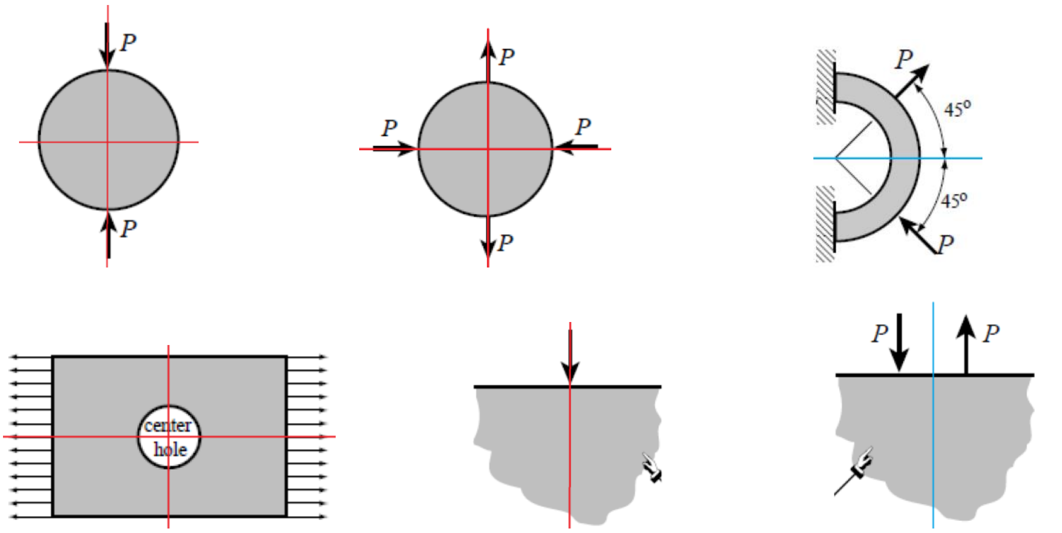
HOMEWORK 2

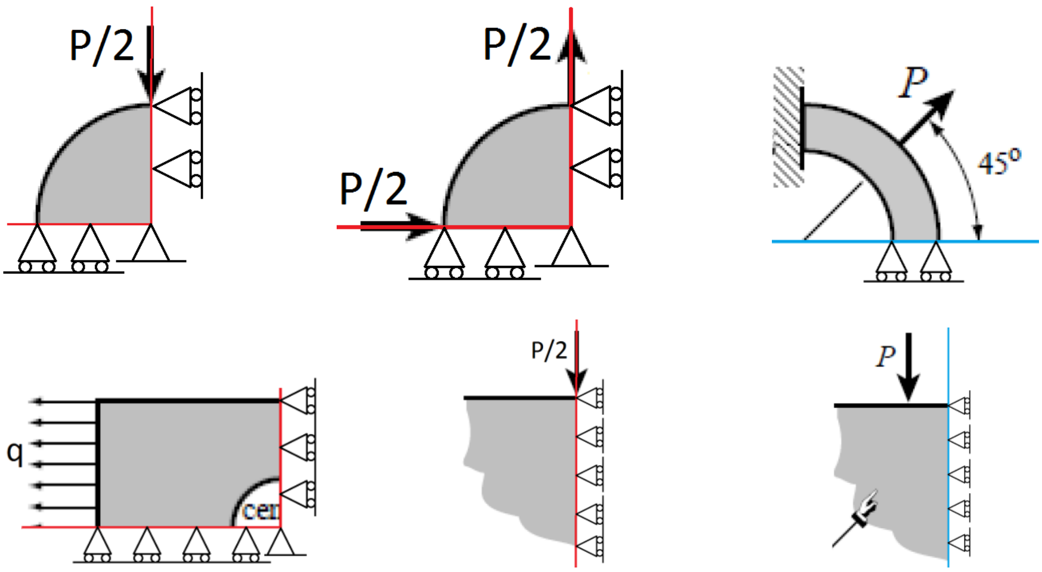
Assignment 2.1:

1) **Identify the symmetry and antisymmetry lines in the two-dimensional problems illustrated in the figure.**

The following figures show the symmetry (red line) and antisymmetry (blue line) lines for each element.



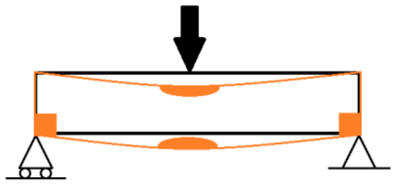
2) **Indicate with rollers or fixed supports which kind of displacements BCs you would specify on the symmetry or antisymmetry lines.**



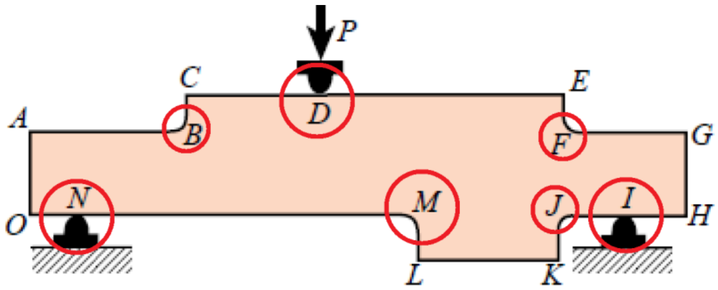
Assignment 2.2:

1) **List what you think are the likely "trouble spots" that would require a locally finer finite element mesh to capture high stress gradients. Identify those spots by its letter and a reason.**

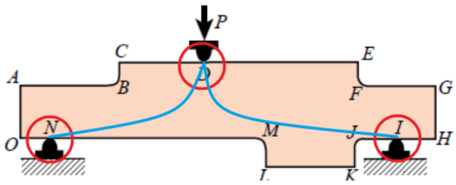
Simplifying the beam helps to find the most stress parts of the beam. The following figure shows the most stress part, which are located in the central part of the beam and in the supports.



Coming back to the original beam, the point under the greater stresses are:



- **Points D, N and I:** Those points are the easiest ones to identify because they are the application points of forces and reactions. The forces are applied in a concentrate points, which generates overloaded areas comparing with the rest of the beam.



- **Points B, F, M and J:** Those points are inner corners where stresses could concentrate. The point M and B, particularly M, could be the most stress point among them, because they are located closer to the application point and the central zone.

The other points are less stress because they are far from the stress concentration zones.

Assignment 2.3:

1) Find the consistent node forces as functions of ρ, Ai, Aj, ω and l, and specialize the result to the prismatic bar A = Ai = Aj.

The nodal force vector is calculated following the next equation:

$$f_{ext} = \int_0^1 q \begin{Bmatrix} 1-\zeta \\ \zeta \end{Bmatrix} L \, d\zeta$$

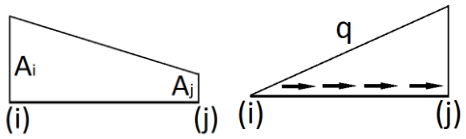
$$\zeta = \frac{\mathrm{x}-\mathrm{x_i}}{\mathrm{L}} \rightarrow x = \zeta \, \mathrm{L} + \mathrm{x}_i$$

Assuming $x_i = 0$,

$$x = \zeta \, \mathrm{L}$$

A. Variable area: $A = A_i(1-\xi) + A_j\xi$

In the following figure it is shown the variation of the area and axial forces.



$$q = \rho A \omega^2 x = \rho\left[\mathrm{A_i}(1-\xi) + \mathrm{A_j}\xi\right]\omega^2 \zeta L$$

Substituting the values and integrating is calculated the nodal forces:

$$f_{ext} = \int_0^1 \rho\left[\mathrm{A_i}(1-\xi) + \mathrm{A_j}\xi\right]\omega^2 \zeta L \begin{Bmatrix} 1-\zeta \\ \zeta \end{Bmatrix} L \, d\zeta$$

$$f_{ext}{}^1 = \int_0^1 \rho\left[\mathrm{A_i}(1-\xi) + \mathrm{A_j}\xi\right]\omega^2 \zeta L^2 (1-\zeta) d\zeta =$$

$$= \int_0^1 \rho\omega^2 L^2 \left(A_i\xi - A_i\xi^2 + A_j\xi^2 - A_i\xi^2 + A_i\xi^3 - A_j\xi^3\right) d\zeta =$$

$$= \rho\omega^2 L^2 \left[A_i\frac{\xi^2}{2} - A_i\frac{\xi^3}{3} + A_j\frac{\xi^3}{3} - A_i\frac{\xi^3}{3} + A_i\frac{\xi^4}{4} - A_j\frac{\xi^4}{4}\right]_0^1 = \rho\omega^2 L^2 \left(\frac{1}{12}A_i + \frac{1}{12}A_j\right)$$

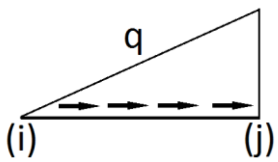
$$f_{ext}{}^2 = \int_0^1 \rho\left[\mathrm{A_i}(1-\xi) + \mathrm{A_j}\xi\right]\omega^2 \zeta^2 L^2 d\zeta = \int_0^1 \rho\omega^2 L^2 \left(A_i\xi^2 - A_i\xi^3 + A_j\xi^3\right) d\zeta =$$

$$= \rho\omega^2 L^2 \left[A_i\frac{\xi^3}{3} - A_i\frac{\xi^4}{4} + A_j\frac{\xi^4}{4}\right]_0^1 = \rho\omega^2 L^2 \left(\frac{1}{12}A_i + \frac{1}{4}A_j\right)$$

$$f_{ext} = \rho\omega^2 L^2 \begin{bmatrix} \frac{1}{12}A_i + \frac{1}{12}A_j \\ \frac{1}{12}A_i + \frac{1}{4}A_j \end{bmatrix}$$

B. Constant area: A = Ai = Aj

In this case, the area is constant and only the axial force (q) varies along the bar:



$$q = \rho A \omega^2 x = \rho A \omega^2 \zeta L$$

Substituting the values in the general equation,

$$f_{ext} = \int_0^1 \rho A \omega^2 \zeta L \begin{Bmatrix} 1-\zeta \\ \zeta \end{Bmatrix} L \, d\zeta = \int_0^1 \rho A \omega^2 L^2 \begin{Bmatrix} \zeta - \zeta^2 \\ \zeta^2 \end{Bmatrix} d\zeta = \rho A \omega^2 L^2 \begin{bmatrix} \dfrac{\zeta^2}{2} - \dfrac{\zeta^3}{3} \\ \zeta^3/_3 \end{bmatrix}_0^1$$

$$= \rho A \omega^2 L^2 \begin{bmatrix} 1/6 \\ 1/3 \end{bmatrix}$$