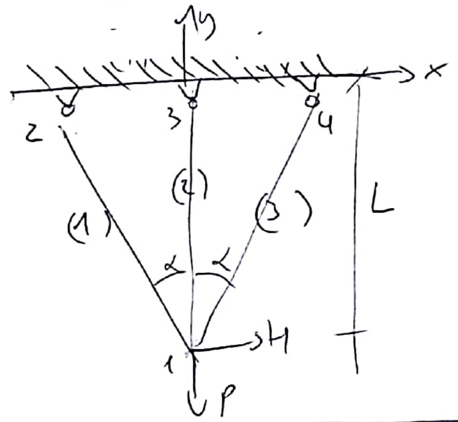
Assignment 1.1 Computational Structural Mech. & Dyn. Marcos Boniquet

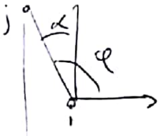
(1) Show that master system eq. are the following and explain why 5^{th} row and colum are all 0's. (Physical meaning)

$$\frac{EA}{L}\begin{bmatrix} 2cs^2 & 0 & -c^2s & c^2s & 0 & 0 & -cs^2 & -c^2s \\ & 1+2c^3 & c^2s & -c^3 & 0 & -1 & -c^2s & -c^3 \\ & & cs^2 & -c^2s & 0 & c & 0 & 0 \\ & & & c^3 & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 \\ & & & & & 1 & 0 & 0 \\ & \text{Sym} & & & & & cs^2 & c^2s \\ & & & & & & & c^3 \end{bmatrix}\begin{bmatrix} u_{x1} \\ u_{y1} \\ u_{x2} \\ u_{y2} \\ u_{x3} \\ u_{y3} \\ u_{x4} \\ u_{y4} \end{bmatrix} = \begin{bmatrix} H \\ -P \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}\begin{matrix} \}1 \\ \\ \}2 \\ \\ \}3 \\ \\ \}4 \\ \\ \end{matrix}$$



$$K^e = \frac{E^eA^e}{L^e}\begin{bmatrix} c^2 & sc & -c^2 & -sc \\ sc & s^2 & -sc & -s^2 \\ -c^2 & -sc & c^2 & sc \\ -sc & -s^2 & sc & s^2 \end{bmatrix}$$

(1)



$\varphi = \alpha + 90$

$\begin{cases} c(90+\alpha) = -s \\ s(90+\alpha) = c \end{cases}$

$L^{(1)} = \frac{L}{c}$

$\frac{E^{(1)}A^{(1)}}{L^{(1)}} = \frac{cEA}{L}$

$$\begin{bmatrix} f_{x1}^{(1)} \\ f_{y1}^{(1)} \\ f_{x2}^{(1)} \\ f_{y2}^{(1)} \end{bmatrix} = \frac{cEA}{L}\begin{bmatrix} s^2 & -cs & -s^2 & cs \\ -cs & c^2 & cs & -c^2 \\ -s^2 & cs & s^2 & sc \\ cs & -c^2 & sc & c^2 \end{bmatrix} = \frac{EA}{L}\begin{array}{c} \begin{matrix} \overbrace{\qquad\quad}^{1} & \overbrace{\qquad\quad}^{2} \end{matrix} \\ \begin{bmatrix} sc^2 & -sc^2 & -cs^2 & c^2s \\ -c^2s & c^3 & c^2s & -c^3 \\ -cs^2 & c^2s & cs^2 & -sc^2 \\ c^2s & -c^3 & -sc^2 & c^3 \end{bmatrix} \end{array}$$

(2) $\varphi = 90°$

$s = 1$ $L^{(2)} = L$

$c = 0$ $\frac{E^{(2)}A^{(2)}}{L^{(2)}} = \frac{EA}{L}$

$$\begin{bmatrix} f_{x1}^{(2)} \\ f_{y1}^{(2)} \\ f_{x3}^{(2)} \\ f_{y3}^{(2)} \end{bmatrix} = \frac{EA}{L}\begin{array}{c} \begin{matrix} \overbrace{\qquad}^{1} & \overbrace{\qquad}^{3} \end{matrix} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \end{array}\begin{matrix} \}1 \\ \\ \}3 \\ \\ \end{matrix}$$

~~(4) $\begin{bmatrix} f_{x2}^{(4)} \\ f_{y2}^{(4)} \\ f_{x3}^{(4)} \\ f_{y3}^{(4)} \end{bmatrix} = \frac{CEA}{LS}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$~~

~~(5) $\begin{bmatrix} f_{x3}^{(5)} \\ f_{y3}^{(5)} \\ f_{x4}^{(5)} \\ f_{y4}^{(5)} \end{bmatrix} = \frac{CEA}{LS}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$~~

(3) $\varphi = 90 - \alpha$

$\sin(90-\alpha) = \cos\alpha$ $\begin{cases} s = c \\ c = s \end{cases}$

$\cos(90-\alpha) = \sin\alpha$

$L^e = \frac{L}{c}$ $\frac{E^{(3)}A^{(3)}}{L^{(3)}} = \frac{cEA}{L}$

$$\begin{bmatrix} f_{x1}^{(3)} \\ f_{y1}^{(3)} \\ f_{x4}^{(3)} \\ f_{y4}^{(3)} \end{bmatrix} = \frac{cEA}{L}\begin{array}{c} \begin{matrix} \overbrace{\qquad\quad}^{1} & \overbrace{\qquad\quad}^{4} \end{matrix} \\ \begin{bmatrix} s^2 & cs & -s^2 & -cs \\ cs & c^2 & -sc & -c^2 \\ -s^2 & -sc & s^2 & cs \\ -cs & -c^2 & cs & c^2 \end{bmatrix} \end{array}\begin{matrix} \}1 \\ \\ \}4 \\ \\ \end{matrix} = \frac{EA}{L}\begin{bmatrix} cs^2 & sc^2 & -s^2c & -c^2s \\ c^2s & c^3 & -sc^2 & -c^3 \\ -s^2c & -sc^2 & s^2c & c^2s \\ -c^2s & -c^3 & c^2s & c^3 \end{bmatrix}$$

$$f = f^{(1)} + f^{(2)} + f^{(3)} = \left(k^{(1)} + k^{(2)} + k^{(3)}\right)u$$

Marcos Berguer

$$
K = \frac{EA}{L}
\begin{array}{c}
\begin{array}{cccc} \quad\quad 1 \quad\quad & \quad\quad 2 \quad\quad & \quad 3 \quad & \quad\quad 4 \quad\quad \end{array} \\
\left[\begin{array}{cccccccc}
s^2c+cs^2 & \cancel{-c^2s+c^2s}\,0 & -sc^2 & c^2s & 0 & 0 & -cs^2 & -c^2s \\
\cancel{-c^2s+c^2s}\,0 & 1+2c^3 & c^2s & -c^3 & 0 & -1 & -c^2s & -c^3 \\
 & & cs^2 & -c^2s & 0 & 0 & 0 & 0 \\
 & & -c^2s & c^3 & 0 & 0 & 0 & 0 \\
 & & 0 & 0 & 0 & 0 & 0 & 0 \\
 & & 0 & 0 & 0 & 1 & 0 & 0 \\
 & \text{SYM} & & & 0 & 0 & s^2c & c^2s \\
 & & & & 0 & 0 & c^2s & c^3
\end{array}\right]
\end{array}
\begin{array}{c} 1 \\ \\ 2 \\ \\ 3 \\ \\ 4 \\ \\ \end{array}
$$

row and colums 5 are 0's because (2) is vertical and it does not have any horizontal displacement, neither horizontal reaction force.

(2) - Boundary conditions:

$$
\begin{cases} u_{x2}=0 \\ u_{y2}=0 \\ u_{x3}=0 \\ u_{y3}=0 \\ u_{x4}=0 \\ u_{y4}=0 \end{cases}
\qquad
u = \begin{pmatrix} u_{x1} \\ u_{y1} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}
$$

2 eq. method final system:

$$
\frac{EA}{L}\begin{pmatrix} 2cs^2 & 0 & -cs^2 & c^2s & 0 & 0 & -cs^2 & -c^2s \\ 0 & 1+2c^3 & c^2s & -c^3 & 0 & -1 & -c^2s & -c^3 \end{pmatrix}\begin{pmatrix} u_{x1} \\ u_{y1} \end{pmatrix} = \begin{pmatrix} H \\ -P \end{pmatrix}
$$

3 - Solve u_{x1}, u_{y1}:

$$
\frac{EA}{L} 2cs^2 u_{x1} = H \rightarrow \quad u_{x1} = \frac{HL}{2cs^2EA}
$$

$$
\frac{EA}{L}(1+2c^3)u_{y1} = -P \rightarrow \quad u_{y1} = \frac{-PL}{(1+2c^3)EA}
$$

$$
\alpha \to 0 \longrightarrow \begin{cases} u_{x1} \to \infty \\ u_{y1} \to \frac{-PL}{3EA} = \frac{1}{3}\left(\frac{-PL}{EA}\right) \end{cases}
$$

$$
\alpha \to \frac{\pi}{2} \longrightarrow \begin{cases} u_{x1} \to \frac{HL}{2EA} \\ u_{y1} \to -\frac{PL}{EA} \end{cases}
$$

If α is smaller, the momentum is higher and node 1 horizontal displacement tends to get bigger. The vertical displacement is 3 times bigger when $\alpha \to \frac{\pi}{2}$, given that proportional stretch is lower in the side bars. When $\alpha \to 0$ and $H \neq 0$ there's no margin and it should break.

(4) - Recover axial forces in the 3 members:

$$
\frac{EA}{L}\begin{bmatrix} \vdots & & \\ -cs^2 & c^2s & \cdots \\ c^2s & -c^3 & \\ 0 & 0 & \\ 0 & -1 & \\ -cs^2 & -c^2s & \\ -c^2s & -c^3 & \end{bmatrix}\begin{bmatrix} u_{x1} \\ u_{y1} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} H \\ -P \\ R_{2x} \\ R_{2y} \\ R_{3x} \\ R_{3y} \\ R_{4x} \\ R_{4y} \end{bmatrix}
$$

$$
\begin{aligned}
&\frac{EA}{L}\left[-cs^2u_{x1} + c^2s u_{y1}\right] = R_{2x} \\
&\frac{EA}{L}\left[c^2s\, u_{x1} - c^3 u_{y1}\right] = R_{2y} \\
&R_{3x} = 0 \\
&-u_{y1}\cdot\frac{EA}{L} = R_{3y} \\
&\frac{EA}{L}\left[-cs^2u_{x1} - c^2s\, u_{y1}\right] = R_{4x} \\
&\frac{EA}{L}\left[-c^2s\, u_{x1} - c^3 u_{y1}\right] = R_{4y}
\end{aligned}
$$

$$U_{x1} = \frac{HL}{2cs^2 EA} \qquad U_{y1} = \frac{-PL}{(1+2c^3)EA}$$

$$\begin{cases} R_{2x} = -\cancel{c}s^2 \dfrac{H}{2\cancel{c}s^2} + \dfrac{c^2 s P}{1+2c^3} = -\left(\dfrac{H}{2} + \dfrac{c^2 s P}{1+2c^3}\right) \\ R_{2y} = c^2 s \dfrac{HL}{2cs^2} + \dfrac{c^3 P}{1+2c^3} = \dfrac{c}{s}\dfrac{H}{2} + \dfrac{c^3 P}{1+2c^3} \\ R_{3x} = 0 \\ R_{3y} = \dfrac{P}{1+2c^3} \\ R_{4x} = R_{2x} \\ R_{4y} = R_{2y} \end{cases}$$

local/global

$$T = \begin{bmatrix} c & s & 0 & 0 \\ -s & c & 0 & 0 \\ 0 & 0 & c & s \\ 0 & 0 & -s & c \end{bmatrix}$$

$$\begin{cases} u^1 = [U_{x1}\ U_{y1}\ U_{x2}\ U_{y2}] \\ u^2 = [U_{x1}\ U_{y1}\ U_{x3}\ U_{y3}] \\ u^3 = [U_{x1}\ U_{y1}\ U_{x4}\ U_{y4}] \end{cases}$$

(1)
$$\begin{pmatrix} \bar{U}_{x1} \\ \bar{U}_{y1} \\ \bar{U}_{x2} \\ \bar{U}_{y2} \end{pmatrix} = \begin{pmatrix} -s & c & 0 & 0 \\ -c & -s & 0 & 0 \\ 0 & 0 & -s & c \\ 0 & 0 & -c & -s \end{pmatrix} \begin{pmatrix} U_{x1} \\ U_{y1} \\ U_{x2} = 0 \\ U_{y2} = 0 \end{pmatrix} \qquad \begin{cases} \bar{U}_{x1} = -sU_{x1} + cU_{y1} \\ \bar{U}_{y1} = -cU_{x1} + sU_{y1} \\ \bar{U}_{x2} = 0 \\ \bar{U}_{y2} = 0 \end{cases}$$

$$\begin{cases} c = -s \\ s = c \end{cases}$$

(2)
$$\begin{pmatrix} \bar{U}_{x1} \\ \bar{U}_{y1} \\ \bar{U}_{x3} \\ \bar{U}_{y3} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} U_{x1} \\ U_{y1} \\ U_{x3} = 0 \\ U_{y3} = 0 \end{pmatrix} \qquad \begin{cases} \bar{U}_{x3} = U_{y3} = 0 \\ \bar{U}_{y3} = -U_{x3} = 0 \end{cases}$$

$$\begin{cases} s = 1 \\ c = 0 \end{cases}$$

(3)
$$\begin{pmatrix} \bar{U}_{x1} \\ \bar{U}_{y1} \\ \bar{U}_{x4} \\ \bar{U}_{y4} \end{pmatrix} = \begin{pmatrix} s & c & 0 & 0 \\ -c & s & 0 & 0 \\ 0 & 0 & s & c \\ 0 & 0 & -c & s \end{pmatrix} \begin{pmatrix} U_{x1} \\ U_{y1} \\ U_{x4} = 0 \\ U_{y4} = 0 \end{pmatrix} \qquad \begin{cases} \bar{U}_{x4} = sU_{x4} + cU_{y4} = 0 \\ \bar{U}_{y4} = -cU_{x4} + sU_{y4} = 0 \end{cases}$$

$$\begin{cases} c = s \\ s = c \end{cases}$$

$$\boxed{\begin{array}{l} d^{(e)} = \bar{U}_{xj} - \bar{U}_{xi} \\ d^{(1)} = \bar{U}_{x2} - \bar{U}_{x1} \\ d^{(2)} = \bar{U}_{x3} - \bar{U}_{x1} \\ d^{(3)} = \bar{U}_{x4} - \bar{U}_{x1} \end{array}}$$

$$\boxed{F^{(e)} = \frac{E^{(e)} A^{(e)}}{L^{(e)}} d^{(e)}}$$

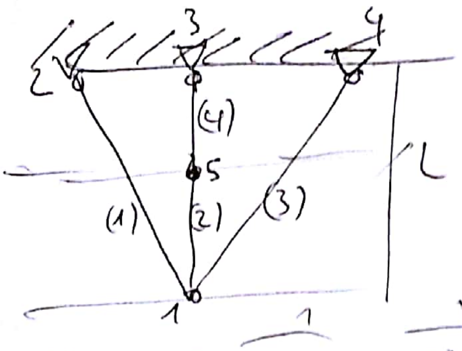
$$\begin{cases} F^{(1)} = \dfrac{c\,EA}{L} d^{(1)} \\ F^{(2)} = \dfrac{EA}{L} d^{(2)} \\ F^{(3)} = \dfrac{c\,EA}{L} d^{(3)} \end{cases}$$

$$\left.\begin{array}{l} d^{(1)} = \cancel{\bar{U}_{x2}} - \bar{U}_{x1} = 0 - \bar{U}_{x1} = -sU_{x1} - cU_{y1} = \\ d^{(2)} = \cancel{U_{x3}} - \bar{U}_{x1} = 0 - \bar{U}_{x1} = -sU_{x1} - cU_{y1} = \\ d^{(3)} = \cancel{U_{x4}} - \bar{U}_{x1} = 0 - \bar{U}_{x1} = -sU_{x1} - cU_{y1} = \end{array}\right\} = -\frac{HL}{2cs\,EA} + \frac{PLc}{(1+2c^3)EA}$$

$$\boxed{F^{(1)} = \frac{-H}{2s} + \frac{Pc^2}{(1+2c^3)} = F^{(3)}}$$

$$\boxed{F^{(2)} = \frac{F^{(1)}}{c} = \frac{-H}{2sc} + \frac{Pc}{1+2c^3}}$$

Assignment 1.2 Computational Structural Mech. & Dyn. Marcus Bourquet



(1) $$\frac{EA}{L}\begin{array}{c} \overbrace{\qquad\qquad}^{1} \; \overbrace{\qquad\qquad}^{2} \\ \begin{bmatrix} sc^2 & -sc^2 & -cs^2 & c^2s \\ -c^2s & c^3 & c^2s & -c^3 \\ -cs^2 & c^2s & cs^2 & -sc^2 \\ c^2s & -c^3 & -sc^2 & c^3 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ \\ 2 \end{array}$$

(2) $$\frac{2EA}{L}\begin{array}{c} \overbrace{\qquad}^{1} \; \overbrace{\qquad}^{5} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ \\ 5 \end{array} = \frac{EA}{L}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 2 \end{bmatrix}$$

(3) $$\frac{EA}{L}\begin{array}{c} \overbrace{\qquad\qquad}^{1} \; \overbrace{\qquad\qquad}^{4} \\ \begin{bmatrix} cs^2 & sc^2 & -s^2c & -c^2s \\ c^2s & c^3 & -sc^2 & -c^3 \\ -s^2c & -sc^2 & s^2c & c^2s \\ -c^2s & -c^3 & c^2s & c^3 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ \\ 4 \end{array}$$

(4) $$\frac{2EA}{L}\begin{array}{c} \overbrace{\qquad}^{5} \; \overbrace{\qquad}^{3} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{c} 5 \\ \\ 3 \end{array} = \frac{EA}{L}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 2 \end{bmatrix}$$

$$\frac{EA}{L}\begin{array}{c} \overbrace{\qquad\qquad}^{1} \; \overbrace{\qquad\qquad}^{2} \; \overbrace{\qquad}^{3} \; \overbrace{\qquad\qquad}^{4} \; \overbrace{\qquad}^{5} \\ \begin{bmatrix} 2cs^2 & 0 & -c^2s & c^2s & 0 & 0 & -cs^2 & -c^2s & 0 & 0 \\ 0 & 2+2c^3 & c^2s & -c^3 & 0 & \boxed{0} & -sc^2 & -c^3 & 0 & -2 \\ -c^2s & c^2s & cs^2 & -c^2s & 0 & 0 & 0 & 0 & 0 & 0 \\ c^2s & -c^3 & -c^2s & c^3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \boxed{2} & 0 & 0 & 0 & -2 \\ -cs^2 & -sc^2 & 0 & 0 & 0 & 0 & -s^2c & c^2s & 0 & 0 \\ -c^2s & -c^3 & 0 & 0 & 0 & 0 & c^2s & c^3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 & \boxed{-2} & 0 & 0 & 0 & 2 \end{bmatrix} \end{array} \begin{bmatrix} u_{1x} \\ u_{1y} \\ u_{2x}=0 \\ u_{2y}=0 \\ u_{3x}=0 \\ u_{3y}=0 \\ u_{4x}=0 \\ u_{4y}=0 \\ u_{5y} \\ u_{5y} \end{bmatrix} = \begin{pmatrix} H \\ -P \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

(sym)

We add a colum and a row of 0's for two more D.F.