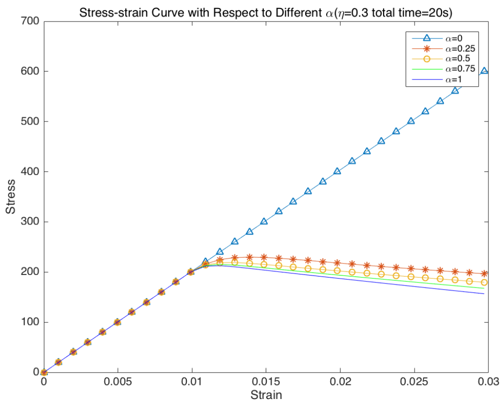
So the C_{11} of the tangent and algorithmic constitutive operator changes non-linearly along the time.



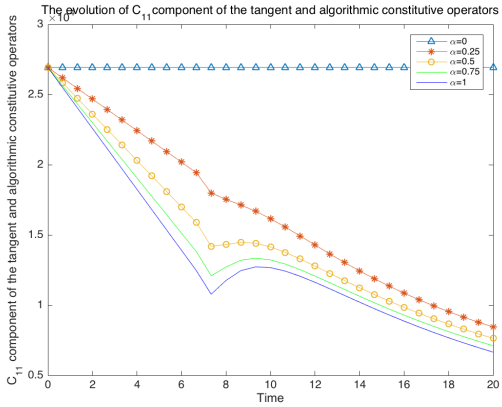
(a) Stress-strain curve with respect to different viscous coefficient$(\alpha = 0.5, totaltime = 20s)$



(b) Stress-strain curve with respect to different strain rate$(\eta = 0.3, \alpha = 0.5)$



(c) Stress-strain curve with respect to different $\alpha(\eta = 0.3, totaltime = 20s)$



(d) The evolution along time of C_{11} component of tangent and algorithmic constitutive operators with respect to $\alpha(\eta = 0.3, totaltime = 20s)$

Figure 8: Effects of $\eta, \dot{\varepsilon}, \alpha$ on the stress-strain behavior of damage models

Conclusion

In this report, we implemented successfully the Matlab code for "non-symmetric tension-compression damage model" and "tension-only" damage model.The correctness is assessed by analyzing three different strain path for both damage model.The result is expressed in stress-strain curve,which behaves as expected, meaning our code works well.The same work has also been done for rate dependent models.We find that the result is affected by viscosity parameters,strain rate and α.

Matlab program available

$https://github.com/Yuyang01/CSM\_HW1$

References

1. Chaves, Eduardo WV. Notes on continuum mechanics. Springer Science & Business Media, 2013.
2. Lecture Slides.