



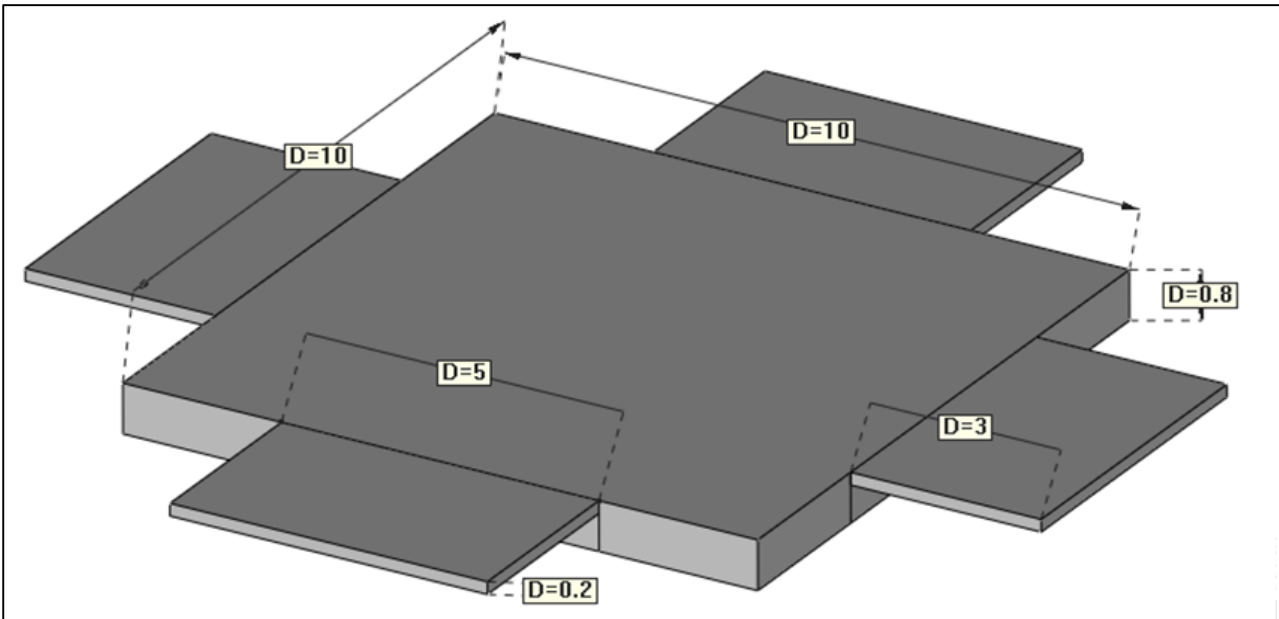
Universitat Politecnica De Catalunya, Barcelona
Masters in Computational Mechanics

Course
Computational Structural Mechanics and Dynamics

Assignment on 'Plates'

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Ques 1. (A) Think first and answer later. What kind of strategy (theory, elements, integration rule, boundary conditions, etc.) will you use for solving the following problems?



Solution:

a. Theory: Reissner-Mindlin Plate Theory

Shear deformations are considered. As the edge plates (thin) are attached to the top surface of the main plate, Plate can rotate independently of the position of the mid surface.

b. Element: Rectangular Element

9 node Lagrange rectangular element represents pure bending. As the vertical displacement (w) have quadratic variation and zero shear deformation results in pure bending rather than zero bending. Each element has 27 degrees of freedom.

c. Integration: Reduced Integration

For integration, the ratio of thickness to characteristic length is considered, the ratio is $8/100$ for main plate (10×0.8) & $2/50$ for edge plates (5×0.2). The reduced integration is more accurate with low computational cost.

d. Boundary Conditions:

Boundary conditions are assumed as force is applied on the main plate at the middle and the edge plates are either clamped or simply supported at their outer edge.

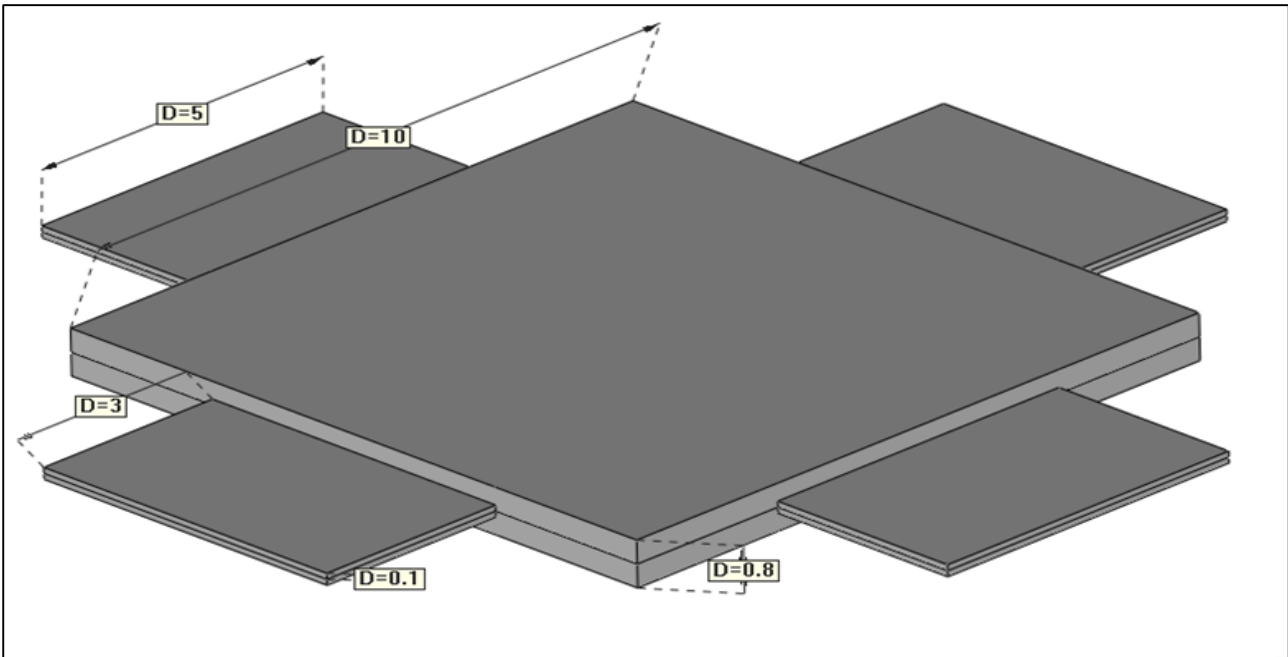
Clamped Condition: $w = \theta_x = \theta_y = 0$

Simply Supported Condition: $w = \theta_x = 0$ or $w = \theta_y = 0$

Weak Supported Condition: $w = 0$

where, w is vertical displacement and θ_x & θ_y are first order partial derivative of w .

Ques 1. (B) Think first and answer later. What kind of strategy (theory, elements, integration rule, boundary conditions, etc.) will you use for solving the following problems?



Solution:

a. Theory: Kirchhoff Plate Theory

The thin plates are attached to the main plate at the middle, therefore plate can't rotate independently of the position of the mid surface. Shear Deformation (transverse shear stress) is zero.

b. Element: Rectangular Element

BFS element of rectangular shape is selected because of its accuracy, low computational cost and perfectly compatibles with the geometry.

c. Integration: Full-exact Integration

For integration, the ratio of thickness to characteristic length is considered, the ratio is $8/100$ for main plate (10×0.8) & $2/50$ for edge plates (5×0.2). The stiffness matrix is not a major issue, so Full Integration is used.

d. Boundary Conditions:

Boundary conditions are assumed as force is applied on the main plate at the middle and the edge plates are either clamped or simply supported at their outer edge.

Clamped Condition: $w = \theta_x = \theta_y = 0$

Simply Supported Condition: $w = \theta_x = 0$ or $w = \theta_y = 0$

Weak Supported Condition: $w = 0$

where, w is vertical displacement and θ_x & θ_y are first order partial derivative of w .

Ques 2. Define and verify a patch test mesh for the MZC element?

Solution:

Non-conforming rectangular Melosh-Zienkiewicz-Cheung (MZC) element:

1. Approximation for w (vertical displacement):

$$w = \alpha_1 + \alpha_2x + \alpha_3y + \alpha_4x^2 + \alpha_5xy + \alpha_6y^2 + \alpha_7x^3 + \alpha_8x^2y + \alpha_9xy^2 + \alpha_{10}y^3 + \alpha_{11}x^3y + \alpha_{12}xy^3$$

2. Arbitrary values of α (alpha):

α	Value
1	0.0002
2	0.0002
3	0.0002
4	0.0002
5	0.0002
6	0.0002

α	Value
7	0.0002
8	0.000201
9	0.000201
10	0.000201
11	0.000201
12	0.000201

The patch of element (Plate geometry) is created in GID, then above mentioned equation is solved explicitly. The displacement values obtained are assigned to origin & edge section, and middle section values are cross-verified with post processing displacement value in GID.

3. Geometry:

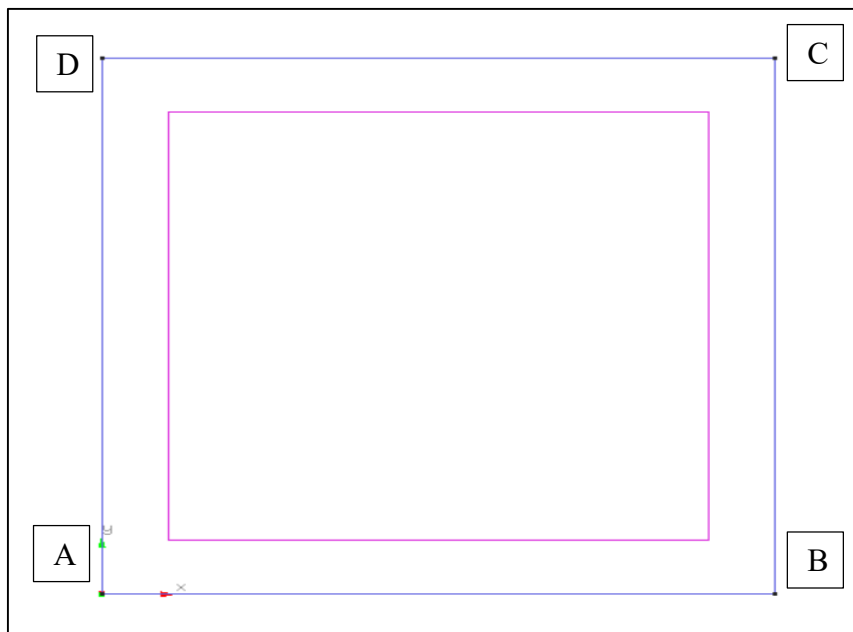
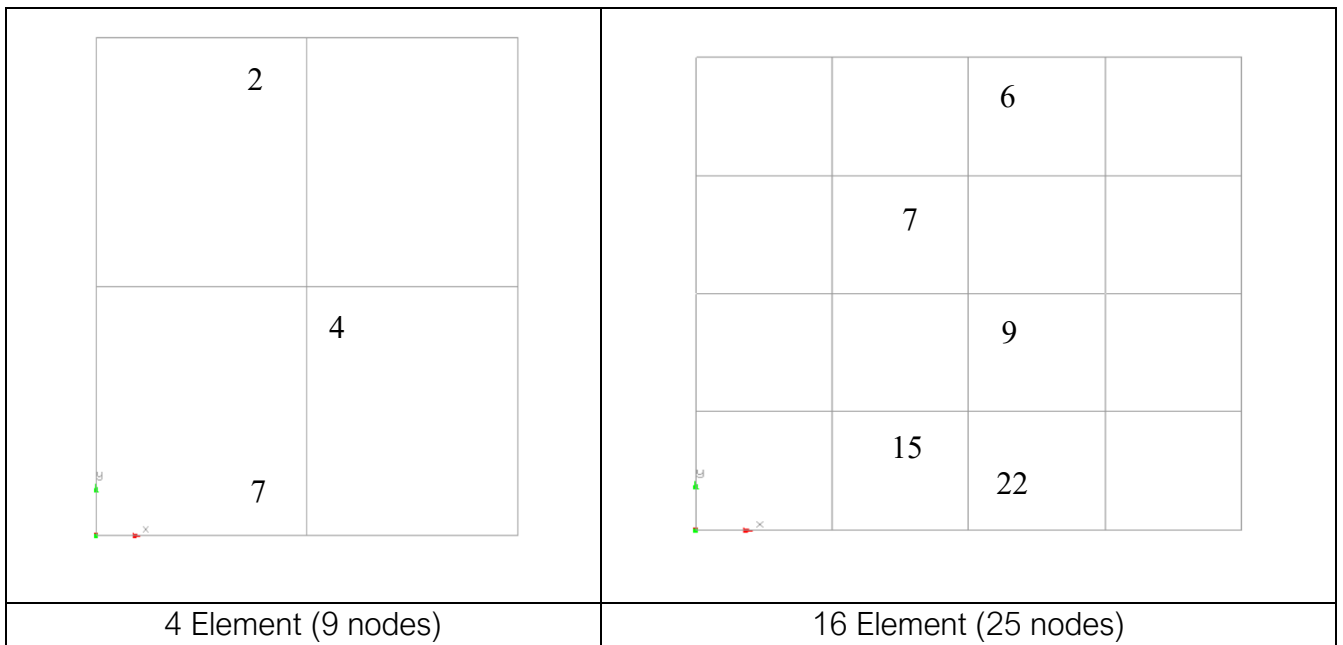


Fig: Plate Geometry

Coordinates: A (0,0); B (1,0); C (1,1); D (0,1).

4. Mesh Geometry:



5. Post Processing (Displacement)

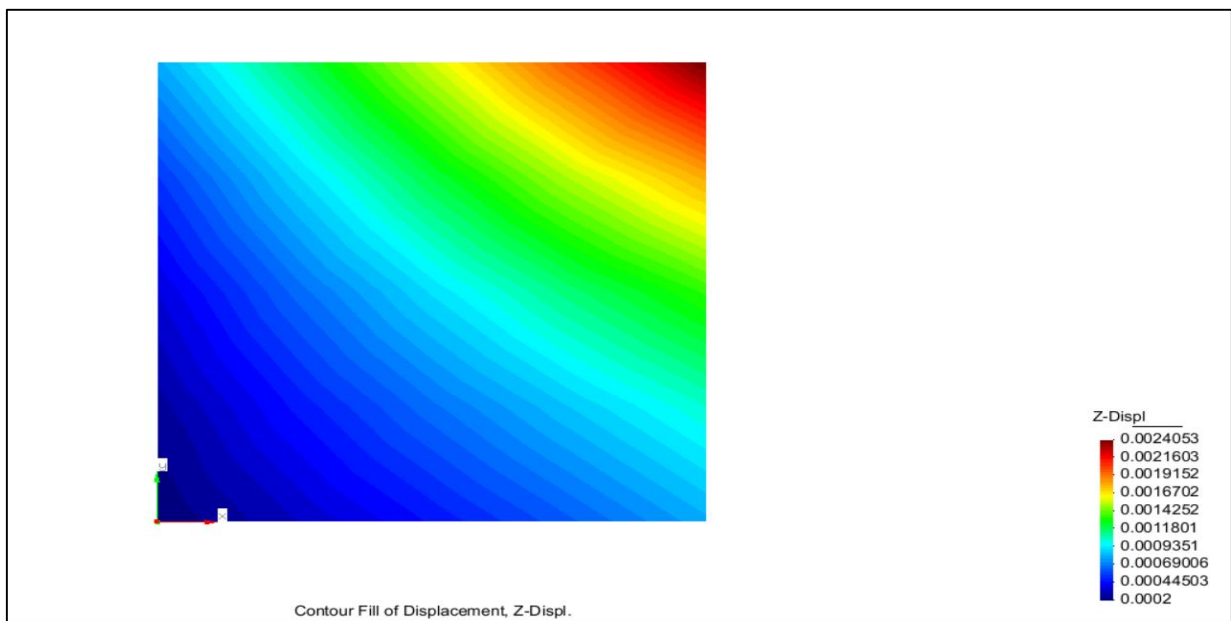


Fig: Z displacement

6. Explicit Data:

For 4 Elements

	x	y	x2	y2	x3	y3	w
Mid	0.5	0	0.25	0	0.125	0	0.000375
	0.5	0.5	0.25	0.25	0.125	0.125	0.000676
	0.5	1	0.25	1	0.125	1	0.001353
Edge	1	0	1	0	1	0	0.000801

	1	0.5	1	0.25	1	0.125	0.001352
	1	1	1	1	1	1	0.002405
Origin	0	0	0	0	0	0	0.0002
	0	0.5	0	0.25	0	0.125	0.000375
	0	1	0	1	0	1	0.000801

For 16 Elements

	x	y	x2	y2	x3	y3	w
Mid	0.5	0	0.25	0	0.125	0	0.000375
	0.5	0.25	0.25	0.0625	0.125	0.015625	0.000493
	0.5	0.5	0.25	0.25	0.125	0.125	0.000676
	0.5	0.75	0.25	0.5625	0.125	0.421875	0.000953
	0.5	1	0.25	1	0.125	1	0.001353
Edge	1	0	1	0	1	0	0.000801
	1	0.25	1	0.0625	1	0.015625	0.001033
	1	0.5	1	0.25	1	0.125	0.001352
	1	0.75	1	0.5625	1	0.421875	0.001797
	1	1	1	1	1	1	0.002405
Origin	0	0	0	0	0	0	0.0002
	0	0.25	0	0.0625	0	0.015625	0.000266
	0	0.5	0	0.25	0	0.125	0.000375
	0	0.75	0	0.5625	0	0.421875	0.000548
	0	1	0	1	0	1	0.000801

7. **Error Calculation:** Error is calculated at Path nodes as:

$$\text{Err} = \text{Abs}\left[\frac{(\text{Cal } w - \text{Obs } w)}{\text{Cal } w}\right]$$

For 4 Element:

Patch Node	Calculated w	Observed w	Error
2	0.000375	0.000479	0.276
4	0.000676	0.000926	0.370
7	0.001353	0.00158	0.169

For 16 Element:

Patch Node	Calculated w	Observed w	Error
6	0.000375	0.000470	0.253
7	0.000493	0.000670	0.361
9	0.000676	0.000911	0.348
15	0.000953	0.00122	0.276
22	0.001353	0.00157	0.161

Conclusion:

It is observed from the error calculation that as the mesh is refined, the error reduces that means the result converges. Thus it can be concluded that the MZC element satisfies the patch test.

-----X-----X-----X-----