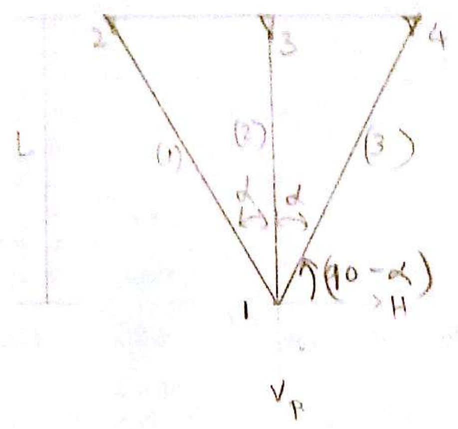

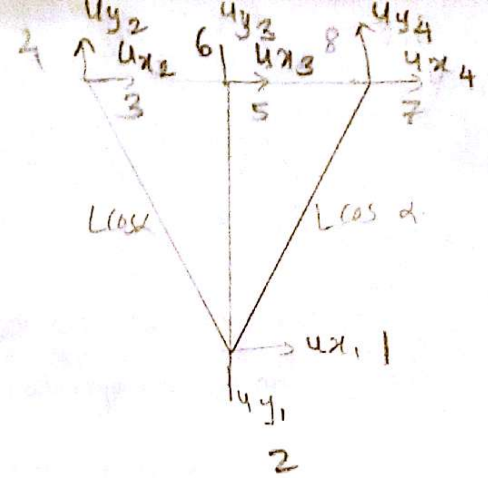

Element	θ	c	s.	Degree of freedom
(1)	$(90+\alpha)$	$\cos(90+\alpha)$	$\cos(90+\alpha)$	1, 2, 3, 4
(2)	90	$\cos(90)$	$\sin(90)$	1, 2, 5, 6
(3)	$(90-\alpha)$	$\cos(90-\alpha)$	$\sin(90-\alpha)$	1, 2, 7, 8

$$K = \frac{A^eE^e}{L^e}\begin{bmatrix} c^2 & sc & -c^2 & -sc \\ sc & s^2 & -sc & -s^2 \\ -c^2 & -sc & c^2 & sc \\ -sc & -s^2 & sc & s^2 \end{bmatrix}$$

For element 1

$$K_1 = \frac{A^1E^1}{\frac{L}{\cos\alpha}}\begin{bmatrix} (\cos(90+\alpha))^2 & \sin(90+\alpha)\cos(90+\alpha) & -(\cos(90+\alpha))^2 & -\sin(90+\alpha)\cos(90+\alpha) \\ \sin(90+\alpha)\cos(90+\alpha) & (\sin(90+\alpha))^2 & -\sin(90+\alpha)\cos(90+\alpha) & -\sin(90+\alpha)^2 \\ -(\cos(90+\alpha))^2 & -\sin(90+\alpha)\cos(90+\alpha) & \cos(90+\alpha)^2 & \sin(90+\alpha)\cos(90+\alpha) \\ -(\sin(90+\alpha)\cos(90+\alpha)) & -\sin(90+\alpha)\sin(90+\alpha) & \sin(90+\alpha)\cos(90+\alpha) & \sin(90+\alpha)^2 \end{bmatrix}$$

$$\cos(90+\alpha) = -\sin\alpha = \underline{\underline{-s}}$$

$$\sin(90+\alpha) = \cos\alpha = \underline{\underline{c}}$$

$$K_1 = \frac{AE}{\frac{L}{\cos\alpha}}\begin{bmatrix} (-\sin\alpha)^2 & \cos\alpha(-\sin\alpha) & -(-\sin\alpha)^2 & -\cos\alpha(-\sin\alpha) \\ \cos\alpha(-\sin\alpha) & (\cos\alpha)^2 & -(\cos\alpha)(-\sin\alpha) & -(\cos\alpha)^2 \\ -(\sin\alpha)^2 & -(\cos\alpha)(-\sin\alpha) & (-\sin\alpha)^2 & \cos\alpha(-\sin\alpha) \\ -(\cos\alpha - \sin\alpha) & -(\cos\alpha)(\cos\alpha) & (-)(\cos\alpha)(\sin\alpha) & (\cos\alpha)^2 \end{bmatrix}$$

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$$K_{(1)} = \frac{AE}{L} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} cs^2 & -c^2 s & -s^2 c & c^2 s \\ & c^3 & c^2 s & -c^3 \\ & & cs^2 & -sc^2 \\ \text{sym} & & & c^3 \end{bmatrix} \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}$$

For element 2

$\cos(90) = 0$ $\sin(90) = 1$

$$k_2 = \frac{AE}{L} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 5 & 6 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ & 1 & 0 & 1 \\ & & 0 & 0 \\ \text{sym.} & & & 1 \end{bmatrix} \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 5 \\ 6 \end{array}$$

For Element 3

$\cos(90-\alpha) = \cos\alpha = \underline{\underline{c}}$ $\sin(90-\alpha) = \sin\alpha = \underline{\underline{s}}$

$$k_3 = \frac{AE}{L} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 7 & 8 \end{array} \\ \begin{bmatrix} c^3 & sc^3 & -c^3 & -sc^2 \\ & s^2 c & -sc^2 & -s^2 c \\ & & c^3 & sc^2 \\ \text{sym.} & & & s^2 c \end{bmatrix} \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 7 \\ 8 \end{array}$$

$$[k]\{u\} = \{f\}$$

Assembling the local stiffness matrix we get global stiffness matrix which has 4 nodes of 2 degree of freedom $\therefore$ $4 \times 2 = 8$ rows & columns

$$
\begin{array}{cccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8
\end{array}
$$

$$
\frac{AE}{L}
\begin{bmatrix}
2cs^2 & 0 & -cs^2 & c^2s & 0 & 0 & -s^2c & -sc^2 \\
 & 1+2c^3 & c^2s & -c^3 & 0 & -1 & -sc^2 & -c^3 \\
 & & s^2c & & 0 & 0 & 0 & 0 \\
 & & & c^3 & 0 & 0 & 0 & 0 \\
 & & & & 0 & 0 & 0 & 0 \\
 & & & & & 1 & 0 & 0 \\
 & & & & & & cs^2 & c^2s \\
 & \text{Symm} & & & & & & c^3
\end{bmatrix}
\begin{array}{c}
1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8
\end{array}
\begin{bmatrix}
u_{x_1} \\ u_{y_1} \\ u_{x_2} \\ u_{y_2} \\ u_{x_3} \\ u_{y_3} \\ u_{x_4} \\ u_{y_4}
\end{bmatrix}
=
\begin{bmatrix}
H \\ -P \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0
\end{bmatrix}
$$

5th row & column expresses the u_{x_3} i.e Horizontal forces acting on pt. 3 which are '0'. It shows that whenever the forces are applied on the beam, the forces gets transfered to point 2 & 4 & therefore there is no displacement on pt. 3

$\alpha \rightarrow \pi/2$

$$u_{x_1} = \frac{HL}{AE(0)} \qquad \text{If. } HL = 0 \quad u_i = 0$$

$$HL \neq 0 \quad u_i = \infty$$

$$u_{y_1} = \frac{-PL}{AE(0)}$$

Displacement transformation

$$u^e = T^e u^e$$

$$\begin{bmatrix} \bar{u}_{xi} \\ \bar{u}_{yi} \\ \bar{u}_{xj} \\ \bar{u}_{yj} \end{bmatrix} = \begin{bmatrix} C & -S & 0 & 0 \\ -S & C & 0 & 0 \\ 0 & 0 & C & S \\ 0 & 0 & -S & C \end{bmatrix} \begin{bmatrix} u_{xi} \\ u_{yi} \\ u_{xj} \\ u_{yj} \end{bmatrix}$$

For Element I

$$\begin{bmatrix} -u_{x_1}S + u_{y_1}C \\ -u_{x_1}C - u_{y_1}S \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -S & C & 0 & 0 \\ -C & -S & 0 & 0 \\ 0 & 0 & -S & C \\ 0 & 0 & -S & C \end{bmatrix} \begin{bmatrix} u_{x_1} \\ u_{y_1} \\ 0 \\ 0 \end{bmatrix}$$

Elongation $d^{(1)} = u_{x_2} - u_{x_1}$

$$= 0 - (-u_{x_1}S - u_{y_1}S)$$

$$= \frac{HL}{2ACSE} + \frac{PLC}{AE(1+2C^3)}$$

(b) Applying the B.C

matrix reduces to

$$\frac{AE}{L}\begin{bmatrix} 2cs^2 & 0 \\ 0 & 1+2c^3 \end{bmatrix}\begin{bmatrix} u_{x_1} \\ u_{y_1} \end{bmatrix} = \begin{bmatrix} H \\ -P \end{bmatrix}$$

$$\boxed{u_{x_1} = \frac{HL}{AE\, 2cs^2_{(\alpha)}}}$$

$$\boxed{u_{y_1} = \frac{-PL}{AE\,(1+2c^3_{(\alpha)})}}$$

(c) $\underline{\alpha \to 0}$

- $u_{x_1} = \dfrac{HL}{AE\,(0)}$. $\Big\}$ if $HL = 0$ $u_{x_1} = 0$; $HL \neq 0$ $u_{x_1} = \infty$

& if $H \neq 0$

The answer '∞' says that if there is force acting in horizontal direction the displacement in 'x' blows up means there is infinite displacement in horizontal direction as there is no reaction

- $u_{y_1} = \dfrac{-PL}{AE(1+2)} = \dfrac{-PL}{3AE}$

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$\therefore$ Axial force

$$F^{(1)} = \frac{EA}{L^{(e)}} d^{(1)} = \frac{EAC}{L} \times \left(\frac{HL}{2AECS} + \frac{PLC}{EA(1+2C^3} \right)$$

$$F^{(1)} = \frac{H}{2S} + \frac{PC^2}{1+2C^3}$$

Element 2

$$\begin{bmatrix} -u_{y_1} \\ -u_{x_1} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} u_{x_1} \\ u_{y_1} \\ 0 \\ 0 \end{bmatrix}$$

$$d^{(2)} = \bar{u}_{x_3} - \bar{u}_{x_1} = -u_{y_1} = \frac{PL}{(1+2C^3)EA}$$

$$F^{(2)} = \frac{P}{1+2C^3}$$

Element 3

$$\begin{bmatrix} u_{x_1}S + u_{y_1} \\ -u_{x_1}C + u_{y_1}S \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} S & C & 0 & 0 \\ -C & -S & 0 & 0 \\ 0 & 0 & S & C \\ 0 & 0 & -C & -S \end{bmatrix} \begin{bmatrix} u_{x_1} \\ u_{y_1} \\ 0 \\ 0 \end{bmatrix}$$

Elongation $d^{(3)} = u^{(3)}_{x_2} - u^{(3)}_{x_1} = -u_{x_1}S - u_{y_1}C$

$$= \frac{-HLS}{2CS^2EA} + \frac{PLC}{(1+2C^3)EA}$$

$$F^{(3)} = \frac{-H}{2S} + \frac{PC^2}{1+2C^3}$$

$\alpha \to 0$ & $H \neq 0$ $\quad F_1 = \infty \quad F_3 = \infty$

There is no resistance so results show that structure will deform.

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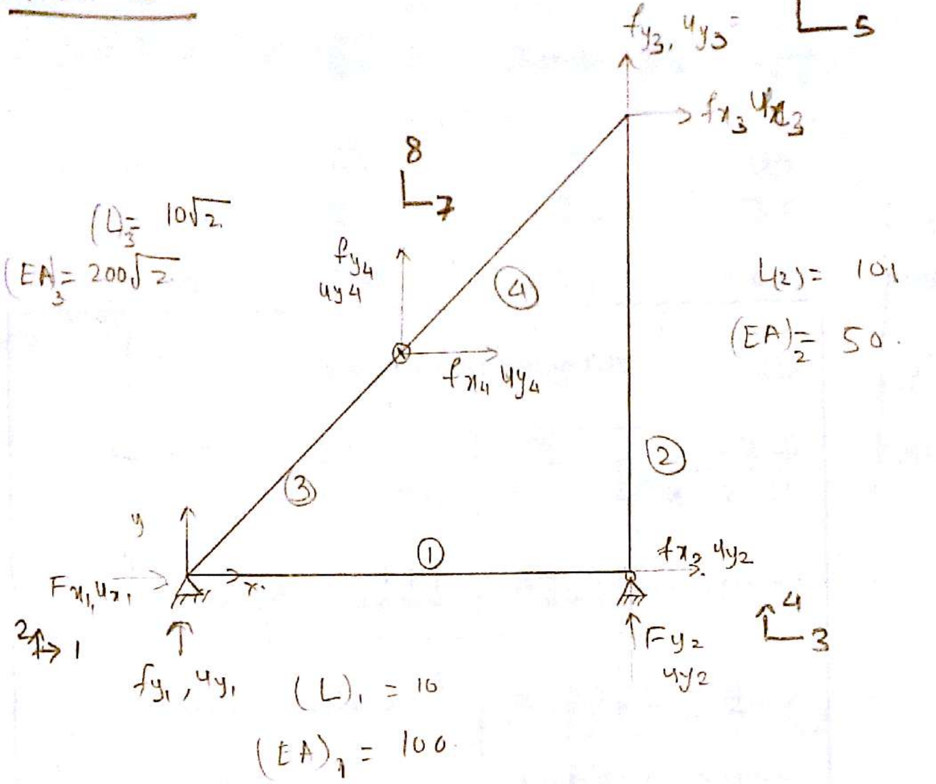
Assignment 2

Adding an extra node in element 5

Element	Degree of freedom			
(1)	1	2	3	4
(2)	3	4	5	6
(3)	1	2	7	8
(4)	7	8	5	6

Assignment 2



Stiffness Matrix -

$$k_{(e)} = \frac{A^{(e)} E^{(e)}}{L^{(e)}} \begin{bmatrix} c^2 & sc & -c^2 & -sc \\ sc & s^2 & -sc & -s^2 \\ -c^2 & -sc & c^2 & sc \\ -sc & -s^2 & sc & s^2 \end{bmatrix}$$

Element 1

$\cos\psi = c = 1 \qquad \sin\psi = s = 0$

$$k_{(1)} = \begin{array}{c} \begin{bmatrix} 10 & 0 & -10 & 0 \\ 0 & 0 & 0 & 0 \\ -10 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} \\ \begin{matrix} 1 & \quad 2 & \quad 3 & \quad 4 \end{matrix} \end{array}$$

Element 2

$\cos\psi = 0 = c \qquad \sin\psi = s = 1$

$$k_{(2)} = \begin{array}{c} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & -5 \\ 0 & 0 & 0 & 0 \\ 0 & -5 & 0 & 5 \end{bmatrix} \begin{matrix} 3 \\ 4 \\ 5 \\ 6 \end{matrix} \\ \begin{matrix} 3 & \quad 4 & \quad 5 & \quad 6 \end{matrix} \end{array}$$

Element 3

$\cos\psi = C = 1/\sqrt{2}$ $\quad\quad$ $\sin\psi = S = 1/\sqrt{2}$

$$K_{(3)} = \begin{array}{c} \begin{array}{cccc} ① & ② & ⑦ & ⑧ \end{array} \\ \begin{bmatrix} 20 & 20 & -20 & -20 \\ 20 & 20 & -20 & -20 \\ -20 & -20 & 20 & 20 \\ -20 & -20 & 20 & 20 \end{bmatrix} \end{array} \begin{array}{c} ① \\ ② \\ ⑦ \\ ⑧ \end{array}$$

Element 4

$\cos\psi = c = 1/\sqrt{2}$ $\quad\quad$ $\sin\psi = S = -1/\sqrt{2}$

$$K_{(4)} = \begin{array}{c} \begin{array}{cccc} ⑦ & ⑧ & ⑤ & ⑥ \end{array} \\ \begin{bmatrix} 20 & 20 & -20 & -20 \\ 20 & 20 & -20 & -20 \\ -20 & -20 & 20 & 20 \\ -20 & -20 & 20 & 20 \end{bmatrix} \end{array} \begin{array}{c} ⑦ \\ ⑧ \\ ⑤ \\ ⑥ \end{array}$$

Assembling global stiffness matrix $\mathcal{S}$

$$K \; u = f.$$

$$\begin{bmatrix} 20 & 20 & 0 & 0 & 0 & 0 & -20 & -20 \\ & 20 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & 0 & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 \\ & & & & & 0 & 0 & 0 \\ \text{Sym.} & & & & & & 20 & 20 \\ & & & & & & & 20 \end{bmatrix} \begin{bmatrix} u_{x_1} \\ u_{y_1} \\ u_{x_2} \\ u_{y_2} \\ u_{x_3} \\ u_{y_3} \\ u_{x_4} \\ u_{y_4} \end{bmatrix} = \begin{bmatrix} f_{x_1} \\ f_{y_1} \\ f_{x_2} \\ f_{y_2} \\ f_{x_3} \\ f_{y_3} \\ f_{x_4} \\ f_{y_4} \end{bmatrix}$$

B.C.

$u_{x_1} = u_{y_1} = u_{y_2} = 0$

$f_{x_2} = f_{x_4} = f_{y_4} = 0$

$f_{x_3} = 2$

$f_{y_3} = 1.$

Reduces to.

$$\begin{bmatrix} -10 & 0 & 0 & 0 & 0 \\ 0 & 20 & 20 & -20 & -20 \\ & & 25 & -20 & -20 \\ & & & 40 & 40 \\ \text{sym} & & & & 40 \end{bmatrix} \begin{bmatrix} u_{x_2} \\ u_{x_3} \\ u_{y_3} \\ u_{x_4} \\ u_{y_4} \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

Elemental stiffness matrix is singular it shows that the displacement is infinite, which means there the structure is deformable.

Furthermore adding as there is 4 links it it has mobility $\underline{\underline{1}}$ so the structure gets converted into <u>4 bar mechanism.</u>