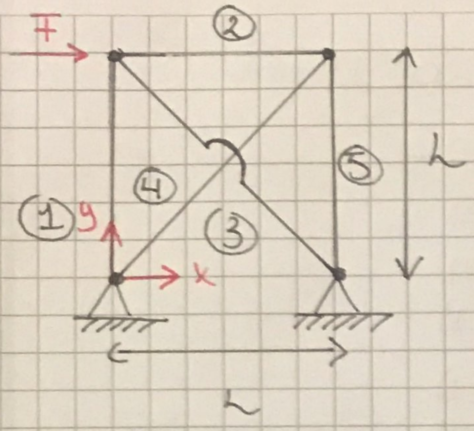


$$K^e = \left(\frac{EA}{L}\right)^e \begin{bmatrix} c^2 & sc & -c^2 & -sc \\ & s^2 & -sc & -s^2 \\ & & c^2 & sc \\ & & & s^2 \end{bmatrix}$$

$L = 6m$, $A = 6cm^2$, $E = 200GPa$, $F = 80kN$

Element 1:

$\phi = 90°$, $L = 6m$ $\Rightarrow$ $K^1 = \left(\frac{EA}{L}\right)^1 \begin{bmatrix} 0 & 0 & 0 & 0 \\ & 1 & 0 & -1 \\ & & 0 & 0 \\ & & & 1 \end{bmatrix}$

$\pi/2$

200GPa, $6cm^2$, 6m

Element 2:

$\phi = 0°$ $\Rightarrow$ $K^2 = \left(\frac{EA}{L}\right) \begin{bmatrix} 1 & 0 & -1 & 0 \\ & 0 & 0 & 0 \\ & & 1 & 0 \\ & & & 0 \end{bmatrix}$

same

Element 3:

$\phi = 135°$, $L = 6\cdot\sqrt{2}\,m$

$3\pi/4$

$$K^3 = \left(\frac{EA}{L}\right) \begin{bmatrix} 1/2 & -1/2 & -1/2 & 1/2 \\ & 1/2 & 1/2 & -1/2 \\ & & 1/2 & -1/2 \\ & & & 1/2 \end{bmatrix}$$

Element 4:

$\phi = 45°$ $\Rightarrow L = 6\cdot\sqrt{2}\,m$

$\rightarrow \pi/4$

$$K^4 = \left(\frac{EA}{L}\right)^4 \begin{bmatrix} 1/2 & 1/2 & -1/2 & -1/2 \\ & 1/2 & -1/2 & -1/2 \\ & & 1/2 & 1/2 \\ & & & 1/2 \end{bmatrix}$$

$EA = 120\cdot10^6\,N$

$L = 6m$ or $L = 6\cdot\sqrt{2}\,m$

$\frac{EA}{L} = 20000\,N/mm$

6m

$\frac{EA}{L} = 14142,14\,N/mm$

Element 5:

$\phi = 270°$

$3\pi/2$

$$K^5 = \left(\frac{EA}{L}\right)^5 \begin{bmatrix} 0 & 0 & 0 & 0 \\ & 1 & 0 & -1 \\ & & 0 & 0 \\ & & & 1 \end{bmatrix}$$

Subtract 1/2 from matrix K^4 and K^3

$$\Rightarrow \quad 7071{,}07 \, N/mm \begin{bmatrix} 1 & 1 & -1 & -1 \\ \vdots & 1 & -1 & -1 \\ & & 1 & 1 \\ \cdot & \cdots & & 1 \end{bmatrix} \Leftarrow (K^4)$$

Expanded element stiffness:

$$\begin{bmatrix} fx_1^{(1)} \\ fy_1^{(1)} \\ fx_2^{(1)} \\ fy_2^{(1)} \\ fx_3^{(1)} \\ fy_3^{(1)} \\ fx_4^{(1)} \\ fy_4^{(1)} \end{bmatrix} = \times 20000 \, N/mm \begin{bmatrix} 0 & 0 & 0 & 0 & & & & \\ 0 & 1 & 0 & -1 & & & 0 & \\ 0 & 0 & 0 & 0 & & & & \\ 0 & -1 & 0 & 1 & & & & \\ & & & & & & & \\ & & 0 & & & & 0 & \\ & & & & & & & \\ & & & & & & & \end{bmatrix} \begin{bmatrix} ux_1^{(1)} \\ uy_1^{(1)} \\ ux_2^{(1)} \\ uy_2^{(1)} \\ ux_3^{(1)} \\ uy_3^{(1)} \\ ux_4^{(1)} \\ uy_4^{(1)} \end{bmatrix}$$

$$\begin{bmatrix} fx_1^{(2)} \\ fy_1^{(2)} \\ fx_2^{(2)} \\ fy_2^{(2)} \\ fx_3^{(2)} \\ fy_3^{(2)} \\ fx_4^{(2)} \\ fy_4^{(2)} \end{bmatrix} = 20\,000 \, N/mm \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} ux_1^{(2)} \\ uy_1^{(2)} \\ ux_2^{(2)} \\ uy_2^{(2)} \\ ux_3^{(2)} \\ uy_3^{(2)} \\ ux_4^{(2)} \\ uy_4^{(2)} \end{bmatrix}$$

$$\begin{bmatrix} fx_1^{(3)} \\ fy_1^{(3)} \\ fx_2^{(3)} \\ fy_2^{(3)} \\ fx_3^{(3)} \\ fy_3^{(3)} \\ fx_4^{(3)} \\ fy_4^{(3)} \end{bmatrix} = 7071{,}07 \, N/mm \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & -1 & 1 \\ 0 & 0 & -1 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -1 & 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} ux_1^{(3)} \\ uy_1^{(3)} \\ ux_2^{(3)} \\ uy_2^{(3)} \\ ux_3^{(3)} \\ uy_3^{(3)} \\ ux_4^{(3)} \\ uy_4^{(3)} \end{bmatrix}$$

Element 4:

$$
\begin{bmatrix} f_{x1}^{(4)} \\ f_{y1}^{(4)} \\ f_{x2}^{(4)} \\ f_{y2}^{(4)} \\ f_{x3}^{(4)} \\ f_{y3}^{(4)} \\ f_{x4}^{(4)} \\ f_{y4}^{(4)} \end{bmatrix} = 7071{,}07 \text{ N/mm} \begin{bmatrix} 1 & 1 & 0 & 0 & -1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 & 1 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_{x1}^{(4)} \\ u_{y1}^{(4)} \\ u_{x2}^{(4)} \\ u_{y2}^{(4)} \\ u_{x3}^{(4)} \\ u_{y3}^{(4)} \\ u_{x4}^{(4)} \\ u_{y4}^{(4)} \end{bmatrix}
$$

Element 5.

$$
\begin{bmatrix} f_{x1}^{(5)} \\ f_{y1}^{(5)} \\ f_{x2}^{(5)} \\ f_{y2}^{(5)} \\ f_{x3}^{(5)} \\ f_{y3}^{(5)} \\ f_{x4}^{(5)} \\ f_{y4}^{(5)} \end{bmatrix} = 20\,000 \text{ N/mm} \begin{bmatrix} 0 & 0 \\ 0 & \begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{matrix} \end{bmatrix} \begin{bmatrix} u_{x1}^{(5)} \\ u_{x1}^{(5)} \\ u_{x2}^{(5)} \\ u_{y2}^{(5)} \\ u_{x3}^{(5)} \\ u_{y3}^{(5)} \\ u_{x4}^{(5)} \\ u_{y4}^{(5)} \end{bmatrix}
$$

Assemble:

$$
\begin{bmatrix} f_{x1} \\ f_{y1} \\ f_{x2} \\ f_{y2} \\ f_{x3} \\ f_{y3} \\ f_{x4} \\ f_{y4} \end{bmatrix} = \begin{bmatrix} 7071{,}07 & 7071{,}07 & 0 & 0 & -7071{,}07 & -7071{,}07 & 0 & 0 \\ 7071{,}07 & 27071{,}07 & 0 & -20000 & -7071{,}07 & -7071{,}07 & 0 & 0 \\ 0 & 0 & 2701{,}07 & -7071{,}07 & -20000 & 0 & -7071{,}07 & 7071{,}07 \\ 0 & -20000 & -7071{,}07 & 27071{,}07 & 0 & 0 & 7071{,}07 & -7071{,}07 \\ -7071{,}07 & -7071{,}07 & -20000 & 0 & 27071{,}07 & 7071{,}07 & 0 & 0 \\ -7071{,}07 & -7071{,}07 & 0 & 0 & 7071{,}07 & 27071{,}07 & 0 & -20000 \\ 0 & 0 & -7071{,}07 & 7071{,}07 & 0 & 0 & 7071{,}07 & -7071{,}07 \\ 0 & 0 & -7071{,}07 & -7071{,}07 & 0 & -20000 & -7071{,}07 & 27071{,}07 \end{bmatrix} \begin{bmatrix} u_{x1} \\ u_{y1} \\ u_{x2} \\ u_{y2} \\ u_{x3} \\ u_{y3} \\ u_{x4} \\ u_{y4} \end{bmatrix}
$$

Introduce boundary conditions:

$u_{x1} = u_{y1} = u_{x4} = u_{y4} = 0 \quad , F = 80000\,N$

$$\begin{bmatrix} 80000 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 27071 & -7071 & -20000 & 0 \\ -7071 & 27071 & 0 & 0 \\ -20000 & 0 & 27071 & 7071 \\ 0 & 0 & 7071 & 27071 \end{bmatrix} \begin{bmatrix} u_{x2} \\ u_{y2} \\ u_{x3} \\ u_{y3} \end{bmatrix}$$

Reduced stiffness due to BC.

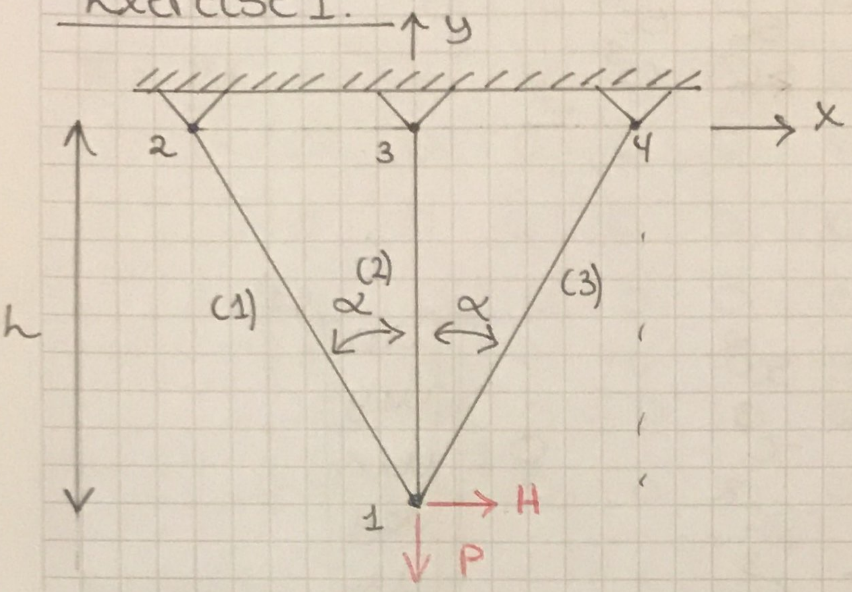
Using a inverting program on computer

$$\Rightarrow \begin{bmatrix} u_{x2} \\ u_{y2} \\ u_{x3} \\ u_{y3} \end{bmatrix} = 10^{-4} \underbrace{\begin{bmatrix} 1{,}068 & 0{,}279 & 0{,}847 & -0{,}221 \\ 0{,}279 & 0{,}442 & 0{,}221 & 0{,}058 \\ 0{,}847 & 0{,}221 & 1{,}068 & -0{,}279 \\ -0{,}221 & 0{,}058 & -0{,}279 & 0{,}442 \end{bmatrix}}_{\mathbb{K}^{-1}} \begin{bmatrix} 80\,000 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} u_{x2} \\ u_{y2} \\ u_{x3} \\ u_{y3} \end{bmatrix} = \begin{bmatrix} 8{,}544 \\ 2{,}232 \\ 6{,}776 \\ -1{,}768 \end{bmatrix} [mm]$$

Assignment 1:

Exercise 1.



Element 1:

$$\left(\frac{EA}{L}\right)^{(1)} \begin{bmatrix} c^2 & sc & -c^2 & -sc \\ sc & s^2 & -sc & -s^2 \\ -c^2 & -sc & c^2 & sc \\ -sc & -s^2 & sc & s^2 \end{bmatrix}$$

$s = \sin\theta$

$c = \cos\theta$

$\sin(\pi/2 - \alpha) = \cos(\alpha)$

$\sin(\pi/2 + \alpha) = \cos(\alpha)$

$\cos(\pi/2 - \alpha) = \sin(\alpha)$

$\cos(\pi/2 + \alpha) = -\sin(\alpha)$

$\theta = \frac{\pi}{2} + \alpha$

$$L^{(1)} = \frac{h}{\cos(\alpha)} = \frac{h}{c}$$

$$\mathbb{K}_1 = \frac{EA}{h} \begin{bmatrix} s^2c & -sc^2 & -s^2c & sc^2 \\ -sc^2 & c^3 & sc^2 & -c^3 \\ -s^2c & sc^2 & s^2c & -sc^2 \\ sc^2 & -c^3 & -sc^2 & c^3 \end{bmatrix}$$

$$\mathbb{K}_2 = \frac{EA}{h} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

$\cos(\pi/2) = 0$

$\sin(\pi/2) = 1$

$$\mathbb{K}_3 = \frac{EA}{L}\begin{bmatrix} sc^2 & sc^2 & -sc^2 & -sc^2 \\ sc^2 & c^3 & -sc^2 & -c^3 \\ -sc^2 & -sc^2 & sc^2 & sc^2 \\ -sc^2 & -c^3 & sc^2 & c^3 \end{bmatrix}$$

Expanded version:

Element (1):

$$\mathbb{F} = \frac{EA}{L} * \begin{bmatrix} s^2c & -sc^2 & -sc^2 & sc^2 & & \\ -sc^2 & c^3 & sc^2 & -c^3 & & 0 \\ -sc^2 & sc^2 & sc^2 & -sc^2 & & \\ sc^2 & -c^3 & -sc^2 & c^3 & & \\ & & & & & \\ & & 0 & & & 0 \\ & & & & & \end{bmatrix} \begin{bmatrix} u_{1x} \\ u_{1y} \\ u_{2x} \\ u_{2y} \\ u_{3x} \\ u_{3y} \\ u_{4x} \\ u_{4y} \end{bmatrix}^{(1)}$$

Element (2):

$$\mathbb{F} = \frac{EA}{L} * \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_{1x} \\ u_{1y} \\ u_{2x} \\ u_{2y} \\ u_{3x} \\ u_{3y} \\ u_{4x} \\ u_{4y} \end{bmatrix}^{(2)}$$

Element 3:

$$\mathbb{F} = \frac{EA}{L}\begin{bmatrix} s^2c & sc^2 & 0 & 0 & 0 & 0 & -sc^2 & -sc^2 \\ sc^2 & c^3 & 0 & 0 & 0 & 0 & -sc^2 & -c^3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -sc^2 & -sc^2 & 0 & 0 & 0 & 0 & sc^2 & sc^2 \\ -sc^2 & -c^3 & 0 & 0 & 0 & 0 & sc^2 & c^3 \end{bmatrix}^{(3)} \begin{bmatrix} u_{1x} \\ u_{1y} \\ u_{2x} \\ u_{2y} \\ u_{3x} \\ u_{3y} \\ u_{4x} \\ u_{4y} \end{bmatrix}$$

Assemble:

Master stiffness:

$$\frac{EA}{L}\begin{bmatrix} 2s^2c & 0 & -sc^2 & sc^2 & 0 & 0 & -sc^2 & -sc^2 \\ & 1+2c^3 & sc^2 & -c^3 & 0 & -1 & -sc^2 & -c^3 \\ & & s^2c & -sc^2 & 0 & 0 & 0 & 0 \\ & & & c^3 & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 \\ & & & & & 1 & 0 & 0 \\ & \text{sym} & & & & & sc^2 & sc^2 \\ & & & & & & & c^3 \end{bmatrix} \begin{bmatrix} u_{1x} \\ u_{1y} \\ u_{2x} \\ u_{2y} \\ u_{3x} \\ u_{3y} \\ u_{4x} \\ u_{4y} \end{bmatrix} = \begin{bmatrix} H \\ -P \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The 5th row and column refers to the displacement in x-direction of node 3 (u_{3x}). The second bar element, which is connected to node is oriented vertically. This means that it has no stiffness in the x-direction

$\Rightarrow$ Row/column 5 in the stiffness matrix $= 0$.

b)
Apply BC (boundary conditions):

$u_{2x} = u_{2y} = u_{3x} = u_{3y} = u_{4x} = u_{4y}$

$\Rightarrow$ reduce the K-matrix to a 2x2.

$$\frac{EA}{L}\begin{bmatrix} 2s^2c & 0 \\ 0 & 1+2c^3 \end{bmatrix}\begin{bmatrix} u_{1x} \\ u_{1y} \end{bmatrix} = \begin{bmatrix} H \\ -P \end{bmatrix}$$

c)

$\alpha \to 0$:

$$u_{1x} = \frac{HL}{EA \cdot (2s^2c)}$$

when $\alpha \to 0$
$\sin(0) = 0$
$\Rightarrow u_{1x} \to \infty$

$$u_{1y} = \frac{-PL}{EA \cdot (1+2c^3)}$$

$\cos(0) = 1$

$$= \frac{-PL}{3EA}$$

This make sence because the stiffness for the whole structure $\to 0$ in x-direction $\Rightarrow u_{1x} \to \infty$. And the deflection in y-dir $\to -\frac{PL}{3EA}$ because of 3 "vertical" bars.

$\alpha \to \pi/2$:

$\sin(\pi/2) = 1$, $\cos(\pi/2) = 0$ This gives

$u_{1x} = \frac{HL}{EA \cdot 0}$ $\Rightarrow u_{1x} \to \infty$. This could be described by the length of the "diagonal" bars $\to \infty$. When $L \to \infty$, the stiffness $L/EA \to 0$

$\Rightarrow u_{1x} \to \infty$.

($L_{diag} = \frac{L}{\cos(\alpha)}$

$u_{1y} = \dfrac{-PL}{EA}$ This make sense because the the "diagonal" bars goes to horizontally orientation. This gives 0 stiffness i y-direction. As well as $L \to \infty \Rightarrow \dfrac{EA}{L} \to 0$.

u_{x1} blows up because the stiffness in x-direction $\to 0$.

d) Using the displacement transformation

$$\bar{u}^c = T^c u^c$$

this is $\underbrace{\cos(\phi)}_{=-\sin(\alpha)}$ and $\underbrace{\sin(\phi)}_{=\cos(\alpha)}$

Element 1:

$$\begin{bmatrix} \bar{u}_{1x} \\ \bar{u}_{1y} \\ \bar{u}_{2x} \\ \bar{u}_{2y} \end{bmatrix} = \begin{bmatrix} c & s & 0 & 0 \\ -s & c & 0 & 0 \\ 0 & 0 & c & s \\ 0 & 0 & -s & c \end{bmatrix} \begin{bmatrix} u_{1x} = \dfrac{HL}{2EAsc^2} \\ u_{1y} = -\dfrac{PL}{EA(1+2c^3)} \\ u_{2x} = 0 \\ u_{2y} = 0 \end{bmatrix}$$

$$\bar{u}_{1x} = \frac{-HL}{2EAsc} - \frac{PLc}{EA(1+2c^3)}$$

$$\bar{u}_{1y} = -\frac{HL}{2EAs^2} + \frac{PLs}{EA(1+2c^3)}$$ this wont affect the force in the bar

$L^{(1)} = L/c$

Hookes law:

$$\Delta L = \underbrace{\bar{u}_{2x}}_{=0} - \bar{u}_{1x} = -\bar{u}_{1x}$$

$$F = \frac{\Delta L \cdot c}{L} EA \Rightarrow F = \frac{H}{2s} + \frac{Pc^2}{(1+2c^3)}$$ both tension

Element 2: $\phi = \pi/2 \Rightarrow c = 0.\ s = 1$

$\bar{u}_{x_1} = u_{y_1}$ (same matrix as for element 1)

$\Delta h = \bar{u}_{x_3} - \bar{u}_{x_1} = -\bar{u}_{x_1}$

$$F = \frac{\Delta h}{h} EA = \underline{\underline{\frac{P}{EA(1+2c^3)}}}$$

Element 3: $\phi = \pi/2 - \alpha \Rightarrow \cos(\phi) = \sin(\alpha)$
$\sin(\phi) = \cos(\alpha)$

$\Delta h = u_{\bar{3}x} - \bar{u}_{1x} = -\bar{u}_{1x}$

$$u_{\bar{x}_1} = \frac{Hh}{2EAsc} - \frac{Phc}{EA(1+2c^3)}$$

gives tension

$$\Rightarrow \text{Hookes law} \Rightarrow F = \underline{-\frac{H}{2s} + \frac{Pc^2}{(1+2c^3)}}$$

gives compression.

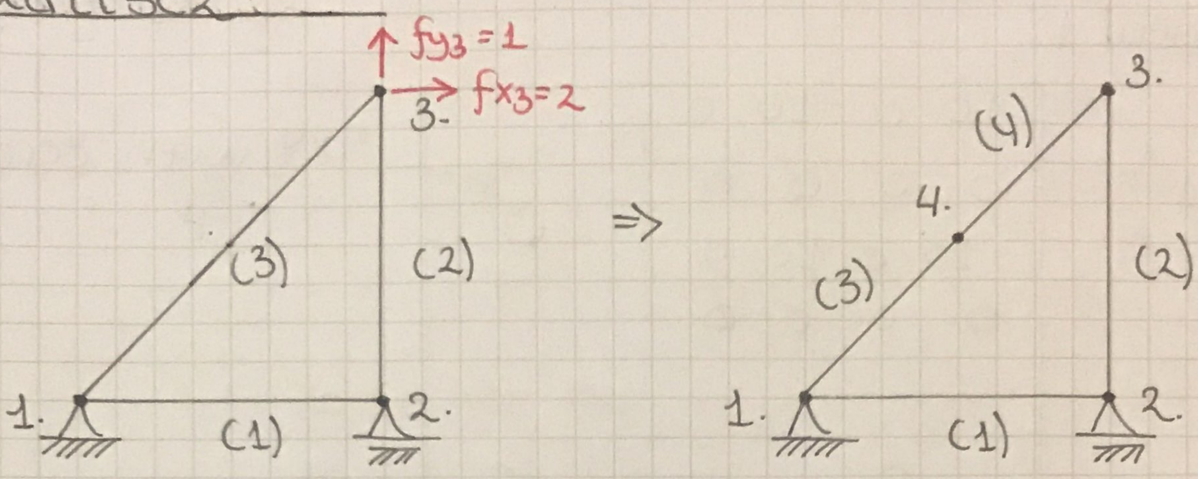
In $F^{(1)}$ and $F^{(3)}$ we have the term

$\frac{H}{2s}$. When $\alpha \to 0 \Rightarrow \sin(\alpha) \to 0$.
This results in the term $\frac{H}{2s} \to \infty$.

The forces blows up because the system can't substain the horizontal force

$\Rightarrow$ unstable system.

Exercise 2 :



Element 1:

$\phi = 0 \Rightarrow \dfrac{EA}{L} = 10$

$$\mathbb{K}_1 = 10 \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Element 2:

$\phi = \pi/2 \Rightarrow \dfrac{EA}{L} = 5$

$$\mathbb{K}_2 = 5 \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

Element 3:

$\phi = \pi/4 \Rightarrow EA = 200\sqrt{2}, \quad L = \dfrac{10\sqrt{2}}{2} \Rightarrow \dfrac{EA}{L} = 40$

$$\mathbb{K}_3 = 40 \begin{bmatrix} 0,5 & 0,5 & -0,5 & -0,5 \\ 0,5 & 0,5 & -0,5 & -0,5 \\ -0,5 & -0,5 & 0,5 & 0,5 \\ -0,5 & -0,5 & 0,5 & 0,5 \end{bmatrix} = 20 \begin{bmatrix} 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 \end{bmatrix}$$

Element 4:

$\phi = \dfrac{5\pi}{4} \Rightarrow \dfrac{EA}{L} = 40 \qquad \mathbb{K}_4 = 20 \begin{bmatrix} 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 \end{bmatrix}$

Expanded element stiffness:

Element 1:

$$\mathbb{K}_1 = \left[\begin{array}{cccc|c} 10 & 0 & -10 & 0 & \\ 0 & 0 & 0 & 0 & \\ -10 & 0 & 10 & 0 & 0 \\ 0 & 0 & 0 & 0 & \\ \hline & 0 & & & 0 \end{array}\right]$$

4x4 with zeros

Element 2:

$$\mathbb{K}_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & -5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -5 & 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Element 3:

$$\mathbb{K}_3 = \begin{bmatrix} 20 & 20 & 0 & 0 & 0 & 0 & -20 & -20 \\ 20 & 20 & 0 & 0 & 0 & 0 & -20 & -20 \\ 0 & 0 & & & & & 0 & 0 \\ 0 & 0 & & & 0 & & 0 & 0 \\ 0 & 0 & & & & & 0 & 0 \\ 0 & 0 & & & & & 0 & 0 \\ -20 & -20 & 0 & 0 & 0 & 0 & 20 & 20 \\ -20 & -20 & 0 & 0 & 0 & 0 & 20 & 20 \end{bmatrix}$$

Element 4:

$$\mathbb{K}_4 = \begin{bmatrix} 0 & 0 \\ 0 & \begin{matrix} 20 & 20 & -20 & -20 \\ 20 & 20 & -20 & -20 \\ -20 & -20 & 20 & 20 \\ -20 & -20 & 20 & 20 \end{matrix} \end{bmatrix}$$

Reconnecting the matrix gives the master stiffness $\mathbb{K}$.

$$\mathbb{K} = \begin{bmatrix} 30 & 20 & -10 & 0 & 0 & 0 & -20 & -20 \\ 20 & 20 & 0 & 0 & 0 & 0 & -20 & -20 \\ -10 & 0 & 10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & -5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 20 & 20 & -20 & -20 \\ 0 & 0 & 0 & -5 & 20 & 25 & -20 & -20 \\ -20 & -20 & 0 & 0 & -20 & -20 & 40 & 40 \\ -20 & -20 & 0 & 0 & -20 & -20 & 40 & 40 \end{bmatrix}$$

introduce boundary conditions

$u_{x1} = u_{y1} = u_{y2} = 0$

$$\begin{bmatrix} 0 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 10 & 0 & 0 & 0 & 0 \\ 0 & 20 & 20 & -20 & -20 \\ 0 & 20 & 25 & -20 & -20 \\ 0 & -20 & -20 & 40 & 40 \\ 0 & -20 & -20 & 40 & 40 \end{bmatrix} \begin{bmatrix} u_{2x} \\ u_{3x} \\ u_{3y} \\ u_{4x} \\ u_{4y} \end{bmatrix}$$

$\mathbb{F}$

The master stiffness matrix is singular because the column 4 and 5 is the same

$\Rightarrow$ linear dependent $\Rightarrow$ singular $\mathbb{K}$.

When adding an extra node we get

$\Rightarrow$ $m + R < 2j$ – joints $\Rightarrow$ $4+3 < 2 \cdot 4$

members reactions $\qquad 7 < 8$

which result in an unstable truss.

Dr. Who's proposal is not preferable.