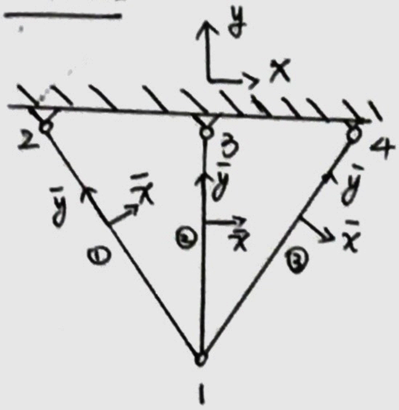
CSMD

Assignment 1

Lei Pan.



We can notice that the coordinats here is different from the lesson 1 example. So the $\underset{\sim}{T}^e$ and $\underset{\sim}{K}^e$ have changed.

$$\begin{bmatrix} \bar{f}_{y_1} \\ \bar{f}_{x_1} \\ \bar{f}_{y_2} \\ \bar{f}_{x_2} \end{bmatrix} = \frac{E^e A^e}{L^e} \cdot \begin{bmatrix} c^2 & sc & -c^2 & -sc \\ sc & s^2 & -sc & -s^2 \\ -c^2 & -sc & c^2 & sc \\ -sc & -s^2 & sc & s^2 \end{bmatrix} \begin{bmatrix} \bar{u}_{y_1} \\ \bar{u}_{x_1} \\ \bar{u}_{y_2} \\ \bar{u}_{x_2} \end{bmatrix} \xrightarrow{\text{transform}}$$

$$\begin{bmatrix} \bar{f}_{x_1} \\ \bar{f}_{y_1} \\ \bar{f}_{x_2} \\ \bar{f}_{y_2} \end{bmatrix} = \frac{E^e A^e}{L^e} \cdot \underbrace{\begin{bmatrix} s^2 & sc & -s^2 & -sc \\ sc & c^2 & -sc & -c^2 \\ -s^2 & -sc & s^2 & sc \\ -sc & -c^2 & sc & c^2 \end{bmatrix}}_{\underset{\sim}{K}^e} \begin{bmatrix} \bar{u}_{x_1} \\ \bar{u}_{y_1} \\ \bar{u}_{x_2} \\ \bar{u}_{y_2} \end{bmatrix}$$

$$\underset{\sim}{K}^{①} = \frac{EA}{L}\cdot c \cdot \left[\begin{array}{cccc|c} s^2 & -sc & -s^2 & sc & \\ -sc & c^2 & sc & -c^2 & \underset{\sim}{0} \\ -s^2 & sc & s^2 & -sc & \\ sc & -c^2 & -sc & c^2 & \\ \hline & \underset{\sim}{0} & & & \underset{\sim}{0} \end{array}\right] \text{(symetric)}$$

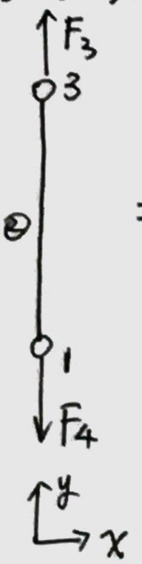
$$\underset{\sim}{K}^{②} = \frac{EA}{L}\cdot \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \text{(symetric)}$$

$$\underset{\sim}{K}^{③} = \frac{EA}{L}\cdot c \cdot \begin{bmatrix} s^2 & sc & 0 & 0 & 0 & 0 & -s^2 & -sc \\ sc & c^2 & 0 & 0 & 0 & 0 & -sc & -c^2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -s^2 & -sc & 0 & 0 & 0 & 0 & s^2 & sc \\ -sc & -c^2 & 0 & 0 & 0 & 0 & sc & c^2 \end{bmatrix} \text{(symetric)}$$

$$\underset{\sim}{K} = \underset{\sim}{K}^{①} + \underset{\sim}{K}^{②} + \underset{\sim}{K}^{③} = \frac{EA}{L} \cdot \begin{bmatrix} 2cs^2 & 0 & -cs^2 & sc^2 & 0 & 0 & -cs^2 & -sc^2 \\ & 1+2c^3 & sc^2 & -c^3 & 0 & -1 & -sc^2 & -c^3 \\ & & cs^2 & -sc^2 & 0 & 0 & 0 & 0 \\ & & & c^3 & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 \\ & & & & & 1 & 0 & 0 \\ \text{Symmetry} & & & & & & cs^2 & sc^2 \\ & & & & & & & c^3 \end{bmatrix}$$

Reason why 5th row and column contain only zeros.

① Because the bar can only stands the y direction forces and 1-3 is on symetric surface, so we get:



=> there is no x direction force on No3 point. so $f_{x_3} = 0$ ~~no matter~~ in any situations, so 5th row only contains zeros.

② because $\underset{\sim}{K}$ is symetric. so $\underset{\sim}{K}^{-1} = \underset{\sim}{K}^{T}$

=> $[\underset{\sim}{u}] = [\underset{\sim}{K}]^{T}[\underset{\sim}{f}]$, because the ② element has forces like above picture, so there is no x direction displacement no matter how the constraint condition is on No. 3 point. so the 5th row of $[\underset{\sim}{K}]^{T}$, which is also the 5th column of $\underset{\sim}{K}$, only contains zeros.

(b) $\frac{EA}{L}\begin{bmatrix} 2cs^2 & 0 \\ 0 & 1+2c^3 \end{bmatrix}\begin{bmatrix} u_{x_1} \\ u_{y_1} \end{bmatrix} = \begin{bmatrix} H \\ -P \end{bmatrix}$ (Modified stiffness system)

(c) $\begin{cases} u_{x_1} = \frac{H}{2cs^2}\cdot\frac{L}{EA} \\ u_{y_1} = -\frac{P}{1+2c^3}\cdot\frac{L}{EA} \end{cases}$

① when $\alpha \to 0$.

$u_{x_1} \to \infty$ and $u_{y_1} = -\frac{P}{3}\cdot\frac{L}{EA}$

– When $\alpha \to 0$, we can consider that the (1), (2), (3) bars become one bar. so $u_{y_1} = -\frac{P}{\frac{E\cdot(3A)}{L}} = -\frac{P}{3}\cdot\frac{L}{EA}$, and the force H will make this ~~when $\alpha \to 0$~~ system ~~not~~ roll, so this system becomes unstable, which means the $u_{x_1} \to \infty$.

② when $\alpha \to \frac{\pi}{2}$.

$u_{x_1} \to \infty$ and $u_{y_1} = -\frac{P}{1}\cdot\frac{L}{EA}$.

– when $\alpha \to \frac{\pi}{2}$, we can say that the (1), (3) bar become horizontal, this condition will cause that there is only (2) bar standing the vertical fore P. So $u_{y_1} = -\frac{P}{\frac{EA}{L}} = -P\cdot\frac{L}{EA}$. At the same time, this system will also be unstable under this situation, which means that the $u_{x_1} \to \infty$

(a)

$$[\underset{\sim}{u}] = \frac{L}{EA} \cdot \begin{bmatrix} \frac{H}{2cs^2} \\ -\frac{P}{1+2c^3} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \bar{u}_{y_1}^{①} \\ \bar{u}_{x_1}^{①} \\ \bar{u}_{y_2}^{①} \\ \bar{u}_{x_2}^{①} \end{bmatrix} = \begin{bmatrix} c & -s & 0 & 0 \\ s & c & 0 & 0 \\ 0 & 0 & c & -s \\ 0 & 0 & s & c \end{bmatrix} \begin{bmatrix} -\frac{P}{1+2c^3} \\ \frac{H}{2cs^2} \\ 0 \\ 0 \end{bmatrix} \times \frac{L}{EA} \Rightarrow \begin{cases} \bar{u}_{y_1}^{①} = -\frac{cP}{1+2c^3} - \frac{H}{2cs} \\ \bar{u}_{y_2}^{①} = 0 \end{cases}$$

$$\begin{bmatrix} \bar{u}_{y_1}^{②} \\ \bar{u}_{x_1}^{②} \\ \bar{u}_{y_3}^{②} \\ \bar{u}_{x_3}^{②} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -\frac{P}{1+2c^3} \\ \frac{H}{2cs^2} \\ 0 \\ 0 \end{bmatrix} \times \frac{L}{EA} \Rightarrow \begin{cases} \bar{u}_{y_1}^{②} = -\frac{P}{1+2c^3} \\ \bar{u}_{y_3}^{②} = 0 \end{cases}$$

$$\begin{bmatrix} \bar{u}_{y_1}^{③} \\ \bar{u}_{x_1}^{③} \\ \bar{u}_{y_4}^{③} \\ \bar{u}_{x_4}^{③} \end{bmatrix} = \begin{bmatrix} c & s & 0 & 0 \\ -s & c & 0 & 0 \\ 0 & 0 & c & s \\ 0 & 0 & -s & c \end{bmatrix} \begin{bmatrix} -\frac{P}{1+2c^3} \\ \frac{H}{2cs^2} \\ 0 \\ 0 \end{bmatrix} \times \frac{L}{EA} \Rightarrow \begin{cases} \bar{u}_{y_1}^{③} = -\frac{P \cdot c}{1+2c^3} + \frac{H}{2cs} \\ \bar{u}_{y_4}^{③} = 0 \end{cases}$$

$$\Rightarrow \begin{cases} F_1 = H/(2s) + \frac{Pc^2}{1+2c^3} \\ F_2 = cP/(1+2c^3) \\ F_3 = -H/(2s) + Pc^2/(1+2c^3) \end{cases}$$

We can see: when $\alpha \to 0$, $H/(2s) \to \infty$

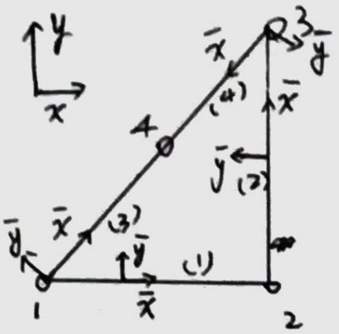
$\Rightarrow$ ~~F_1 and F_3~~ $F_1 \to \infty$ and $F_3 \to -\infty$

Reason why F_1 and F_3 blow up

because when $\alpha \to 0$, $u_{x_1} = \frac{H}{2CS^2} \cdot \frac{L}{EA} \to \infty$. which means "blow up."

~~And~~. u_{x_1} will influences the amount of F_1 and F_2 and the ~~influence~~ influence will $\to \infty$, so the F_1 and F_2 will "blow up".

CSMD | Assignment 2 | Lei Pan.



(1) : $\alpha_1 = 0$, $c = 1$, $s = 0$

(2) : $\alpha_2 = 90°$, $c = 0$, $s = 1$

(3) : $\alpha_3 = 45°$, $c = \frac{\sqrt{2}}{2}$, $s = \frac{\sqrt{2}}{2}$

(4) : $\alpha_4 = -135°$, $c = -\frac{\sqrt{2}}{2}$. $s = -\frac{\sqrt{2}}{2}$

because :

$$\underset{\sim}{k}^e = \frac{E^e A^e}{L^e} \begin{bmatrix} c^2 & sc & -c^2 & -sc \\ sc & s^2 & -sc & -s^2 \\ -c^2 & -sc & c^2 & sc \\ -sc & -s^2 & sc & s^2 \end{bmatrix}$$

so: $$K^{①} = 10 \times \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} ; \quad K^{②} = 5 \times \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

$$K^{③} = 40 \times \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} ; \quad K^{④} = 40 \times \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$
[\underset{\sim}{f}^{①}] = \left[\begin{array}{cccc|c}
10 & 0 & -10 & 0 & \\
0 & 0 & 0 & 0 & \underset{\sim}{0} \\
-10 & 0 & 10 & 0 & \\
0 & 0 & 0 & 0 & \\
\hline
 & & \underset{\sim}{0} & & \underset{\sim}{0}
\end{array}\right] [\underset{\sim}{u}^{①}]
$$

$$
[\underset{\sim}{f}^{②}] = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 5 & 0 & -5 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -5 & 0 & 5 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} [\underset{\sim}{u}^{②}]
$$

$$
[\underset{\sim}{f}^{③}] = \begin{bmatrix}
20 & 20 & 0 & 0 & 0 & 0 & -20 & -20 \\
20 & 20 & 0 & 0 & 0 & 0 & -20 & -20 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-20 & -20 & 0 & 0 & 0 & 0 & 20 & 20 \\
-20 & -20 & 0 & 0 & 0 & 0 & 20 & 20
\end{bmatrix} [\underset{\sim}{u}^{③}]
$$

$$
[\underset{\sim}{f}^{④}] = \left[\begin{array}{c|cccc}
\underset{\sim}{0} & & \underset{\sim}{0} & & \\
\hline
 & 20 & 20 & -20 & -20 \\
\underset{\sim}{0} & 20 & 20 & -20 & -20 \\
 & -20 & -20 & 20 & 20 \\
 & -20 & -20 & 20 & 20
\end{array}\right] [\underset{\sim}{u}^{③}]
$$

$$
[\underset{\sim}{f}] = \begin{bmatrix} 30 & 20 & -10 & 0 & 0 & 0 & -20 & -20 \\ 20 & 20 & 0 & 0 & 0 & 0 & -20 & -20 \\ -10 & 0 & 10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & -5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 20 & 20 & -20 & -20 \\ 0 & 0 & 0 & -5 & 20 & 25 & -20 & -20 \\ -20 & -20 & 0 & 0 & -20 & -20 & 40 & 40 \\ -20 & -20 & 0 & 0 & -20 & -20 & 40 & 40 \end{bmatrix} [\underset{\sim}{u}]
$$

$\det(\underset{\sim}{k}) = 0$, so $\underset{\sim}{k}$ is singular . ~~and the solution~~

~~Reason~~ becaus $u_{x1} = u_{y1} = u_{y2} = 0$, so modified maste stiffness is :

$$
\begin{bmatrix} 10 & 0 & 0 & 0 & 0 \\ 0 & 20 & 20 & -20 & -20 \\ 0 & 20 & 25 & -20 & -20 \\ 0 & -20 & -20 & 40 & 40 \\ 0 & -20 & -20 & 40 & 40 \end{bmatrix} \begin{bmatrix} u_{x2} \\ u_{x3} \\ u_{y3} \\ u_{x4} \\ u_{y4} \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 1 \\ f_{x4} \\ f_{y4} \end{bmatrix}
$$

$\det(\underset{\sim}{k_M}) = 0$. So $\underset{\sim}{k_M}$ is singular. the solutions are infinite.

Reason : We add a node 4 , but we don't add corresponding constraints, which causes that the DoFs > constraints, so this system becomes free system. and the solutions become infinite.

So from this. We get to know that for bars or truss problem, we just need to replace every bar with 1 element, else the simplified model is different from real situation.