Plate

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Assignment a)

In assignment a), we have been given two types of plates. One is a thick plate with four thin plates connecting at the top plane of the thick plate. And another one is a thick plate with four thin plates connecting at the central plane so all the sub-plates' middle planes are aligned while a.1's plates are not. But a.1 and a.2 are equal because no matter which plate theory we are going to choose, both Reissner-Mindlin plate theory and Kirchho plate theory have promised that the points along the same normal to the middle plane have the same vertical displacement. Thus, the continuity along the connecting lines has been promised.

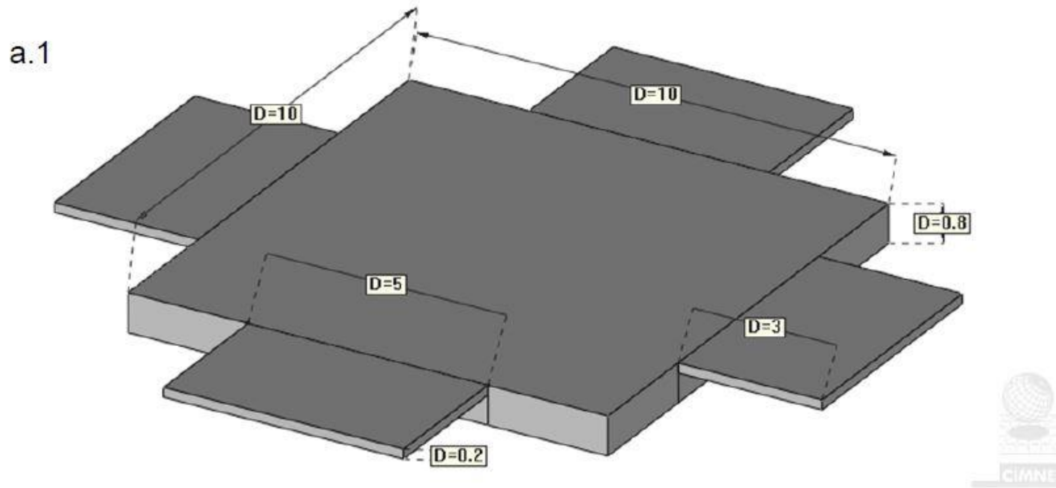

Figure 1. Plate a.1

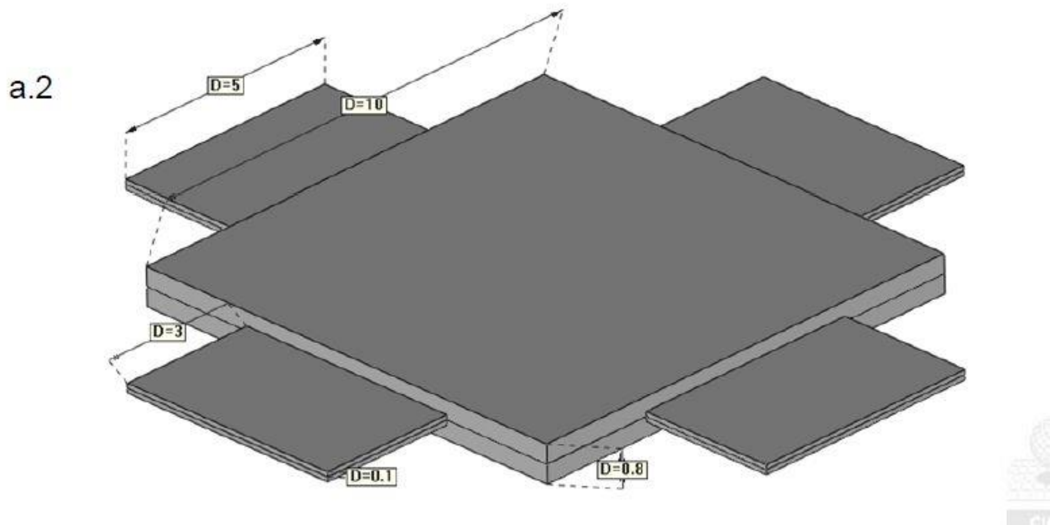

Figure 2. Plate 2

1. The symmetry

Because both plates are symmetric, so we can divide them into 1/4 parts and then imposing proper boundary conditions. This simplification will make the problem easier to be solved.

2. Boundary Conditions

Firstly, on the dividing line which is parallel to x label, we will impose $\theta_y = 0$. And another dividing line which is parallel to y label, we will impose $\theta_x = 0$. For other edges, we are supposed to apply camped boundary conditions with zero vertical displacement and zero rotations in x and y directions or other boundary conditions which can make the structure system not free.

Secondly, on the connecting lines, we also need to impose the corresponding restrictions. Along the connecting lines, the plates have same vertical displacement and rotations.

3. Theory

After calculation, we can find that thickness/width of sub-plates of a) and b) are all less than 0.1. The Kirchho plate theory holds when $\frac{\text{thickness}}{\text{width}} \leq 0.10$, so here we can implement Kirchho plate theory for a) and b). And because Reissner-Mindlin plate theory holds for both thin and thick plate, so we can also apply Reissner-Mindlin plate theory if we want to consider the shear stress to get more accurate results.

4. Element

The elements designed for Kirchho plate theory and Reissner-Mindlin plate theory are all having good convergence. Consequently, all of them can produce accurate results with enough elements amount.

5. Integration rule

As we have discussed, this is a thin plate problem which can be solved by using both Reissner-Mindlin plate theory and Kirchho plate theory. For thin plate problem we can ignore the shear stress.
For Reissner-Mindlin plate theory, we need to use reduced integration rule when we want to ignore the influence of shear stress. But if we want to get more accurate results, we should apply full integration rule.
For Kirchho plate theory, we need to use full integration rule to get satisfied results.

Assignment b)

1. The analytical results

We know the displacements is:

$$u = [\omega, \frac{\partial \omega}{\partial x}, \frac{\partial \omega}{\partial y}]$$

When we make the external load equal be zero:

$$\omega = x + y$$

So:

$$u = [x + y, 1,1]$$

2. The results of MCZ element

We have utilized the code of Clamp_UL_1.m.

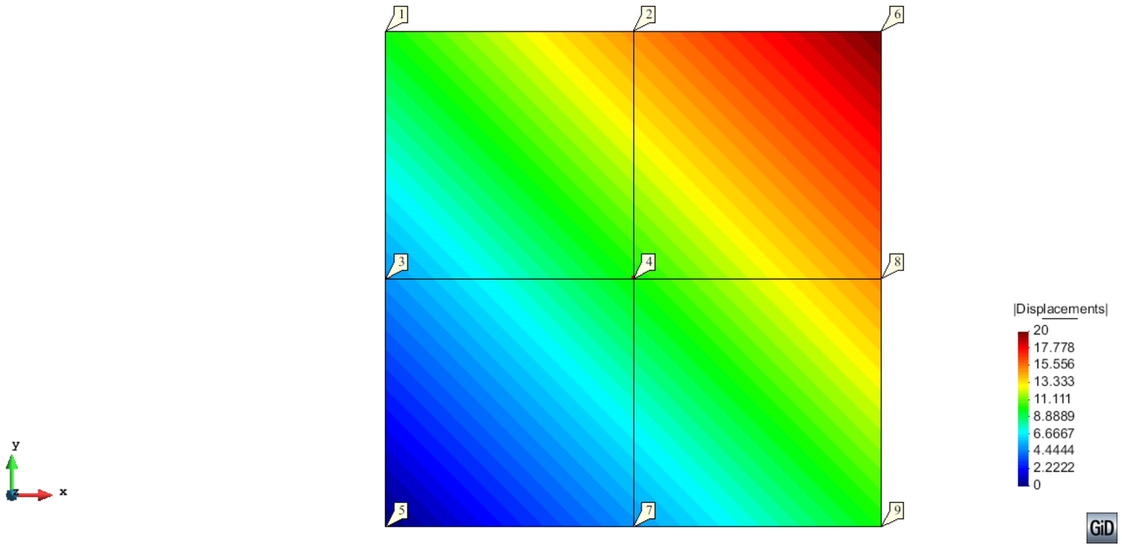

Figure 3. The result of displacement ω

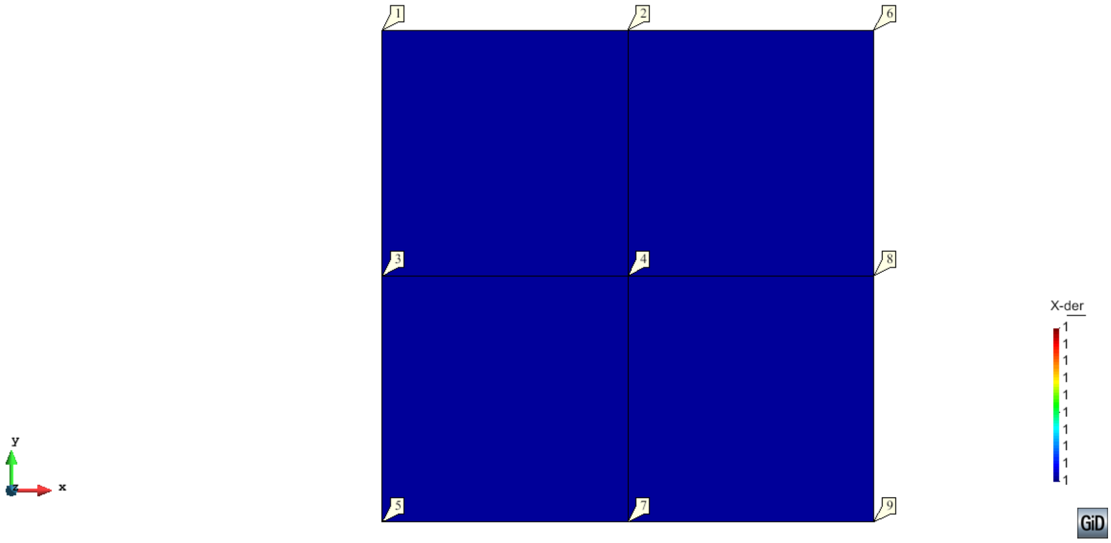

Figure 4. The rotation x

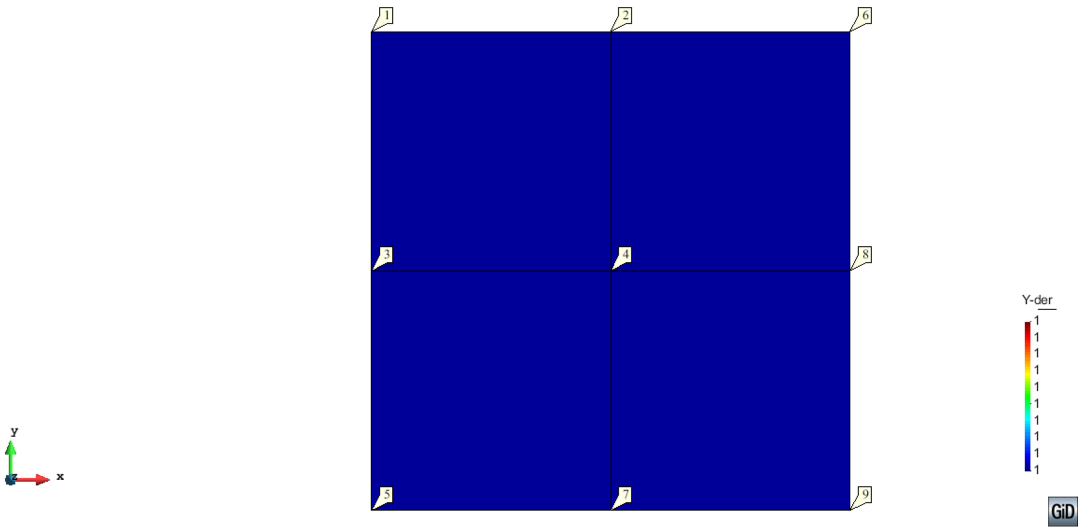

Figure 5. The rotation y

From above results, we can see that the MCZ element satisfy the patch test.