MAESTRÍA EN INGENIERÍA ESTRUCTURAL Y DE CONSTRUCCIÓN
U N I V E R S I T A T   P O L I T É C N I C A   D E   C A T A L U N Y A

TRABAJO N°02:
FEM Modelling: Introduction
Variational Formulation

Student:

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Assignment 2.1

On "FEM Modelling: Introduction":

1. Identify the symmetry and antisymmetry lines in the two-dimensional problems illustrated in the figure. They are:

(a) a circular disk under two diametrically opposite point forces (the famous "Brazilian test" for concrete)
(b) the same disk under two diametrically opposite force pairs
(c) a clamped semiannulus under a force pair oriented as shown
(d) a stretched rectangular plate with a central circular hole.
(e) and (f) are half-planes under concentrated loads.

2. Having identified those symmetry/antisymmetry lines, state whether it is possible to cut the complete structure to one half or one quarter before laying out a finite element mesh. Then draw a coarse FE mesh indicating, with rollers or fixed supports, which kind of displacement BCs you would specify on the symmetry or antisymmetry lines.

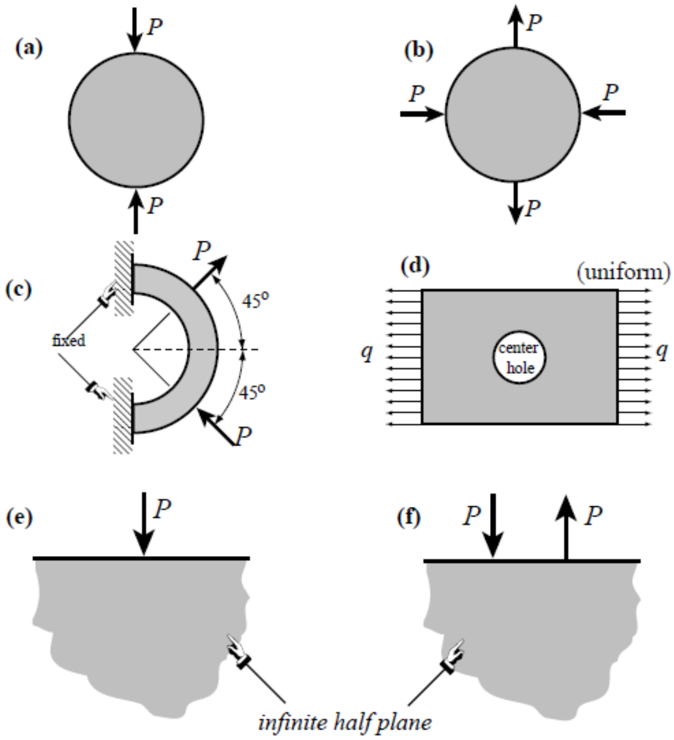


Figure 2.1.- Problems for assignment 2.1

Assignment 2.2

On “FEM Modelling: Introduction”:

1. The plate structure shown in the figure is loaded and deforms in the plane of the paper. The applied load at *D* and the supports at *I* and *N* extend over a fairly narrow area. List what you think are the likely “trouble spots” that would require a locally finer finite element mesh to capture high stress gradients. Identify those spots by its letter and a reason.

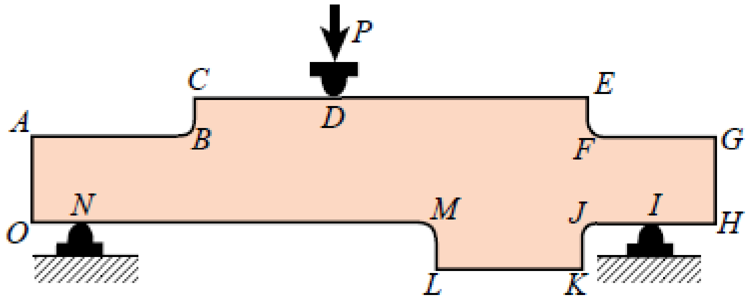


Figure 2.2.- Inplane bent plate

Assignment 2.3

On “Variational Formulation”:

1. A tapered bar element of length l and areas A_i and A_j with A interpolated as

$$A = A_i(1-\xi) + A_j\xi$$

and constant density ρ rotates on a plane at uniform angular velocity ω (rad/sec) about node i. Taking axis x along the rotating bar with origin at node i , the centrifugal axial force is $q(x)=\rho A\omega^2 x$ along the length in which x is the longitudinal coordinate $x=x^e$.

Find the consistent node forces as functions of ρ, A_i, A_j, ω and l, and specialize the result to the prismatic bar $A = A_i = A_j$.

Date of Assignment: 12 / 02 / 2018
Date of Submission: 19 / 02 / 2018

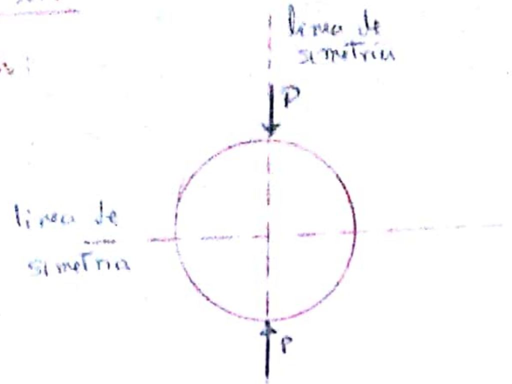
The assignment must be submitted as a pdf file named **As2-Surname.pdf** to the CIMNE virtual center.

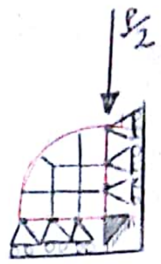
Problema 2.1

Solución:

(1) (2)

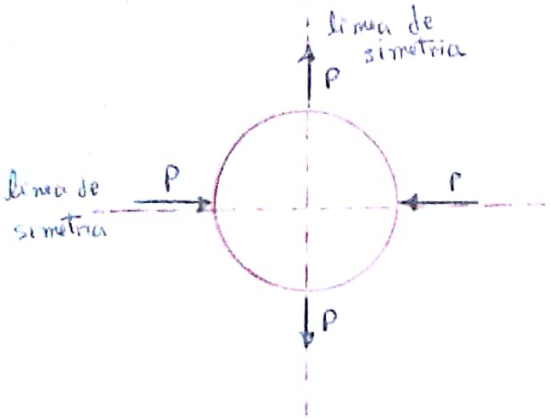
(a)

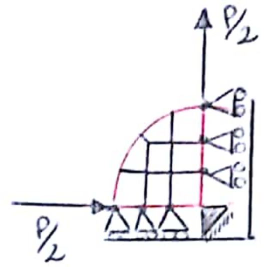




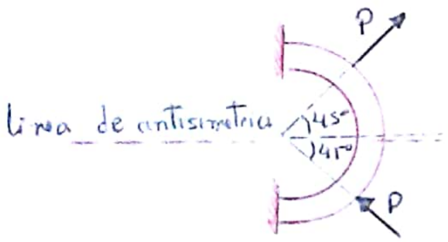
En mallado de curvas mediante coord. isoparamétricas Zienkiewicz, O.C y D.V. Philips.

(b)

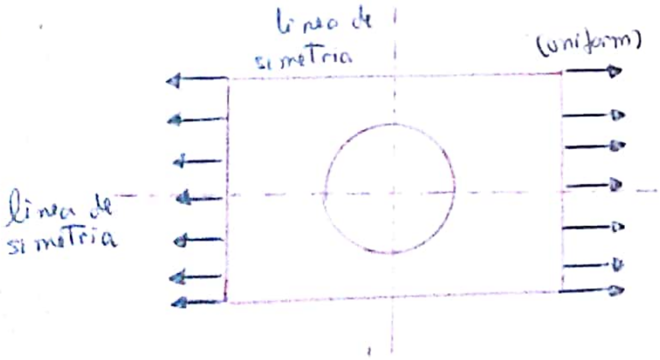


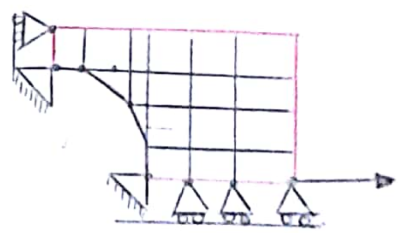


(c)

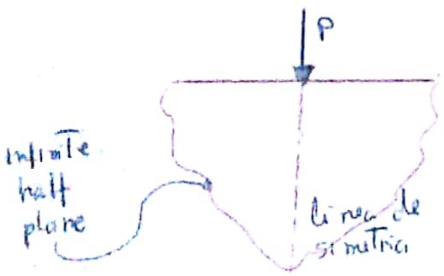


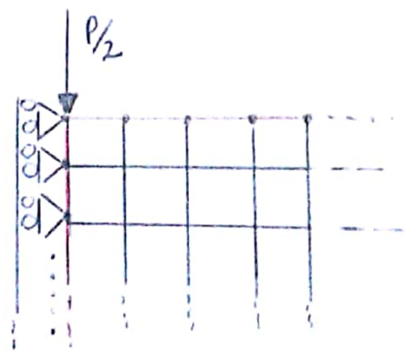
(d)



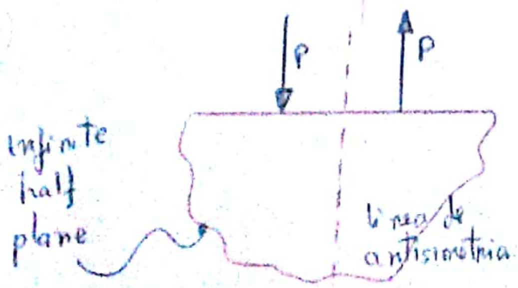


(e)



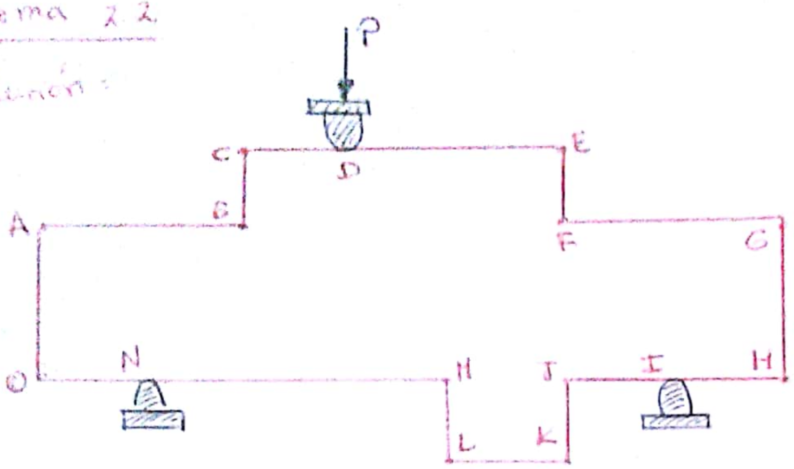


(f)

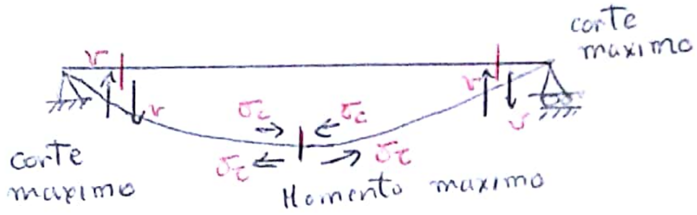


Problema 2.2

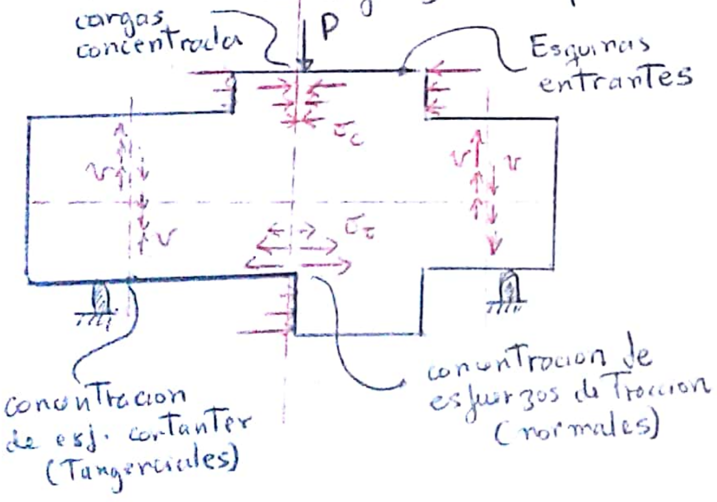
Solución:



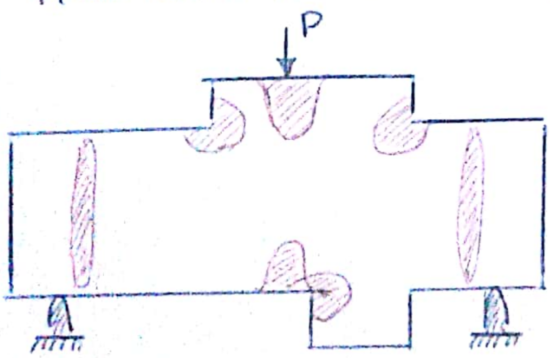
1° Para conocer la distribucion de esfuerzos, recordaremos el comportamiento a flexion de una viga; aproximadamente; el caso solicitado será similar:



2° Situamos los esfuerzos a presentarse:



3° Finalmente indicamos las zonas de gradientes de alto estres:



Problema 23

Solución:

* Considerando el area interpolada de la barra conica:

$$A = A_i(1-\xi) + A_j\xi \quad \ldots (I)$$

* Considerando la representacion o variación de la fuerza centrifuga en función de la posición, ademas de otros parametros:

$$q(x) = \rho \cdot A \cdot \omega^2 \cdot x \quad \ldots (II)$$

* Considerando los metodos variacionales, para determinar el vector de fuerzas ext. nodales, Tenemos:

$$f_{ext} = \int_0^1 q \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} l\, d\xi \quad \ldots (III)$$

Reemplazando (II) en (III):

$$f_{ext} = \int_0^1 \rho A \omega^2 l \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} l\, d\xi$$

$$f_{ext} = \int_0^1 \rho(A_i(1-\xi) + A_j\xi) \cdot \omega^2 \cdot l \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} \cdot l\, d\xi$$

$$f_{ext} = \rho \omega^2 l^2 \int_0^1 (A_i - A_i\xi + A_j\xi) \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} d\xi$$

$$f_{ext} = \rho \omega^2 l^2 \int_0^1 \begin{bmatrix} A_i - A_i\xi + A_j\xi - A_i\xi + A_i\xi^2 - A_j\xi^2 \\ A_i\xi - A_i\xi^2 + A_j\xi^2 \end{bmatrix} d\xi$$

$$f_{ext} = \rho \omega^2 l^2 \left(\begin{bmatrix} A_i\xi - \frac{2A_i\xi^2}{2} + \frac{A_j\xi^2}{2} + \frac{A_i\xi^3}{3} - \frac{A_j\xi^3}{3} \\ \frac{A_i\xi^2}{2} - \frac{A_i\xi^3}{3} + \frac{A_j\xi^3}{3} \end{bmatrix} \right) \Bigg|_0^1$$

$$f_{ext} = \rho \omega^2 l^2 \begin{bmatrix} A_i(1) - \frac{2(A_i)}{2} + \frac{A_j}{2} + \frac{A_i}{3} - \frac{A_j}{3} \\ \frac{A_i}{2} \quad \frac{A_i}{3} \quad \frac{A_j}{3} \end{bmatrix}$$

$$f_{ext} = \rho \omega^2 l^2 \begin{bmatrix} \frac{A_i}{3} + \frac{A_j}{6} \\ \frac{A_i}{6} + \frac{A_j}{3} \end{bmatrix}$$

* finalmente para el caso $A = A_i = A_j$: $\quad f_{ext} = \rho \omega^2 l^2 \begin{bmatrix} \frac{A}{2} \\ \frac{A}{2} \end{bmatrix}$