Ms. Kimey K Wazare

Master of Science in Computational Mechanics 2017-2018

Computational Structural Mechanics and Dynamics

Assignment 1

On "The Direct Stiffness Method":

Consider the truss problem defined in the Figure. All geometric and material properties: L, α, E and A, as well as the applied forces P and H, are to be kept as variables. This truss has 8 degrees of freedom, with six of them removable by the fixed-displacement conditions at nodes 2, 3 and 4. This structure is statically indeterminate as long as $\alpha \neq 0$.

K.

(a) Show that the master stiffness equations are

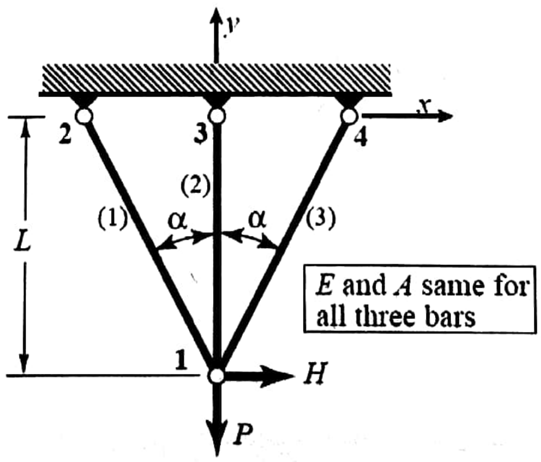
$$\frac{EA}{L}\begin{bmatrix} 2cs^2 & 0 & -cs^2 & c^2s & 0 & 0 & -cs^2 & -c^2s \\ & 1+2c^3 & c^2s & -c^3 & 0 & -1 & -c^2s & -c^3 \\ & & cs^2 & -c^2s & 0 & 0 & 0 & 0 \\ & & & c^3 & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 \\ & & & & & 1 & 0 & 0 \\ & & & & & & cs^2 & c^2s \\ \text{symm} & & & & & & & c^3 \end{bmatrix}\begin{bmatrix} u_{x1} \\ u_{y1} \\ u_{x2} \\ u_{y2} \\ u_{x3} \\ u_{y3} \\ u_{x4} \\ u_{y4} \end{bmatrix} = \begin{bmatrix} H \\ -P \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

in which $c = \cos\alpha$ and $s = \sin\alpha$. Explain from physics why the 5th row and column contain only zeros.

(b) Apply the BCs and show the 2-equation modified stiffness system.

(c) Solve for the displacements u_{x1} and u_{y1}. Check that the solution makes physical sense for the limit cases $\alpha \to 0$ and $\alpha \to \pi/2$. Why does u_{x1} "blow up" if $H \neq 0$ and $\alpha \to 0$?

(d) Recover the axial forces in the three members. Partial answer: $F^{(3)} = -H/(2s) + Pc^2/(1+2c^3)$. Why do $F^{(1)}$ and $F^{(3)}$ "blow up" if $H \neq 0$ and $\alpha \to 0$?



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Mr. Kimey Kishor Wazare
Sub - CSMD.

Assignment 1
(Direct Stiffness Matrix)

Q1) Truss problem :

→ Solution:

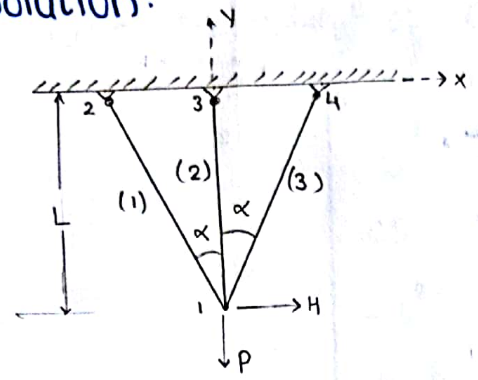


fig - Given problem

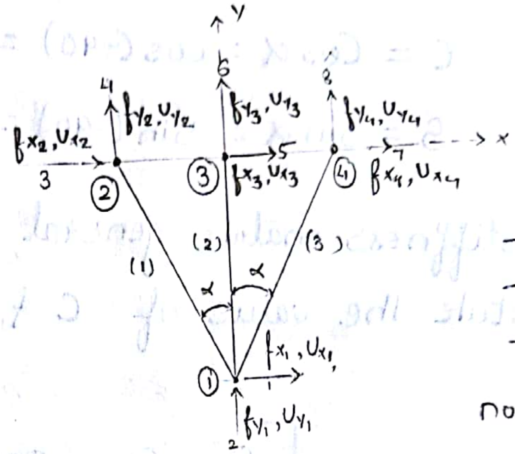


fig - FEM Model.

→ $\alpha \neq 0$
→ 8 DOF
→ Forces at node ① $(H, -P)$
→ E, A, L & α are variables (material & geometric properties)

Step ① Globalization :

$$\boxed{\bar{K}^e \bar{U}^e = \bar{f}^e}$$

where, Displacement transformation - $\bar{U}^e = T^e U^e$

force transformation - $f^e = (T^e)^T \bar{f}^e$

Stiffness - $K^e = (T^e)^T \bar{K}^e (T^e)$

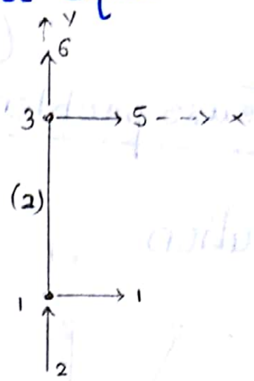
Therefore, Stiffness Matrix is :

$$K^e = \frac{E^e A^e}{L^e} \begin{bmatrix} C^2 & SC & -C^2 & -SC \\ SC & S^2 & -SC & -S^2 \\ -C^2 & -SC & C^2 & SC \\ -SC & -S^2 & SC & S^2 \end{bmatrix}$$

where , $C = \cos\alpha$ & $S = \sin\alpha$.

a

step ② Element wise ~~mass~~ stiffness matrix and equilibrium eq^n.



(i) Element ② $\alpha = -90$, $L^{(2)} = L$

$$C = \cos\alpha = \cos(-90) = 0$$

$$S = \sin\alpha = \sin(-90) = -1$$

Using stiffness matrix general formula, substitute the value of C & S resp.

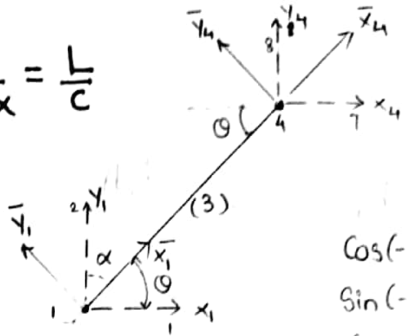
$$K_2 = \frac{EA}{L} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 5 & 6 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 5 \\ 6 \end{array}$$

The element stiffness equation –

$$f^{(2)} = K_2 \, U^{(2)}$$

$$\begin{bmatrix} f_{x_1}^{(2)} \\ f_{y_1}^{(2)} \\ f_{x_3}^{(2)} \\ f_{y_3}^{(2)} \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} U_{x_1}^{(2)} \\ U_{y_1}^{(2)} \\ U_{x_3}^{(2)} \\ U_{y_3}^{(2)} \end{bmatrix}$$

(ii) Element 3 $\quad \theta = (-90-\alpha)\ , \ L^{(3)} = \dfrac{L}{\cos\alpha} = \dfrac{L}{C}$



$\cancel{\cos\theta = +\sin}$

$\cos\theta = \cos(-90-\alpha) = -\sin\alpha = -S$

$\sin\theta = \sin(-90-\alpha) = -\cos\alpha = -C$

$\cos(-\theta) = \cos\theta$

$\sin(-\theta) = -\sin\theta$

$\cos(90-\theta) = \sin\theta$

$\sin(90-\theta) = \cos\theta$

Using stiffness matrix general formula, substitute the values of C & S resp.

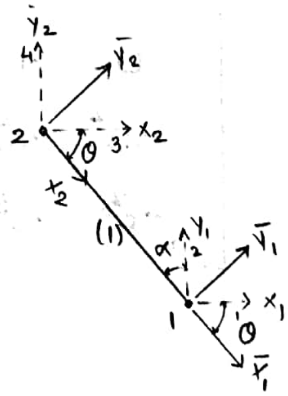
$$K_3 = \frac{EAC}{L}\ \begin{array}{c c} \begin{array}{cccc} 1 & 2 & 7 & 8 \end{array} & \\ \begin{bmatrix} S^2 & CS & -S^2 & -CS \\ CS & C^2 & -CS & -C^2 \\ -S^2 & -CS & S^2 & CS \\ -CS & -C^2 & CS & C^2 \end{bmatrix} & \begin{array}{c} 1 \\ 2 \\ 7 \\ 8 \end{array} \end{array}$$

$\therefore$, The element equation,

$$\begin{bmatrix} f_{x_1}^{(3)} \\ f_{y_1}^{(3)} \\ f_{x_4}^{(3)} \\ f_{y_4}^{(3)} \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} CS^2 & C^2S & -CS^2 & -C^2S \\ C^2S & C^3 & -C^2S & -C^3 \\ -CS^2 & -C^2S & CS^2 & C^2S \\ -C^2S & -C^3 & C^2S & C^3 \end{bmatrix} \begin{bmatrix} U_{x_1}^{(3)} \\ U_{y_1}^{(3)} \\ U_{x_4}^{(3)} \\ U_{y_4}^{(3)} \end{bmatrix}$$

(iii) Element 1

$\theta = (-90+\alpha)\ , \ L^{(1)} = \dfrac{L}{\cos\alpha} = \dfrac{L}{C}$



$\cos\theta = \cos(-90+\alpha) = \sin\alpha = S$

$\sin\theta = \sin(-90+\alpha) = -\cos\alpha = -C$

Using stiffness matrix general formula, substituting the values of C & S resp.

$$K_1 = \frac{EA}{L} \begin{array}{c} \\ \\ \\ \\ \\ \end{array} \begin{array}{cccc|c} 1 & 2 & 3 & 4 & \\ \hline \end{array}$$

$$K_1 = \frac{EA}{L}\begin{array}{cc} \begin{array}{cccc} \;\;1 & \;\;\;\;2 & \;\;\;\;3 & \;\;\;\;4 \end{array} & \\ \begin{bmatrix} cs^2 & -c^2s & -cs^2 & c^2s \\ -c^2s & c^3 & c^2s & -c^3 \\ -cs^2 & c^2s & cs^2 & -c^2s \\ c^2s & -c^3 & c^2s & c^3 \end{bmatrix} & \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} \end{array}$$

The element equation is,

$$\begin{bmatrix} f_{x_1}^{(1)} \\ f_{y_1}^{(1)} \\ f_{x_2}^{(1)} \\ f_{y_2}^{(1)} \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} cs^2 & -c^2s & -cs^2 & c^2s \\ -c^2s & c^3 & c^2s & -c^3 \\ -cs^2 & c^2s & cs^2 & -c^2s \\ c^2s & -c^3 & c^2s & c^3 \end{bmatrix} \begin{bmatrix} U_{x_1}^{(1)} \\ U_{y_1}^{(1)} \\ U_{x_2}^{(1)} \\ U_{y_2}^{(1)} \end{bmatrix}$$

step ③ The globalized stiffness Matrix - $[K = K_1 + K_2 + K_3]$

$$K = \begin{array}{cc} \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} & \\ \begin{bmatrix} 2cs^2 & 0 & -cs^2 & c^2s & 0 & 0 & -cs^2 & -c^2s \\ 0 & 1+2c^3 & c^2s & -c^3 & 0 & -1 & -c^2s & -c^3 \\ -cs^2 & c^2s & cs^2 & -c^2s & 0 & 0 & 0 & 0 \\ c^2s & -c^3 & -c^2s & c^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ -cs^2 & -c^2s & 0 & 0 & 0 & 0 & cs^2 & c^2s \\ -c^2s & -c^3 & 0 & 0 & 0 & 0 & c^2s & c^3 \end{bmatrix} & \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{array} \end{array} \frac{EA}{L}$$

Step ④ Master Stiffness Equations -

$$\begin{bmatrix} f_{x_1} \\ f_{y_1} \\ f_{x_2} \\ f_{y_2} \\ f_{x_3} \\ f_{y_3} \\ f_{x_4} \\ f_{y_4} \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} 2cs^2 & 0 & -cs^2 & c^2s & 0 & 0 & -cs^2 & -c^2s \\ 0 & 1+2c^3 & c^2s & -c^3 & 0 & -1 & -c^2s & -c^3 \\ -cs^2 & c^2s & cs^2 & -c^2s & 0 & 0 & 0 & 0 \\ c^2s & -c^3 & -c^2s & c^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ -cs^2 & -c^2s & 0 & 0 & 0 & 0 & cs^2 & c^2s \\ -c^2s & -c^3 & 0 & 0 & 0 & 0 & c^2s & c^3 \end{bmatrix} \begin{bmatrix} U_{x_1} \\ U_{y_1} \\ U_{x_2} \\ U_{y_2} \\ U_{x_3} \\ U_{y_3} \\ U_{x_4} \\ U_{y_4} \end{bmatrix}$$

Explaination: The 5^{th} row & column contains only zeros.

(i) According to the given problem only element ① ~~can~~ forces acts on node 3. in x direction. But joints are hinges, so forces acting on element ① will not contribute in x dir^n of node 3.

(ii) Due to its perpendicular configuration of x axis. So ~~&~~ 5^{th} row is zero.

(iii) Similarly, Node 3 in x dir^n will not contribute to forces on other nodes.

(b)

step ⑤ Boundary Conditions :

(i) Displacement boundary conditions -

$$U_{x_2} = U_{y_2} = U_{x_3} = U_{y_3} = U_{x_4} = U_{y_4} = 0$$

(ii) Force boundary conditions -

$$F_{x_1} = H \quad , \quad F_{y_1} = -P$$

∴, The remaining all forces at node 2, 3, 4 are reaction forces.

step ⑥ Apply BCs on step ④

$$\begin{bmatrix} H \\ -P \\ f_{x_2} \\ f_{y_2} \\ f_{x_3} \\ f_{y_3} \\ f_{x_4} \\ f_{y_4} \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} 2cs^2 & 0 & -cs^2 & c^2s & 0 & 0 & -cs^2 & -c^2s \\ 0 & 1+2c^3 & c^2s & -c^3 & 0 & -1 & -c^2s & -c^3 \\ -cs^2 & c^2s & cs^2 & -c^2s & 0 & 0 & 0 & 0 \\ c^2s & -c^3 & -c^2s & c^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ -cs^2 & -c^2s & 0 & 0 & 0 & 0 & cs^2 & c^2s \\ -c^2s & -c^3 & 0 & 0 & 0 & 0 & c^2s & c^3 \end{bmatrix} \begin{bmatrix} U_{x_1} \\ U_{y_1} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

step ⑦ Modified stiffness system,

$$\begin{bmatrix} H \\ -P \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} 2cs^2 & 0 \\ 0 & 1+2c^3 \end{bmatrix} \begin{bmatrix} U_{x_1} \\ U_{y_1} \end{bmatrix}$$

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step 8 For displacements-

(i)

$$\begin{bmatrix} U_{x_1} \\ U_{y_1} \end{bmatrix} = \frac{L}{EA} \begin{bmatrix} 2cs^2 & 0 \\ 0 & 1+2c^3 \end{bmatrix}^{-1} \begin{bmatrix} H \\ -P \end{bmatrix}$$

$$\begin{bmatrix} U_{x_1} \\ U_{y_1} \end{bmatrix} = \frac{L}{EA(2cs^2)(1+2c^3)} \begin{bmatrix} 1+2c^3, & 0 \\ 0 & 2cs^2 \end{bmatrix} \begin{bmatrix} H \\ -P \end{bmatrix}$$

(ii) Checking for limit $\underline{\underline{\alpha \to 0}}$

$$U_{x_1} = \lim_{\alpha \to 0} \frac{LH}{EA(2cs^2)}$$

$\therefore$, $U_{x_1} \longrightarrow \infty$

$$U_{y_1} = \lim_{\alpha \to 0} \frac{-LP}{EA(1+2c^3)} = \frac{-LP}{3EA}$$

(iii) checking for limit $\alpha \to \pi/2$

$$U_{x_1} = \lim_{\alpha \to \pi/2} \frac{LH}{2EAcs^2}$$

$U_{x_1} \longrightarrow \infty$

$$U_{y_1} = \lim_{\alpha \to \pi/2} \frac{-LP}{EA(1+2c^3)} = \frac{-LP}{EA}$$

(iv) Reason: The element ① & ③ merges with element ② to form a oscillating ~~pe~~ system, which rotate about the node 3. Because U_{x_1} blows up as $\alpha \to 0$ & $H \neq 0$.

(d)

step (9) Axial forces in the 3 members.

$f_L \rightarrow$ local forces

$K_L \rightarrow$ local stiffness

$U_g \rightarrow$ global displacement

$$[f_L] = [K_L][T][U_g]$$

(i) Element (1)

$$\begin{bmatrix} f_{x_1}^{(1)} \\ f_{y_1}^{(1)} \\ f_{x_2}^{(1)} \\ f_{y_2}^{(1)} \end{bmatrix} = \frac{EAc}{L} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} s & -c & 0 & 0 \\ c & s & 0 & 0 \\ 0 & 0 & s & -c \\ 0 & 0 & c & s \end{bmatrix} \begin{bmatrix} \frac{LH}{2EAcs^2} \\ \frac{-LP}{EA(1+2c^3)} \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} f_{x_1}^{(1)} \\ f_{y_1}^{(1)} \\ f_{x_2}^{(1)} \\ f_{y_2}^{(1)} \end{bmatrix} = \begin{bmatrix} \frac{H}{2s} + \frac{Pc^2}{1+2c^3} \\ 0 \\ \frac{-H}{2s} - \frac{Pc^2}{1+2c^3} \\ 0 \end{bmatrix}$$

(ii) Element (2)

$$\begin{bmatrix} f_{x_1}^{(2)} \\ f_{y_1}^{(2)} \\ f_{x_3}^{(2)} \\ f_{y_3}^{(2)} \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} \frac{LH}{2EAcs^2} \\ \frac{-LP}{EA(1+2c^3)} \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} f_{x_1}^{(2)} \\ f_{y_1}^{(2)} \\ f_{x_3}^{(2)} \\ f_{y_3}^{(2)} \end{bmatrix} = \begin{bmatrix} -\frac{P}{1+2c^3} \\ 0 \\ \frac{P}{1+2c^3} \\ 0 \end{bmatrix}$$

(iii) <u>Element ③</u>

$$\begin{bmatrix} f_{x_1}^{(3)} \\ f_{y_1}^{(3)} \\ f_{x_4}^{(3)} \\ f_{y_4}^{(3)} \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} c & s & 0 & 0 \\ -s & c & 0 & 0 \\ 0 & 0 & c & s \\ 0 & 0 & -s & c \end{bmatrix} \begin{bmatrix} \frac{LH}{2EAcs^2} \\ \frac{-LP}{EA(1+2c^3)} \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} f_{x_1}^{(3)} \\ f_{y_1}^{(3)} \\ f_{x_4}^{(3)} \\ f_{y_4}^{(3)} \end{bmatrix} = \begin{bmatrix} -\frac{H}{2s} - \frac{Pc^2}{1+2c^3} \\ 0 \\ \frac{H}{2s} + \frac{Pc^2}{1+2c^3} \\ 0 \end{bmatrix}$$

(iv) <u>Reason</u>: $f^{(1)}$ & $f^{(3)}$ blows up because when α tends to zero, the structure couldn't resist the force in x direction. And starts rotating at node 3, which result in giving up the stability. The structure becomes unstable & collapses.

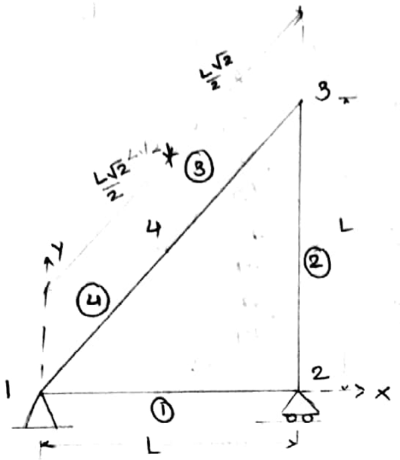
Assignment 2

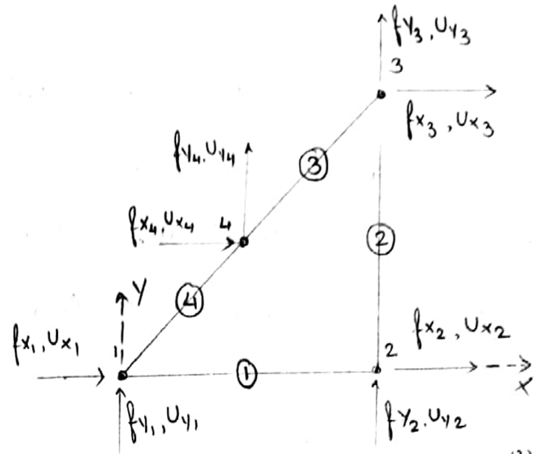
Dr. Who proposes "improving" the result for the example truss of the 1st lesson by putting one extra node, 4 at the midpoint of member (3) 1-3, so that it is subdivided in two different members: (3) 1-4 and (4) 3-4. His "reasoning" is that more is better. Try Dr. Who's suggestion by hand computations and verify that the solution "blows up" because the modified master stiffness is singular. Explain physically.

Date of Assignment: 05/02/2018
Date of Submission: 12/02/2018

Q2)

→ Solution:





$L^{(1)} = 10$

$L^{(2)} = 10$

$L^{(3)} = 5\sqrt{2}$

$L^{(4)} = 5\sqrt{2}$

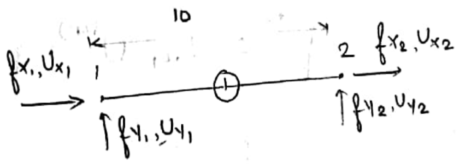
$E^{(1)}A^{(1)} = 100$

$E^{(2)}A^{(2)} = 50$

$E^{(3)}A^{(3)} = E^{(4)}A^{(4)} = 200\sqrt{2}$

Step ① Element stiffness Matrix.

Element ① :

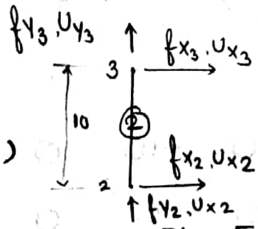


$$f^{(1)} = K_1 U^{(1)}$$

$$\left(\frac{EA}{L}\right)^{(1)} = \frac{100}{10} = 10$$

$$\begin{bmatrix} f_{x_1}^{(1)} \\ f_{y_1}^{(1)} \\ f_{x_2}^{(1)} \\ f_{y_2}^{(1)} \end{bmatrix} = 10 \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} U_{x_1}^{(1)} \\ U_{y_1}^{(1)} \\ U_{x_2}^{(1)} \\ U_{y_2}^{(1)} \end{bmatrix}$$

Element ② :

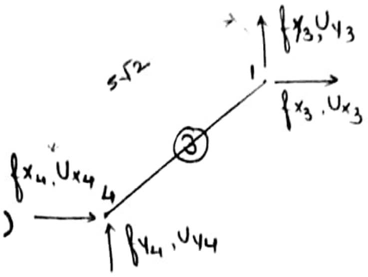


$$f^{(2)} = K_2 U^{(2)}$$

$$\left(\frac{EA}{L}\right)^{(2)} = \frac{50}{10} = 5$$

$$\begin{bmatrix} f_{x_2}^{(2)} \\ f_{y_2}^{(2)} \\ f_{x_3}^{(2)} \\ f_{y_3}^{(2)} \end{bmatrix} = 5 \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} U_{x_2}^{(2)} \\ U_{y_2}^{(2)} \\ U_{x_3}^{(2)} \\ U_{y_3}^{(2)} \end{bmatrix}$$

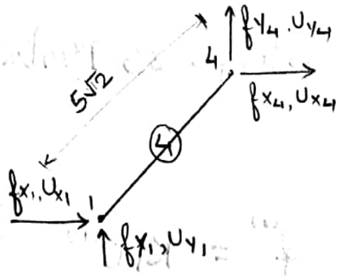
Element ③



$$f^{(3)} = K_3\, U^{(3)} \qquad \left(\frac{EA}{L}\right)^{(3)} = \frac{200\sqrt{2}}{5\sqrt{2}} = 40$$

$$\begin{bmatrix} f_{x_3}^{(3)} \\ f_{y_3}^{(3)} \\ f_{x_4}^{(3)} \\ f_{y_4}^{(3)} \end{bmatrix} = 40 \begin{bmatrix} 0.5 & 0.5 & -0.5 & -0.5 \\ 0.5 & 0.5 & -0.5 & -0.5 \\ -0.5 & -0.5 & 0.5 & 0.5 \\ -0.5 & -0.5 & 0.5 & 0.5 \end{bmatrix} \begin{bmatrix} U_{x_3}^{(3)} \\ U_{y_3}^{(3)} \\ U_{x_4}^{(3)} \\ U_{y_4}^{(3)} \end{bmatrix}$$

Element ④



$$f^{(4)} = K_4\, U^{(4)} \qquad \left(\frac{EA}{L}\right)^{(4)} = 40$$

$$\begin{bmatrix} f_{x_1}^{(4)} \\ f_{y_1}^{(4)} \\ f_{x_4}^{(4)} \\ f_{y_4}^{(4)} \end{bmatrix} = 40 \begin{bmatrix} 0.5 & 0.5 & -0.5 & -0.5 \\ 0.5 & 0.5 & -0.5 & -0.5 \\ -0.5 & -0.5 & 0.5 & 0.5 \\ -0.5 & -0.5 & 0.5 & 0.5 \end{bmatrix} \begin{bmatrix} U_{x_1}^{(4)} \\ U_{y_1}^{(4)} \\ U_{x_4}^{(4)} \\ U_{y_4}^{(4)} \end{bmatrix}$$

step ② Global Stiffness Matrix : $(K = K_1 + K_2 + K_3 + K_4)$

$$K = \begin{array}{c} \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\ \begin{bmatrix} 30 & 20 & -10 & 0 & 0 & 0 & -20 & -20 \\ 20 & 20 & 0 & 0 & 0 & 0 & -20 & -20 \\ -10 & 0 & 10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & -5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 20 & 20 & -20 & -20 \\ 0 & 0 & 0 & -5 & 20 & 25 & -20 & -20 \\ -20 & -20 & 0 & 0 & -20 & -20 & 40 & 40 \\ -20 & -20 & 0 & 0 & -20 & -20 & 40 & 40 \end{bmatrix} \\ \begin{array}{cccccccc} \times & \times & & \times & & & & \end{array} \end{array} \begin{array}{cc} 1 & \times \\ 2 & \times \\ 3 & \\ 4 & \times \\ 5 & \\ 6 & \\ 7 & \\ 8 & \end{array}$$

Step ④ Assemble.

$$\boxed{f = K\,U}$$

$$\begin{bmatrix} f_{x_1} \\ f_{y_1} \\ f_{x_2} \\ f_{y_2} \\ f_{x_3} \\ f_{y_3} \\ f_{x_4} \\ f_{y_4} \end{bmatrix} = \begin{bmatrix} 30 & 20 & -10 & 0 & 0 & 0 & -20 & -20 \\ 20 & 20 & 0 & 0 & 0 & 0 & -20 & -20 \\ -10 & 0 & 10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & -5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 20 & 20 & -20 & -20 \\ 0 & 0 & 0 & -5 & 20 & 25 & -20 & -20 \\ -20 & -20 & 0 & 0 & -20 & -20 & 40 & 40 \\ -20 & -20 & 0 & 0 & -20 & -20 & 40 & 40 \end{bmatrix} \begin{bmatrix} U_{x_1} \\ U_{y_1} \\ U_{x_2} \\ U_{y_2} \\ U_{x_3} \\ U_{y_3} \\ U_{x_4} \\ U_{y_4} \end{bmatrix}$$

Step ④ Applying Support and loading.

(i) Displacement Boundary Conditions -

$$U_{x_1} = U_{y_1} = U_{y_2} = 0$$

(ii) Force Boundary Conditions -

$$f_{x_2} = 0 \;,\; f_{x_4} = f_{y_4} = 0 \;,\; f_{x_3} = 2 \;,\; f_{y_3} = 1$$

$$f_{x_1} = R_{x_1} \;,\; f_{y_1} = R_{y_1} \;,\; f_{y_2} = R_{y_2}$$

Substituting the boundary conditions -

$$\begin{bmatrix} R_{x_1} \\ R_{y_1} \\ 0 \\ R_{y_2} \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} & & \\ & K & \\ & & \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ U_{x_2} \\ 0 \\ U_{x_3} \\ U_{y_3} \\ U_{x_4} \\ U_{y_4} \end{bmatrix}$$

step ⑤

Reduced stiffness equation -

$$\begin{bmatrix} 0 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 10 & 0 & 0 & 0 & 0 \\ 0 & 20 & 20 & -20 & -20 \\ 0 & 20 & 25 & -20 & -20 \\ 0 & -20 & -20 & 40 & 40 \\ 0 & -20 & -20 & 40 & 40 \end{bmatrix} \begin{bmatrix} U_{x_2} \\ U_{x_3} \\ U_{y_3} \\ U_{x_4} \\ U_{y_4} \end{bmatrix}$$

Step ⑥ Physical Significance –

(i) In the solved example (without extranode), there is no redundancy. If any member is knocked off, then the entire system collapses as a mechanism.

(ii) In the solved problem (with one extra node), there is redundancy. Its necessary to strengthen the structure by adding members, for stability.

(iii) Therefore, the truss secures a high strength and additional equations is satisfied by displacement compatibility conditions.

(iv) In general for practical structures with a high redundancy number, its ~~ccssy~~ vital to carryout displacement formulation, (equilibrium of forces & displacement compatibility at joints).

—— x —— x ——