Assignment 6

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Assignment a)

To program in Matlab the Timoshenko 2 Nodes Beam element with reduce integration for the shear stiffness matrix

Solution:
A new stiffness matrix added is according to the solution for Reduced Integration for the two node Timoshenko beam element, acheived by using only one integration point. The resultant matrices are,

$$K_b^(e) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

and,

$$K_b^(e) = \begin{bmatrix} 1 & \frac{l}{2} & -1 & \frac{l}{2} \\ \frac{l}{2} & \frac{l^2}{4} & -\frac{l}{2} & \frac{l^2}{4} \\ -1 & -\frac{l}{2} & 1 & -\frac{l}{2} \\ \frac{l}{2} & \frac{l^2}{4} & -\frac{l}{2} & \frac{l^2}{4} \end{bmatrix}$$

To acheive the above objective, the following lines of code were added/edited to the Matlab Program

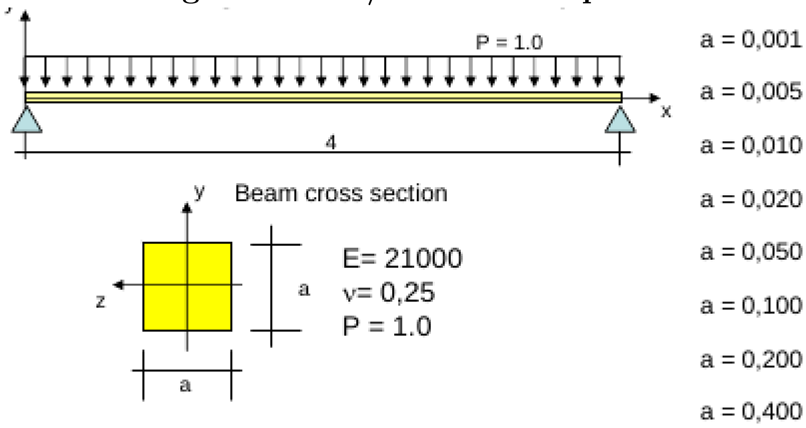
```
  ttim = timing('Time needed to  set initial values',ttim); %Reporting time

  % 0 to use Timoshenko 1 to use Reduced Stiffness Method
  reducedStiffness=0;

% Element cycle
  for ielem = 1 : nelem
```

and,

```
% Element cycle
  for ielem = 1 : nelem

    lnods(1:nnode) = elements(ielem,1:nnode);

    coor_x(1:nnode) = coordinates(lnods(1:nnode),1);   % Elem. X coordinate

    len = coor_x(2) - coor_x(1);  % x_j > x_i

    const = D_matb/len;

    K_b = [ 0 ,  0 ,  0 ,  0 ;
            0 ,  1 ,  0 , -1 ;
            0 ,  0 ,  0 ,  0 ;
            0 , -1 ,  0 ,  1 ];

    K_b = K_b * const;

    if reducedStiffness==0

        const = D_mats/len;

        K_s = [   1    ,   len/2 ,   -1    ,   len/2 ;
                len/2 , len^2/3 , -len/2 , len^2/6 ;
                -1    ,  -len/2 ,    1    ,  -len/2 ;
                len/2 , len^2/6 , -len/2 , len^2/3 ];

        K_s = K_s * const;
    else

        const = D_matsReduced/len;

        K_s = [   1    ,   len/2 ,   -1    ,   len/2 ;
                len/2 , len^2/4 , -len/2 , len^2/4 ;
                -1    ,  -len/2 ,    1    ,  -len/2 ;
                len/2 , len^2/4 , -len/2 , len^2/4 ];

        K_s = K_s * const;
    end
```

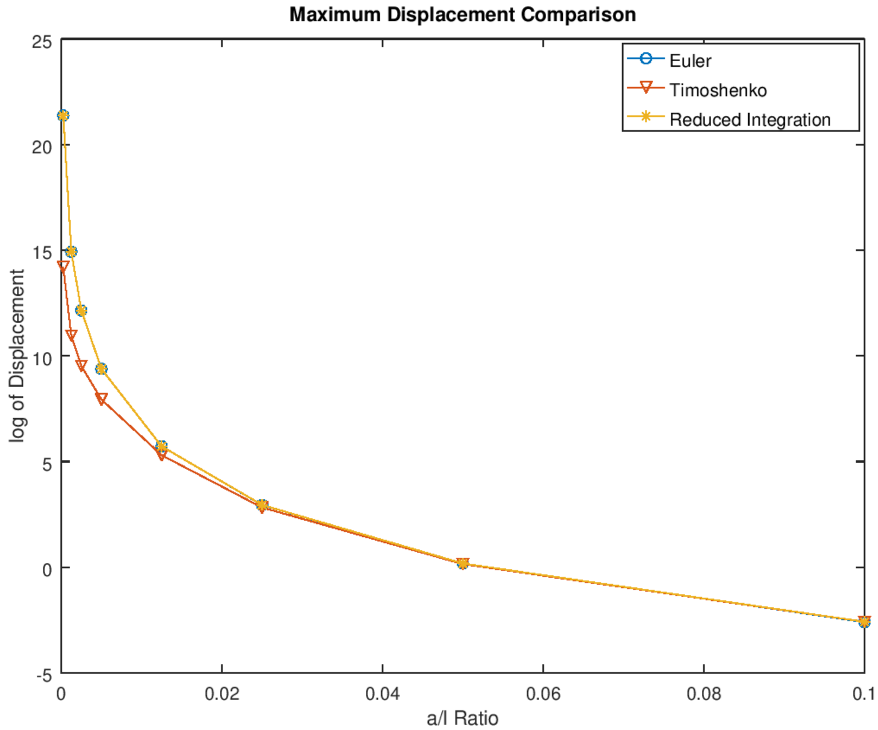
Assignment b)

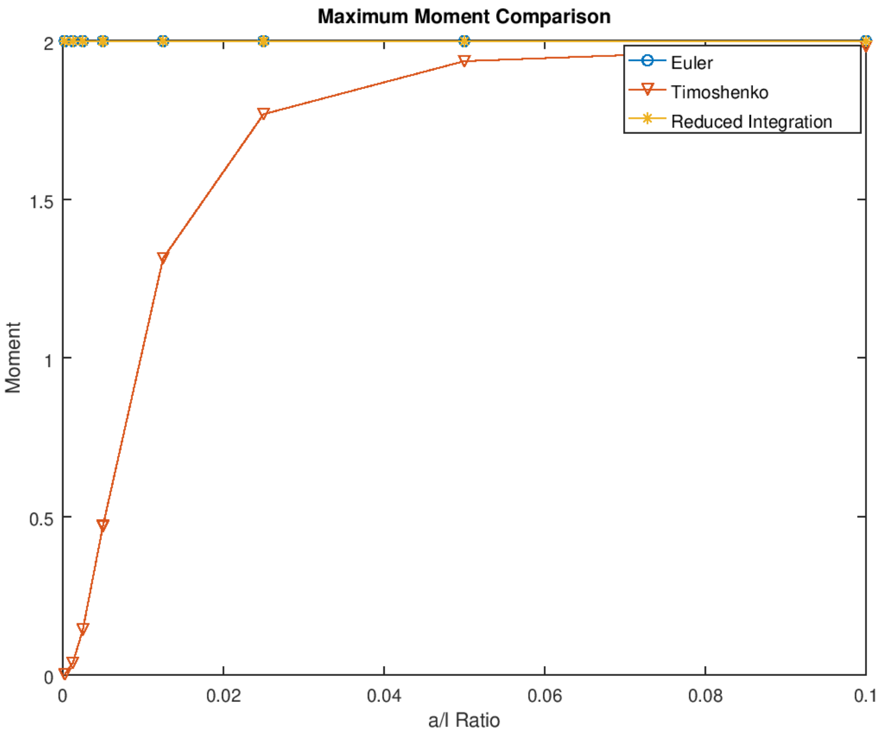
To solve the following problem with a 64 element mesh with the
2 nodes Euler Bernulli element
2 nodes Timoshenko Full Integrate element
2 nodes Timoshenko Reduce Integration element.
and, compare maximum displacements, moments and shear for the 3 elements against the a/L relationship

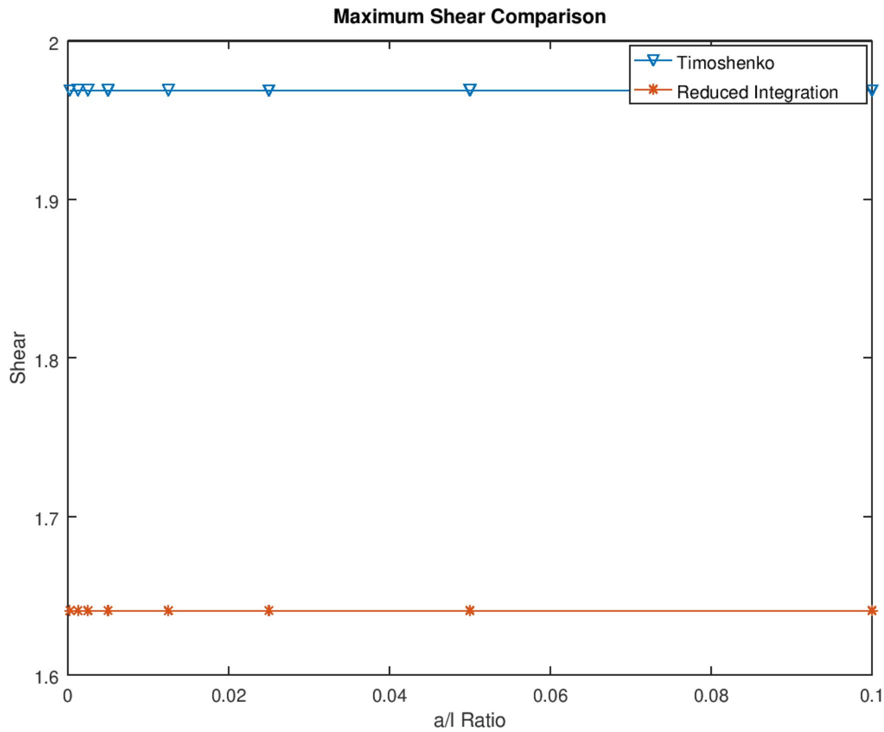


Solution:

Timoshenko beam element typically results in a stiffer approximation. This typically occurs at low $\frac{a}{L}$ ($\frac{a}{L} < \frac{1}{10}$) ratio. This is mitigated by using Reduced Integration Method where the number of gauss points used are less than that required. This was observed in the obtained graphs. The values obtained by Reduced Integration Method matches the results obtained by Euler Method.


Maximum Displacement Comparison
Euler
Timoshenko
Reduced Integration
log of Displacement
a/l Ratio
25
20
15
10
5
0
-5
0
0.02
0.04
0.06
0.08
0.1


Maximum Moment Comparison
Euler
Timoshenko
Reduced Integration
Moment
a/l Ratio
2
1.5
1
0.5
0
0
0.02
0.04
0.06
0.08
0.1


Maximum Shear Comparison
Timoshenko
Reduced Integration
Shear
a/l Ratio
2
1.9
1.8
1.7
1.6
0
0.02
0.04
0.06
0.08
0.1