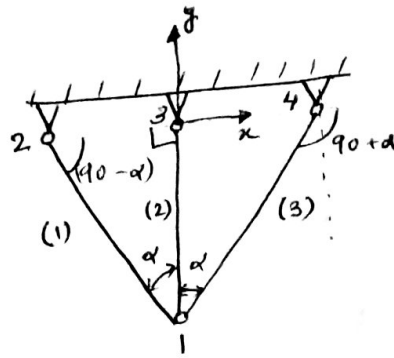


1a)



(1)

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For Element -2.

$$\cos \theta = \cos(-90) = 0$$

$$\sin \theta = \sin(-90) = -1$$

$$\begin{bmatrix} F_{1x}^{(2)} \\ F_{1y}^{(2)} \\ F_{3x}^{(2)} \\ F_{3y}^{(2)} \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} U_{1x} \\ U_{1y} \\ U_{3x} \\ U_{3y} \end{bmatrix}$$

which we get from general eqⁿ,

$$\begin{bmatrix} F_{1x}^{(2)} \\ F_{1y}^{(2)} \\ F_{2x}^{(2)} \\ F_{2y}^{(2)} \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} c_2^2 & c_2 s_2 & -c_2^2 & c_2 s_2 \\ c_2 s_2 & s_2^2 & -c_2 s_2 & -s_2^2 \\ -c_2^2 & -c_2 s_2 & c_2^2 & c_2 s_2 \\ -c_2 s_2 & -s_2^2 & c_2 s_2 & s_2^2 \end{bmatrix} \begin{bmatrix} U_{1x} \\ U_{1y} \\ U_{2x} \\ U_{2y} \end{bmatrix}$$

where $c_2 = \cos \theta$ $s_2 = \sin \theta$

For Element 1:

(2)

~~$\cos \theta = \cos \alpha = \sin \alpha$~~

$$\cos \theta = \cos(-90 + \alpha) = \sin \alpha = c_1 = s$$

$$\sin \theta = \sin(-90 + \alpha) = -\cos \alpha = s_1 = -c$$

$$L_1 = \frac{L}{\cos \alpha} = \frac{L}{c}$$

$$\begin{bmatrix} F_{1x}^{(1)} \\ F_{1y}^{(1)} \\ F_{2x}^{(1)} \\ F_{2y}^{(1)} \end{bmatrix} = \frac{EAC}{L} \begin{bmatrix} c_1^2 & c_1 s_1 & -c_1^2 & -c_1 s_1 \\ c_1 s_1 & s_1^2 & -c_1 s_1 & -s_1^2 \\ -c_1^2 & -c_1 s_1 & c_1^2 & c_1 s_1 \\ -c_1 s_1 & -s_1^2 & c_1 s_1 & s_1^2 \end{bmatrix} \begin{bmatrix} U_{1x} \\ U_{1y} \\ U_{2x} \\ U_{2y} \end{bmatrix}$$

$$\begin{bmatrix} F_{1x}^{(1)} \\ F_{1y}^{(1)} \\ F_{2x}^{(1)} \\ F_{2y}^{(1)} \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} U_{1x} & U_{1y} & U_{2x} & U_{2y} \\ c s^2 & -c^2 s & -c s^2 & +c^2 s \\ -c^2 s & c^3 & c^2 s & -c^3 \\ -c s^2 & c^2 s & c s^2 & -c^2 s \\ c^2 s & -c^3 & -c^2 s & c^3 \end{bmatrix} \begin{bmatrix} U_{1x} \\ U_{1y} \\ U_{2x} \\ U_{2y} \end{bmatrix}$$

For Element 3.

$$\cos\theta = \cos(-90-\alpha) = -\sin\alpha = c_3 = -s$$

$$\sin\theta = \sin(-90-\alpha) = -\cos\alpha = s_3 = -c$$

$$L_3 = \frac{L}{\cos\alpha} ; c_3^2 = s^2$$

$$-c_3 s_3 = -cs ; c_3 s_3 = cs ; s_3^2 = c^2 ;$$

$$-s_3^2 = -c^2 ; -c_3^2 = -s^2$$

$$\begin{bmatrix} F_{1x} \\ F_{1y} \\ F_{4x} \\ F_{4y} \end{bmatrix} = \frac{EAc}{L} \begin{bmatrix} c_3^2 & c_3 s_3 & -c_3^2 & -c_3 s_3 \\ c_3 s_3 & s_3^2 & -c_3 s_3 & -s_3^2 \\ -c_3^2 & -c_3 s_3 & c_3^2 & c_3 s_3 \\ -c_3 s_3 & -s_3^2 & c_3 s_3 & s_3^2 \end{bmatrix} \begin{bmatrix} \theta_{1x} \\ U_{1y} \\ U_{4x} \\ U_{4y} \end{bmatrix}$$

$$= \frac{EA}{L} \begin{bmatrix} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ -c^2 & -cs & c^2 & cs \\ -cs & -s^2 & cs & s^2 \end{bmatrix} \begin{bmatrix} U_{1x} \\ U_{1y} \\ U_{4x} \\ U_{4y} \end{bmatrix}$$

~~For element 4~~

(4)

On Assembly of Global Matrix, we obtain,

$$\begin{bmatrix} F_{1x} \\ F_{1y} \\ F_{2x} \\ F_{2y} \\ F_{3x} \\ F_{3y} \\ F_{4x} \\ F_{4y} \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} U_{1x} & U_{1y} & U_{2x} & U_{2y} & U_{3x} & U_{3y} & U_{4x} & U_{4y} \\ 2cs^2 & c^2s & -cs^2 & +c^2s & 0 & 0 & -cs^2 & -c^2s \\ c^2s & 2c^3+1 & c^2s & -c^3 & 0 & -1 & -c^2s & -c^3 \\ -cs^2 & c^2s & cs^2 & -c^2s & 0 & 0 & 0 & 0 \\ c^2s & -c^3 & -c^2s & c^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ -cs^2 & -c^2s & 0 & 0 & 0 & 0 & cs^2 & c^2s \\ -c^2s & -c^3 & 0 & 0 & 0 & 0 & c^2s & c^3 \end{bmatrix} \begin{bmatrix} U_{1x} \\ U_{1y} \\ U_{2x} \\ U_{2y} \\ U_{3x} \\ U_{3y} \\ U_{4x} \\ U_{4y} \end{bmatrix}$$

Only element one (1) can contribute to forces on node '3' in the x-direction. Since it is assumed that the joints acts as hinged hinges, any force acting on element one(1) will not contribute to forces in the x-direction of node 3. This is due to its perpendicular configuration to the x-axis. Hence the row 5 is zero (0). It is also due to the same configuration that displacement if any, of node '3' in x-direction will not contribute to forces on the other nodes. Hence the column 5 is zero.

(b) Applying boundary condition, we get,

(5)

$$\frac{EA}{L} \begin{bmatrix} 2cs^2 & 0 & -cs^2 & c^2s & 0 & 0 & -cs^2 & -c^2s \\ & 2c^3+1 & c^2s & -c^3 & 0 & -1 & -c^2s & -c^3 \\ & & cs^2 & -c^2s & 0 & 0 & 0 & 0 \\ & & & c^3 & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 \\ & & & & & 1 & 0 & 0 \\ & & & & & & cs^2 & c^2s \\ & & & & & & & c^3 \end{bmatrix} \begin{bmatrix} U_{1x} \\ U_{1y} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} H \\ -P \\ F_{2x} \\ F_{2y} \\ F_{3x} \\ F_{3y} \\ F_{4x} \\ F_{4y} \end{bmatrix}$$

Sym

The two equations are

$$\begin{bmatrix} -H \\ P \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} 2cs^2 & 0 \\ 0 & 1+2c^3 \end{bmatrix} \begin{bmatrix} U_{1x} \\ U_{1y} \end{bmatrix}$$

$$c) \quad \frac{L}{EA} \begin{bmatrix} 2cs^2 & 0 \\ 0 & 1+2c^3 \end{bmatrix}^{-1} \begin{bmatrix} H \\ -P \end{bmatrix} = \begin{bmatrix} U_{1x} \\ U_{1y} \end{bmatrix}$$

For $\alpha \rightarrow 0$,

$$\frac{L}{EA} \Rightarrow \frac{L}{EA(2cs^2)(1+2c^3)} \begin{bmatrix} 1+2c^3 & 0 \\ 0 & 2cs^2 \end{bmatrix} \begin{bmatrix} H \\ -P \end{bmatrix}$$

For $\alpha \rightarrow 0$,

$$U_{ix} = \lim_{\alpha \rightarrow 0} \frac{LH}{EA 2cs^2}$$

\therefore we have, $U_{ix} \rightarrow \infty$

$$\& U_{iy} = \lim_{\alpha \rightarrow 0} \frac{-LP}{EA(1+2c^3)}$$

$$U_{iy} = -\frac{LP}{EA(1+2c^3)} \Rightarrow \frac{-LP}{3EA} = U_{iy}$$

For $\alpha \rightarrow \frac{\pi}{2}$

Let distance ~~between nodes~~ = ~~L~~

$$U_{ix} = \lim_{\alpha \rightarrow \frac{\pi}{2}} \frac{LH}{EA(2cs^2)}$$

$$= \lim_{\alpha \rightarrow \frac{\pi}{2}} \frac{LH}{EA \sin 2\alpha \sin \alpha}$$

$\therefore U_{ix} \rightarrow \infty$ as $\alpha \rightarrow \frac{\pi}{2}$

$$U_{iy} = \lim_{\alpha \rightarrow \frac{\pi}{2}} \frac{-LP}{EA(1+2c^3)}$$

$$= -\frac{LP}{EA}$$

U_{ix} 'blows up' as $\alpha \rightarrow 0$ & if $H \neq 0$ because the element (1) & (3) merge with (2) to form a pendulum like system which can rotate about the node '3'.

(d) For axial forces we use the following equation,

(7)

$$[f_e] = [K_L][T][u_g]$$

$[f_e]$ → local forces

$[u_g]$ → global displacement.

$[K_L]$ → local Stiffness Matrix.

For element 1.

$$\begin{bmatrix} f_{1x}^{(1)} \\ f_{1y}^{(1)} \\ f_{2x}^{(1)} \\ f_{2y}^{(1)} \end{bmatrix} = [K_L] \begin{bmatrix} U_{1x} \\ U_{1y} \\ U_{2x} \\ U_{2y} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} f_{1x}^{(1)} \\ f_{1y}^{(1)} \\ f_{2x}^{(1)} \\ f_{2y}^{(1)} \end{bmatrix} = \frac{EA c}{L} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} s & -c & 0 & 0 \\ c & s & 0 & 0 \\ 0 & 0 & s & -c \\ 0 & 0 & c & s \end{bmatrix} \begin{bmatrix} \frac{LH}{2EAcs^2} \\ \frac{-LP}{EA(1+2c^3)} \\ 0 \\ 0 \end{bmatrix}$$

$$= \frac{EA}{L} \begin{bmatrix} s & -c & -s & c \\ 0 & 0 & 0 & 0 \\ -s & c & s & -c \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{LH}{2EAcs^2} \\ \frac{-LP}{EA(1+2c^3)} \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{pmatrix} f_{1x}^{(1)} \\ f_{1y}^{(1)} \\ f_{2x}^{(1)} \\ f_{2y}^{(1)} \end{pmatrix} = \begin{pmatrix} \frac{H}{2cs} + \frac{Pc^2}{1+2c^3} \\ 0 \\ -\frac{H}{2s} - \frac{Pc^2}{1+2c^3} \\ 0 \end{pmatrix}$$

Element 2

$$\begin{pmatrix} f_{1x}^{(2)} \\ f_{1y}^{(2)} \\ f_{3x}^{(2)} \\ f_{3y}^{(2)} \end{pmatrix} = \frac{EA}{L} \begin{pmatrix} 1 & 0 & -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} \frac{LH}{2cs^2 \times EA} \\ -\frac{PK}{(1+2c^3) \times EA} \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} f_{1x}^{(2)} \\ f_{1y}^{(2)} \\ f_{3x}^{(2)} \\ f_{3y}^{(2)} \end{pmatrix} = \begin{pmatrix} \frac{-P}{1+2c^3} \\ 0 \\ \frac{P}{1+2c^3} \\ 0 \end{pmatrix}$$

Element 3

$$\begin{pmatrix} f_{1x}^{(3)} \\ f_{1y}^{(3)} \\ f_{4x}^{(3)} \\ f_{4y}^{(3)} \end{pmatrix} = \frac{EA_c}{L} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} c_3 & s_3 & 0 & 0 \\ -s_3 & c_3 & 0 & 0 \\ 0 & 0 & c_3 & s_3 \\ 0 & 0 & -s_3 & c_3 \end{bmatrix} \begin{bmatrix} u_{1x} \\ u_{1y} \\ u_{2x} \\ u_{2y} \end{bmatrix}$$

$$= \begin{bmatrix} -s & -c & s & c \\ 0 & 0 & 0 & 0 \\ s & c & -s & -c \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{Ac}{2cs^2} \\ \frac{Pc}{1+2c^3} \\ 0 \\ 0 \end{bmatrix}$$

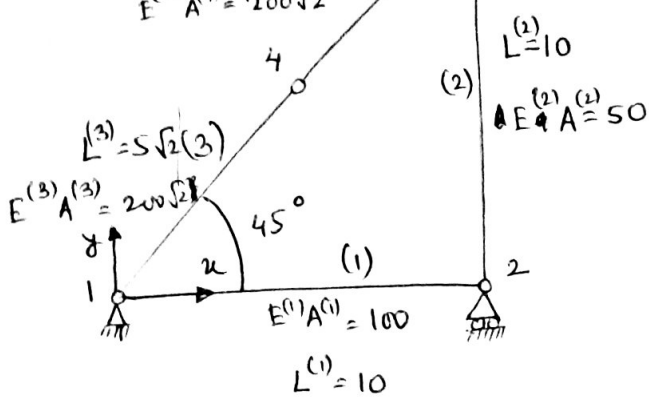
$$= \begin{bmatrix} -\frac{H}{2s} & -\frac{Pc^2}{1+2c^3} \\ 0 & 0 \\ \frac{H}{2s} & +\frac{Pc^2}{1+2c^3} \\ 0 & 0 \end{bmatrix}$$

⊗ $f^{(1)}$ & $f^{(3)}$ "blow up" if $H \neq 0$ and $\alpha \rightarrow 0$ because
 where when $\alpha \rightarrow 0$ the structure does not resist the
~~force~~ force in x -direction, but starts rotating ~~the~~
 about the node '3'

Assignment 1

(10)

Qns 2 $L^{(4)} = 5\sqrt{2}$
 $E^{(4)}A^{(4)} = 200\sqrt{2}$ (4)



For element 1

$\theta = 0^\circ; \cos \theta = 1; \sin \theta = 0$

$$\begin{pmatrix} f_{1x}^{(1)} \\ f_{1y}^{(1)} \\ f_{2x}^{(1)} \\ f_{2y}^{(1)} \end{pmatrix} = \frac{10\phi}{2} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} U_{1x} \\ U_{1y} \\ U_{2x} \\ U_{2y} \end{pmatrix}$$

For element 2

For element 2

$\theta = 90^\circ$

$\cos \theta = 0$

$\sin \theta = 1$

$$\Rightarrow \begin{pmatrix} f_{1x}^{(2)} \\ f_{1y}^{(2)} \\ f_{2x}^{(2)} \\ f_{2y}^{(2)} \end{pmatrix} = \begin{pmatrix} 10 & 0 & -10 & 0 \\ 0 & 0 & 0 & 0 \\ -10 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} U_{1x} \\ U_{1y} \\ U_{2x} \\ U_{2y} \end{pmatrix}$$

$$\begin{pmatrix} f_{2x} \\ f_{2y} \\ f_{3x} \\ f_{3y} \end{pmatrix} = \frac{5\phi}{10} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} U_{2x} \\ U_{2y} \\ U_{3x} \\ U_{3y} \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} f_{2x} \\ f_{2y} \\ f_{3x} \\ f_{3y} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & -5 \\ 0 & 0 & 0 & 0 \\ 0 & -5 & 0 & 5 \end{pmatrix} \begin{pmatrix} U_{2x} \\ U_{2y} \\ U_{3x} \\ U_{3y} \end{pmatrix}$$

For element 3.

$$\theta = 45^\circ; \cos \theta = \frac{1}{\sqrt{2}}; \sin \theta = \frac{1}{\sqrt{2}}$$

(1)

$$\begin{pmatrix} f_{1x} \\ f_{21y} \\ f_{4x} \\ f_{4y} \end{pmatrix} = \frac{40}{\sqrt{2}} \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} u_{1x} \\ u_{1y} \\ u_{4x} \\ u_{4y} \end{pmatrix} \Rightarrow \begin{pmatrix} f_{1x} \\ f_{1y} \\ f_{4x} \\ f_{4y} \end{pmatrix} = \begin{pmatrix} 20 & 20 & -20 & -20 \\ 20 & 20 & -20 & -20 \\ -20 & -20 & 20 & 20 \\ -20 & -20 & 20 & 20 \end{pmatrix} \begin{pmatrix} u_{1x} \\ u_{1y} \\ u_{4x} \\ u_{4y} \end{pmatrix}$$

For element 4.

$$\begin{pmatrix} f_{4x} \\ f_{4y} \\ f_{3x} \\ f_{3y} \end{pmatrix} = \frac{40}{\sqrt{2}} \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} u_{4x} \\ u_{4y} \\ u_{3x} \\ u_{3y} \end{pmatrix} \Rightarrow \begin{pmatrix} f_{4x} \\ f_{4y} \\ f_{3x} \\ f_{3y} \end{pmatrix} = \begin{pmatrix} 20 & 20 & -20 & -20 \\ 20 & 20 & -20 & -20 \\ -20 & -20 & 20 & 20 \\ -20 & -20 & 20 & 20 \end{pmatrix} \begin{pmatrix} u_{4x} \\ u_{4y} \\ u_{3x} \\ u_{3y} \end{pmatrix}$$

Global Matrix

$$\begin{pmatrix} F_{1x} \\ F_{1y} \\ 0 \leftarrow F_{2x} \\ F_{2y} \\ 2 \leftarrow F_{3x} \\ 1 \leftarrow F_{3y} \\ 0 \leftarrow F_{4x} \\ 0 \leftarrow F_{4y} \end{pmatrix} = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix} \begin{pmatrix} u_{1x} \\ u_{1y} \\ u_{2x} \\ u_{2y} \\ u_{3x} \\ u_{3y} \\ u_{4x} \\ u_{4y} \end{pmatrix} \Rightarrow \begin{pmatrix} u_{1x} \rightarrow 0 \\ u_{1y} \rightarrow 0 \\ u_{2x} \rightarrow 0 \\ u_{2y} \rightarrow 0 \\ u_{3x} \\ u_{3y} \\ u_{4x} \\ u_{4y} \end{pmatrix}$$

$$\begin{pmatrix} 10 & 0 & 0 \\ 0 & 20 & 20 \\ 0 & 20 & 25 \end{pmatrix}$$

$$\begin{pmatrix} F_{2x}=0 \\ F_{3x}=2 \\ F_{3y}=1 \\ F_{4x}=0 \\ F_{4y}=0 \end{pmatrix} = \begin{pmatrix} 10 & 0 & 0 & 0 & 0 \\ 0 & 20 & 20 & -20 & -20 \\ 0 & 20 & 25 & -20 & -20 \\ 0 & -20 & -20 & 40 & 40 \\ 0 & -20 & -20 & 40 & 40 \end{pmatrix} \begin{pmatrix} U_{2x} \\ U_{3x} \\ U_{3y} \\ U_{4x} \\ U_{4y} \end{pmatrix}$$

Since the last two rows are same, the rank of the matrix is ~~sing~~ 4. So the matrix is singular.

One of the basic assumption of truss system is that truss has to be composed of $m = 3 + 2(j - 3)$ members, where $m =$ number of members & $j =$ number of joint

So for 4 joint the number of members should be 5 where as in reality it is 4. So the structure has a tendency to rotate about node 4 which makes the solution ~~unstable~~ "blow up".