Computational Structural Mechanics and Dynamics

Assignment 2
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Assignment 2.1

1. Identify the symmetry and antisymmetry lines in the two-dimensional problems illustrated in the figure.

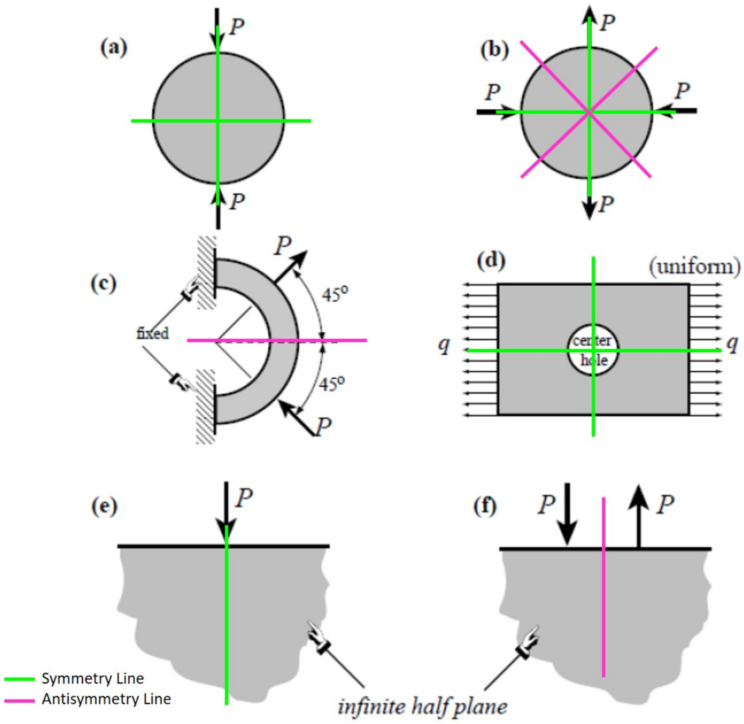
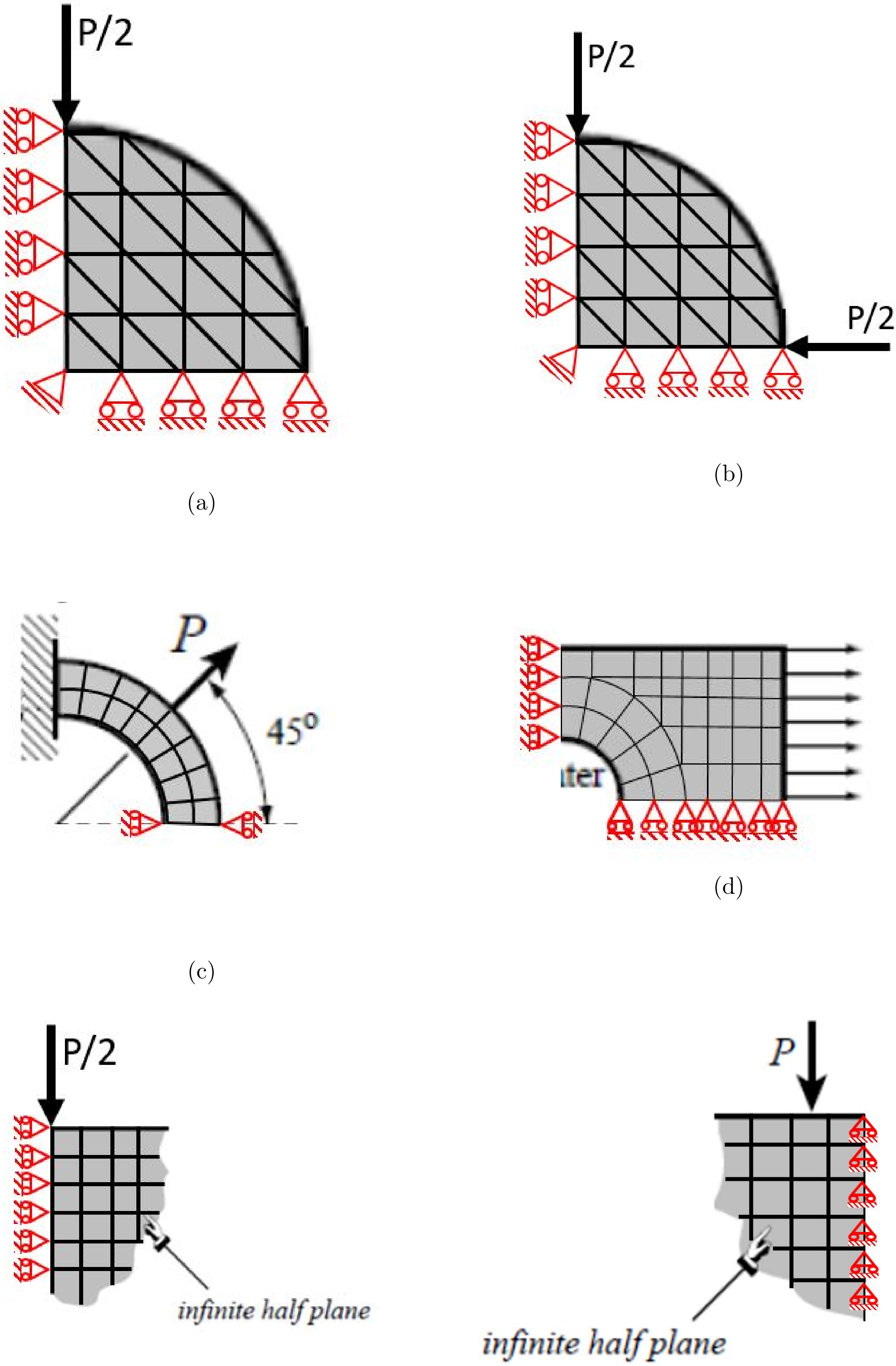


Figure 1: symmetry and antisymmetry lines

2. Having identified those symmetry/antisymmetry lines, state whether it is possible to cut the complete structure to one half or one quarter before laying out a finite element mesh. Then draw a coarse FE mesh indicating, with rollers or fixed supports, which kind of displacement BCs you would specify on the symmetry or antisymmetry lines.



(a) (b) (c) (d) (e) (f)

Figure 2: FE mesh

Assignment 2.2

1. The plate structure shown in the figure is loaded and deforms in the plane of the paper. The applied load at D and the supports at I and N extend over a fairly narrow area. List what you think are the likely “trouble spots” that would require a locally finer finite element mesh to capture high stress gradients. Identify those spots by its letter and a reason.

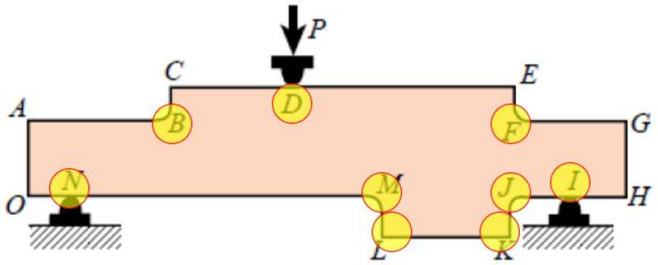


Figure 3: inplane bent plate

Spots N, I and D need mesh refinement because they are support points and tolerating critical loads. Spots B, F, M, L, K and J have to have refined mesh because they are corners regarded as critical geometry points because of changes happening in area.

Assignment 2.3

1. A tapered bar element of length l and areas Ai and Aj with A interpolated as:

$$A = A_i(1-\xi) + A_j\xi$$

and constant density ρ rotates on a plane at uniform angular velocity ω (rad/sec) about node i. Taking axis x along the rotating bar with origin at node i, the centrifugal axial force is $q(x) = \rho A\omega^2 x$ along the length in which x is the longitudinal coordinate $x = x^e$.

Find the consistent node forces as functions of ρ, A_i, A_j, ω and l, and specialize the result to the prismatic bar $A = A_i = A_j$.

The consistency nodal force vector is definded as:

$$f_{ext} = \int_0^1 q \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} l \, d\xi$$

$$q = \rho A\omega^2 x = \rho A\omega^2 \xi l = \rho\omega^2 \xi l (A_i(1-\xi) + A_j\xi)$$

After normalize the axial centrifugal force:

$$
\begin{aligned}
f_{ext} &= \int_0^1 \rho l^2 \omega^2 \xi (A_i(1-\xi) + A_j \xi) \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} d\xi \\
&= \rho l^2 \omega^2 \int_0^1 \begin{bmatrix} A_i(\xi^3 - 2\xi^2 + \xi) + A_j(\xi^2 - \xi^3) \\ A_i(\xi^2 - \xi^3) + A_j \xi^3 \end{bmatrix} d\xi \\
&= \rho l^2 \omega^2 \begin{bmatrix} A_i(\xi^4/4 - 2/3\,\xi^3 + \xi^2/2) + A_j(\xi^3/3 - \xi^4/4) \\ A_i(\xi^3/3 - \xi^4/4) + A_j \xi^4/4 \end{bmatrix} \\
&= \rho l^2 \omega^2 \begin{bmatrix} A_i(1/4 - 2/3 + 1/2) + A_j(1/3 - 1/4) \\ A_i(1/3 - 1/4) + A_j\, 1/4 \end{bmatrix} \\
&= \rho l^2 \omega^2 \begin{bmatrix} A_i/12 + A_j/12 \\ A_i/12 + A_j/4 \end{bmatrix} \longrightarrow f_{ext} = \rho l^2 \omega^2 \begin{bmatrix} A/6 \\ A/3 \end{bmatrix}
\end{aligned}
$$