Assignment 5

Samadrita Karmakar

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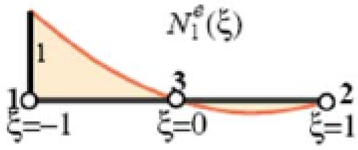
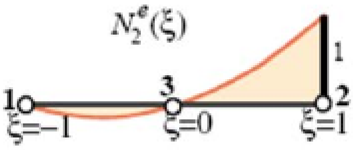
PROBLEM 5.1

a)

The Formulation of Shape Function has been done by Direct Method

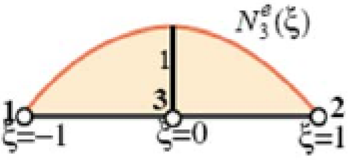


$N_1^e(\xi) = a_0 + a_1\xi + a_2\xi$ Let, $N_1^e = c_{f1}\xi(\xi - 1)$ at node 1, $N_1^e = 1$ Hence,
$\rightarrow 1 = c_{f1}\xi(\xi - 1)$
putting $\xi = -1$, the value of Node 1,
we have,
$\rightarrow c_{f1} = \frac{1}{2}$
so, $N_1^e = \frac{-1}{2}\xi + \frac{1}{2}\xi^2$



$$N_2^e(\xi) = b_0 + b_1\xi + b_2\xi$$

Let, $N_2^e = c_{f2}\xi(\xi + 1)$ at node 2, $N_2^e = 1$ Hence,
$\rightarrow 1 = c_{f2}\xi(\xi + 1)$
putting $\xi = 1$, the value of Node 2,
we have,
$\rightarrow c_{f2} = \frac{1}{2}$
so, $N_2^e = \frac{1}{2}\xi + \frac{1}{2}\xi^2$



$$N_2^e(\xi) = c_0 + c_1\xi + c_2\xi$$

Let, $N_2^e = c_{f3}(\xi - 1)(\xi + 1)$ at node 3, $N_3^e = 1$ Hence,
$\rightarrow 1 = c_{f3}(\xi - 1)(\xi + 1)$
putting $\xi = 0$, the value of Node 3,
we have,

$\rightarrow c_{f3} = -1$
so, $N_3^e = 1 - \xi^2$

Hence we have,
$a_0 = 0; a_1 = -\frac{1}{2}; a_2 = \frac{1}{2}$
$b_0 = 0; b_1 = \frac{1}{2}; b_2 = \frac{1}{2}$
$c_0 = 1; c_1 = 0; c_2 = -1$

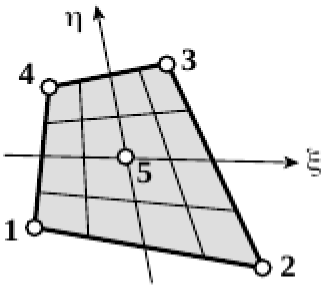
b)

Sum of Shape functions $= N_1^e + N_2^e + N_3^e$
$= -\frac{1}{2}\xi + \frac{1}{2}\xi^2 + \frac{1}{2}\xi + \frac{1}{2}\xi^2 + 1 - \xi^2$
$= 1$

c)

$\frac{dN_1^e}{d\xi} = -\frac{1}{2} + \xi$
$\frac{dN_2^e}{d\xi} = \frac{1}{2} + \xi$
$\frac{dN_2^e}{d\xi} = -2\xi$

PROBLEM 5.2



We will be using the hierarchial approach for this problem We first solve for N_5^e
Let, $N_5^e = c_5(\xi + 1)(\xi - 1)(\eta - 1)(\eta + 1)$
putting $N_5^e = 1$ and $\xi = 1$ we have,
$c_5 = 1$
so, $N_5^e = (\xi + 1)(\xi - 1)(\eta - 1)(\eta + 1)$

Now,
Let, $N_1^e = c_1(\xi - 1)(\eta - 1)$
putting $N_1^e = 1$, $\xi = -1$ and $\eta = -1$ we have

$\rightarrow c_1 = \frac{1}{4}$
so, $N_1^e = \frac{1}{4}(\xi - 1)(\eta - 1)$

Now,
Let, $N_2^e = c_2(\xi + 1)(\eta - 1)$
putting $N_2^e = 1$, $\xi = 1$ and $\eta = -1$ we have
$\rightarrow c_2 = -\frac{1}{4}$
so, $N_2^e = -\frac{1}{4}(\xi + 1)(\eta - 1)$

Now,
Let, $N_3^e = c_3(\xi + 1)(\eta + 1)$
putting $N_3^e = 1$, $\xi = 1$ and $\eta = 1$ we have
$\rightarrow c_3 = \frac{1}{4}$
so, $N_3^e = \frac{1}{4}(\xi + 1)(\eta + 1)$

Now,
Let, $N_4^e = c_4(\xi - 1)(\eta + 1)$
putting $N_4^e = 1$, $\xi = -1$ and $\eta = 1$ we have
$\rightarrow c_4 = -\frac{1}{4}$
so, $N_4^e = -\frac{1}{4}(\xi - 1)(\eta + 1)$

Considering N_1 over node 5,
$N_1^e = \frac{1}{4}(\xi - 1)(\eta - 1) + g_1(\eta^2 - 1)(\xi^2 - 1)$
at node 5, $\xi = 0$, $\eta = 0$, $N_1^e = 0$
we get
$g_1 = -\frac{1}{4}$
so, $N_1^e = \frac{1}{4}(\xi - 1)(\eta - 1) - \frac{1}{4}(\eta^2 - 1)(\xi^2 - 1)$

Considering N_2 over node 5,
$N_2^e = -\frac{1}{4}(\xi + 1)(\eta - 1) + g_2(\eta^2 - 1)(\xi^2 - 1)$
at node 5, $\xi = 0$, $\eta = 0$, $N_2^e = 0$
we get
$g_2 = -\frac{1}{4}$
so, $N_2^e = -\frac{1}{4}(\xi + 1)(\eta - 1) - \frac{1}{4}(\eta^2 - 1)(\xi^2 - 1)$

Considering N_3 over node 5,
$N_3^e = \frac{1}{4}(\xi+1)(\eta+1) + g_3(\eta^2-1)(\xi^2-1)$
at node 5, $\xi = 0$, $\eta = 0$, $N_3^e = 0$
we get
$g_3 = -\frac{1}{4}$
so,$N_3^e = +\frac{1}{4}(\xi+1)(\eta+1) - \frac{1}{4}(\eta^2-1)(\xi^2-1)$

Considering N_4 over node 5,
$N_4^e = -\frac{1}{4}(\xi-1)(\eta+1) + g_4(\eta^2-1)(\xi^2-1)$
at node 5, $\xi = 0$, $\eta = 0$, $N_4^e = 0$
we get
$g_4 = -\frac{1}{4}$
so,$N_4^e = + - \frac{1}{4}(\xi-1)(\eta+1) - \frac{1}{4}(\eta^2-1)(\xi^2-1)$

$N_1^e + N_2^e + N_3^e + N_4^e + N_5^e =$
$\frac{1}{4}(\xi-1)(\eta-1) - \frac{1}{4}(\eta^2-1)(\xi^2-1)$
$-\frac{1}{4}(\xi+1)(\eta-1) - \frac{1}{4}(\eta^2-1)(\xi^2-1)$
$+\frac{1}{4}(\xi+1)(\eta+1) - \frac{1}{4}(\eta^2-1)(\xi^2-1)$
$-\frac{1}{4}(\xi-1)(\eta+1) - \frac{1}{4}(\eta^2-1)(\xi^2-1)$
$+(\eta^2-1)(\xi^2-1) = 1$

Problem 5.3

$n_E n_G >= n_F - n_R$
For Hexahedron,
$n_E = 6$, order of the stress-stain matrix $\mathbf{E}$
n_G is the Number of Gauss Points
$n_F = n \times 3$ Number of Degrees of Freedom where n is the number of nodes
$n_R = 6$ Number of Independent Rigid Body Modes

n	n_F	$n_F - 6$	Recomended Rule
8	24	18	$2 \times 2 \times 2$
20	60	54	$3 \times 3 \times 3$
27	81	75	$3 \times 3 \times 3$
64	192	186	$4 \times 4 \times 4$