Computational Structural Mechanics and Dynamics

Assignment 1

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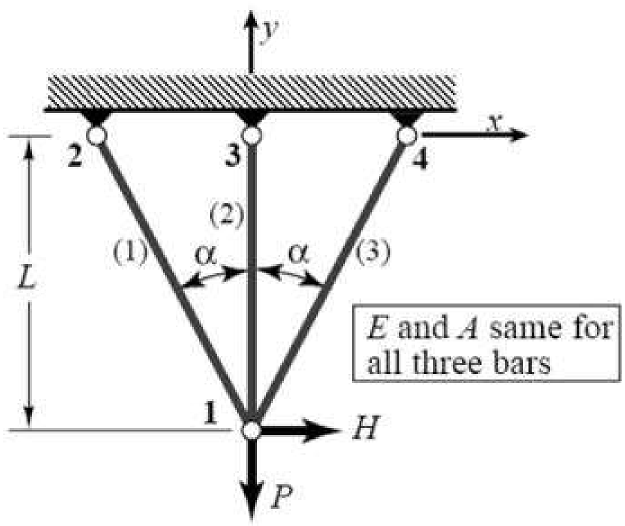
Assignment: Problem 1

Consider the truss problem defined in the figure 1.1. All geometric and material properties: L, α, E and A, as well as the applied forces P and H are to be kept as variables. This truss has 8 degrees of freedom, with six of them removable by the fixed displacement conditions at nodes 2, 3 and 4. This structure is statically indeterminate as long as $\alpha \neq 0$.



a) Show that the master stiffness equations are

$$\frac{EA}{L}\begin{bmatrix} 2cs^2 & 0 & -cs^2 & c^2s & 0 & 0 & -cs^2 & -c^2s \\ & 1+2c^3 & c^2s & -c^3 & 0 & -1 & -c^2s & -c^3 \\ & & cs^2 & -c^2s & 0 & 0 & 0 & 0 \\ & & & c^3 & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 \\ & & & & & 1 & 0 & 0 \\ & & & & & & cs^2 & c^2s \\ symm & & & & & & & c^3 \end{bmatrix}\begin{bmatrix} u_{x1} \\ u_{y1} \\ u_{x2} \\ u_{y2} \\ u_{x3} \\ u_{y3} \\ u_{x4} \\ u_{y4} \end{bmatrix} = \begin{bmatrix} H \\ -P \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

In order to demonstrate this, we have to consider the elemental stiffness matrix for each element: in our case three elements. Each matrix will be $4x4$ as we are using linear elements in $2-D$. This means that each node has two degrees of freedom (DOF): the displacements along the $x-axis$ and $y-axis$. The system to be solve is $Ku = f$. Considering that the elements are rotated respect to the reference system, it is needed to introduce a relation between the local and the global coordinates:

$$\begin{aligned} u_x^e &= u_x^{glob}c + u_y^{glob}s \\ u_y^e &= -u_x^{glob}s + u_y^{glob}c \end{aligned} \tag{1}$$

in which $c = \cos\alpha$ and $s = \sin\alpha$. Writing it in matrix formulation:

$$\begin{bmatrix} u_{xi}^e \\ u_{yi}^e \\ u_{xj}^e \\ u_{yj}^e \end{bmatrix} = \begin{bmatrix} c & s & 0 & 0 \\ -s & c & 0 & 0 \\ 0 & 0 & c & s \\ 0 & 0 & -s & c \end{bmatrix}\begin{bmatrix} u_{xi}^{glob} \\ u_{yi}^{glob} \\ u_{xj}^{glob} \\ u_{yj}^{glob} \end{bmatrix}$$

An equivalent transformation can be written for the forces and writing both of them in compressed formulation:

$$\begin{aligned} f^e &= Tf^{glob} \\ u^e &= Tu^{glob} \end{aligned} \tag{2}$$

in which T is the transformation matrix defined above. At the end, my system to be solved will be:

$$K^{glob} = (T)^T K^e T \tag{3}$$

This will lead to a rotated stiffness matrix that allows to represent the rotated system per each element in the reference system. This matrix is the following:

$$K^e = \frac{EA}{L^e}\begin{bmatrix} c^2 & sc & -c^2 & -sc \\ & s^2 & -sc & -s^2 \\ & & c^2 & sc \\ symm & & & s^2 \end{bmatrix}$$

in which $c = cos\alpha$ and $s = sin\alpha$.

So we can write the system of equations for each element as:

$$\begin{bmatrix} f_{x1}^{(e)} \\ f_{y1}^{(e)} \\ f_{x2}^{(e)} \\ f_{y2}^{(e)} \end{bmatrix} = \frac{EA}{L^{(e)}}\begin{bmatrix} c^2 & sc & -c^2 & -sc \\ & s^2 & -sc & -s^2 \\ & & c^2 & sc \\ symm & & & s^2 \end{bmatrix}\begin{bmatrix} u_{x1}^{(e)} \\ u_{y1}^{(e)} \\ u_{x2}^{(e)} \\ u_{y2}^{(e)} \end{bmatrix}$$

in which $u_{x1}, u_{y1}...$ are the displacements at the nodes of the element, and $f_{x1}, f_{y1}...$ are the respective forces at the nodes.

Considering the trigonometric relations:

$$\begin{aligned} sin(90+\alpha) &= cos\alpha \\ cos(90+\alpha) &= -sin\alpha \\ sin(90-\alpha) &= cos\alpha \\ cos(90-\alpha) &= sin\alpha \end{aligned} \tag{4}$$

we can write the system of equations per each element.

Element 1:

$$\begin{bmatrix} f_{x1}^{(1)} \\ f_{y1}^{(1)} \\ f_{x2}^{(1)} \\ f_{y2}^{(1)} \end{bmatrix} = \frac{EA}{L^{(1)}} \begin{bmatrix} s^2 & -sc & -s^2 & sc \\ & c^2 & sc & -c^2 \\ & & s^2 & -sc \\ symm & & & c^2 \end{bmatrix} \begin{bmatrix} u_{x1}^{(1)} \\ u_{y1}^{(1)} \\ u_{x2}^{(1)} \\ u_{y2}^{(1)} \end{bmatrix}$$

Element 2:

$$\begin{bmatrix} f_{x1}^{(2)} \\ f_{y1}^{(2)} \\ f_{x2}^{(2)} \\ f_{y2}^{(2)} \end{bmatrix} = \frac{EA}{L^{(2)}} \begin{bmatrix} 0 & 0 & 0 & 0 \\ & 1 & 0 & -1 \\ & & 0 & 0 \\ symm & & & 1 \end{bmatrix} \begin{bmatrix} u_{x1}^{(2)} \\ u_{y1}^{(2)} \\ u_{x2}^{(2)} \\ u_{y2}^{(2)} \end{bmatrix}$$

Element 3:

$$\begin{bmatrix} f_{x1}^{(3)} \\ f_{y1}^{(3)} \\ f_{x2}^{(3)} \\ f_{y2}^{(3)} \end{bmatrix} = \frac{EA}{L^{(3)}} \begin{bmatrix} s^2 & sc & -s^2 & -sc \\ & c^2 & -sc & -c^2 \\ & & s^2 & sc \\ symm & & & c^2 \end{bmatrix} \begin{bmatrix} u_{x1}^{(3)} \\ u_{y1}^{(3)} \\ u_{x2}^{(3)} \\ u_{y2}^{(3)} \end{bmatrix}$$

Taking into account that the length of elements 1 and 3 is:

$$L^{(e)} = \frac{L^{(2)}}{c};\ e = 1,3 \tag{5}$$

$$c = cos\alpha \tag{6}$$

the stiffness matrix for elements 1 and 3 becomes:

$$K^{(1)} = \frac{EA}{L} \begin{bmatrix} cs^2 & -sc^2 & -cs^2 & sc^2 \\ & c^3 & sc^2 & -c^3 \\ & & cs^2 & -sc^2 \\ symm & & & c^3 \end{bmatrix}$$

$$K^{(3)} = \frac{EA}{L} \begin{bmatrix} cs^2 & sc^2 & -cs^2 & -sc^2 \\ & c^3 & -sc^2 & -c^3 \\ & & cs^2 & sc^2 \\ symm & & & c^3 \end{bmatrix}$$

Now, applying the compatibility condition, we can say that the displacements in each local node, converging on the same global node, has to be the same.

$$\begin{aligned} u_{x1}^{(1)} = u_{x1}^{(2)} = u_{x1}^{(3)} = u_{x1} \\ u_{y1}^{(1)} = u_{y1}^{(2)} = u_{y1}^{(3)} = u_{y1} \end{aligned} \tag{7}$$

Moving from the local coordinates to the global ones:

$$\begin{aligned} u_{x2}^{(1)} &= u_{x2} \\ u_{y2}^{(1)} &= u_{y2} \\ u_{x2}^{(2)} &= u_{x3} \\ u_{y2}^{(2)} &= u_{y3} \\ u_{x2}^{(3)} &= u_{x4} \\ u_{y2}^{(3)} &= u_{y4} \end{aligned} \tag{8}$$

and applying the equilibrium condition, so the forces are balanced, we'll obtain this force vector. Moving in the same way as above from local to global coordinates:

$$\begin{bmatrix} H \\ -P \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

We have defined the displacement and the force vector; now we have to assemble the global matrix in order to demonstrate the master stiffness equation. To assemble the global matrix we are going to take each component of the local stiffness matrix defined above and sum with the respective ones, acting on the same global node in the same global reference direction, x or y. For instance, the element in position 1,1 of the global stiffness matrix will be the sum of all the elements acting on the global node 1 in the x-direction from each of the local systems. Assembling the global stiffness matrix:

$$\frac{EA}{L}\begin{bmatrix} 2cs^2 & 0 & -cs^2 & c^2s & 0 & 0 & -cs^2 & -c^2s \\ & 1+2c^3 & c^2s & -c^3 & 0 & -1 & -c^2s & -c^3 \\ & & cs^2 & -c^2s & 0 & 0 & 0 & 0 \\ & & & c^3 & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 \\ & & & & & 1 & 0 & 0 \\ & & & & & & cs^2 & c^2s \\ symm & & & & & & & c^3 \end{bmatrix}$$

It was easy to predict that in the 5^{th} row and column there all zeros. That's because it is related to the x-direction of the 3^rd global node, which is not connected on that direction to any other nodes. This makes that the global node number three doesn't have any force acting on the horizontal direction, that's the reason of the all zeros in the respective column and row.

As we wanted to demonstrate, the master stiffness equations are:

$$\frac{EA}{L}\begin{bmatrix} 2cs^2 & 0 & -cs^2 & c^2s & 0 & 0 & -cs^2 & -c^2s \\ & 1+2c^3 & c^2s & -c^3 & 0 & -1 & -c^2s & -c^3 \\ & & cs^2 & -c^2s & 0 & 0 & 0 & 0 \\ & & & c^3 & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 \\ & & & & & 1 & 0 & 0 \\ & & & & & & cs^2 & c^2s \\ symm & & & & & & & c^3 \end{bmatrix}\begin{bmatrix} u_{x1} \\ u_{y1} \\ u_{x2} \\ u_{y2} \\ u_{x3} \\ u_{y3} \\ u_{x4} \\ u_{y4} \end{bmatrix} = \begin{bmatrix} H \\ -P \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

b) Apply the BCs and show the 2-equation modified stiffness system

As we can see, we have 8 equations and 8 unknowns, so the system can be solved (a part from some values of α). This system is too big to be solved by hand in a fast way, but as far as we know some prescribed values because of the constraints applied in nodes 2, 3 and 4, we can reduce it as the unknown will be fewer.

We have three hinges, so the displacements in both the x and y directions at each node are prescribed and equal to zero.

$$u_{x2} = u_{x3} = u_{x4} = u_{y2} = u_{y3} = u_{y4} = 0 \tag{9}$$

To simplify the system, we can delete the rows and columns related to the known values. If the prescribed value is $\neq 0$, it has to be added the product with the respective value in the stiffness matrix in the force vector with the opposite sign.

As expected we will have a system $2x2$ as the only unknown values are the displacements in both directions of the global node 1. The system is the following:

$$\frac{EA}{L}\begin{bmatrix} 2cs^2 & 0 \\ 0 & 1+2c^3 \end{bmatrix}\begin{bmatrix} u_{x1} \\ u_{y1} \end{bmatrix} = \begin{bmatrix} H \\ -P \end{bmatrix}$$

c) Solve for the displacements u_{x1} and u_{y1}. Check that the solution makes physical sense for the limit cases $\alpha \longrightarrow 0$ and $\alpha \longrightarrow \frac{\pi}{2}$. Why does u_{x1} "blow up" if $H \neq 0$ and $\alpha \longrightarrow 0$?

Solving the above system, we'll obtain:

$$\begin{aligned}\frac{EA}{L}(2\cos\alpha\sin^2\alpha)u_{x1} &= H\\ \frac{EA}{L}(1+2\cos^3\alpha)u_{y1} &= -P\end{aligned} \tag{10}$$

so the solutions for u_{x1} and u_{y1} are:

$$\begin{aligned}u_{x1} &= \frac{LH}{2EA\cos\alpha\sin^2\alpha}\\ u_{y1} &= -\frac{PL}{EA(1+2\cos^3\alpha)}\end{aligned} \tag{11}$$

If $\alpha \longrightarrow 0$ we obtain:

$$\begin{aligned}u_{x1} &= \frac{LH}{0} \longrightarrow \infty\\ u_{y1} &= \frac{PL}{3EA}\end{aligned} \tag{12}$$

u_{x1} blows up because when α tends to 0, the system starts not to be statically indeterminate as the centre of rotation of each elements are going to align themselves permitting a rotation on the hinge in the global node 3. If the forces are applied only on the y-directions, the system doesn't move as it transmits everything along the element to the hinge. If there is a force applied on the node 1 ($H \neq 0$), it generates a momentum acting on the element that makes u_{x1} "blows up". Physically makes sense that the displacement along y is three times smaller than before, as when $\alpha = 0$ the three elemnts are going to coincide and the stiffness will be the sum of the elements.

If $\alpha \longrightarrow \frac{\pi}{2}$ we obtain:

$$\begin{aligned}u_{x1} &= \frac{LH}{0} \longrightarrow \infty\\ u_{y1} &= \frac{PL}{EA}\end{aligned} \tag{13}$$

that's because it means that to satisfy the condition of $\alpha = \frac{\pi}{2}$ the elements 1 and 3 will be horizontal and their length tends to ∞. That makes sense because the stiffness matrix will tends to ∞ as it depends from the inverse of the length of the element.

d) Recover the axial forces in the three members. Why do $F^{(1)}$ and $F(3)$ "blow up" if $H \neq 0$ and $\alpha \longrightarrow 0$?

In order to find the axial forces of each element, we have to find the reaction forces on the hinges in the nodes 2, 3 and 4. To do this, we recover the element stiffness matrix using the displacements computed before and the unknowns will be the hinge reaction.

Element 1:

$$\begin{bmatrix} f_{x1}^{(1)}\\ f_{y1}^{(1)}\\ f_{x2}^{(1)}\\ f_{y2}^{(1)}\end{bmatrix} = \frac{EA}{L}\begin{bmatrix} cs^2 & -sc^2 & -cs^2 & sc^2\\ & c^3 & sc^2 & -c^3\\ & & cs^2 & -sc^2\\ symm & & & c^3\end{bmatrix}\begin{bmatrix}\frac{LH}{2EA\cos\alpha\sin^2\alpha}\\ -\frac{PL}{EA(1+2\cos^3\alpha)}\\ 0\\ 0\end{bmatrix}$$

Element 2:

$$\begin{bmatrix} f_{x1}^{(2)}\\ f_{y1}^{(2)}\\ f_{x2}^{(2)}\\ f_{y2}^{(2)}\end{bmatrix} = \frac{EA}{L}\begin{bmatrix} 0 & 0 & 0 & 0\\ & 1 & 0 & -1\\ & & 0 & 0\\ symm & & & 1\end{bmatrix}\begin{bmatrix}\frac{LH}{2EA\cos\alpha\sin^2\alpha}\\ -\frac{PL}{EA(1+2\cos^3\alpha)}\\ 0\\ 0\end{bmatrix}$$

Element 3:

$$\begin{bmatrix} f_{x1}^{(3)}\\ f_{y1}^{(3)}\\ f_{x2}^{(3)}\\ f_{y2}^{(3)}\end{bmatrix} = \frac{EA}{L}\begin{bmatrix} cs^2 & sc^2 & -cs^2 & -sc^2\\ & c^3 & -sc^2 & -c^3\\ & & cs^2 & sc^2\\ symm & & & c^3\end{bmatrix}\begin{bmatrix}\frac{LH}{2EA\cos\alpha\sin^2\alpha}\\ -\frac{PL}{EA(1+2\cos^3\alpha)}\\ 0\\ 0\end{bmatrix}$$

The axial force for each element will be the f_{y2}^e for $e = 1, 2, 3$ divided by the cosinus of the angle. To find the f_{y2}^e we have to solve the last two rows of each above matrix.

For the element 1:

$$f_{a1} = \frac{f_{y2}^{(1)}}{\cos\alpha} = \frac{H}{2\sin\alpha} - \frac{P\cos^2\alpha}{1 + 2\cos^3\alpha} \tag{14}$$

For the element 2:

$$f_{a2} = \frac{f_{y2}^{(2)}}{1} = \frac{P}{1 + 2\cos^3\alpha} \tag{15}$$

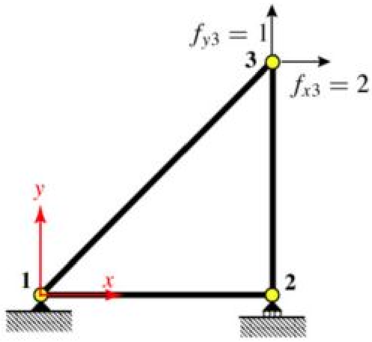
For the element 3:

$$f_{a3} = \frac{f_{y2}^{(3)}}{\cos\alpha} = -\frac{H}{2\sin\alpha} + \frac{P\cos^2\alpha}{1 + 2\cos^3\alpha} \tag{16}$$

As expected, the axial force of the element 2 does not depend on H as they are perpendicular. Otherwise, the axial forces acting on element 1 and 3 are depending both on H, P and α. As we can see from the results, the values of these two forces blow up to ∞ as $\alpha \longrightarrow 0$. This is because, for this value of the angle, the three elements will coincide with the horizontal one and it becomes a structure not statically indeterminate anymore and the single horizontal can rotate around the node 3.

Assignment: Problem 2

Dr. Who proposes "improving" the result for the example truss of the 1st lesson by putting one extra node, 4 at the midpoint of member (3) 1-3, so that it is subdivided in two different members: (3) 1-4 and (4) 3-4. His "reasoning" is that more is better. Try Dr. Who's suggestion by hand computations and verify that the solution "blows up" because the modified master stiffness is singular. Explain physically.



Answer

Adding a node between nodes 1 and 2, means to split the element between these two nodes in two smaller elements. From this we can already say that the system will not have a unique solution as we are adding more DOF than degrees of constraints. In fact these two elements can have small displacements even if all the rest of the structure won't move. Solving the new problem, we will demonstrate what said by words before if the final matrix will be singular, so the determinant will be equal to zero and some columns or rows will be linearly dependent.

Having explained in the 1^{st} problem the equilibrium and compatibility condition, the local systems are:

Element 1:

$$\begin{bmatrix} f_{x1} \\ f_{y1} \\ f_{x2} \\ f_{y2} \end{bmatrix} = 10 \begin{bmatrix} 1 & 0 & -1 & 0 \\ & 0 & 0 & 0 \\ & & 1 & 0 \\ symm & & & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

in which $EA = 100$ and $L = 10$, so $\frac{EA}{L} = 10$.

Element 2:

$$\begin{bmatrix} f_{x2} \\ f_{y2} \\ f_{x3} \\ f_{y3} \end{bmatrix} = 5 \begin{bmatrix} 0 & 0 & 0 & 0 \\ & 1 & 0 & -1 \\ & & 0 & 0 \\ symm & & & 1 \end{bmatrix} \begin{bmatrix} u_{x2} \\ 0 \\ u_{x3} \\ u_{y3} \end{bmatrix}$$

in which $EA = 50$ and $L = 10$, so $\frac{EA}{L} = 5$.

Element 3:

$$\begin{bmatrix} f_{x3} \\ f_{y3} \\ f_{x4} \\ f_{y4} \end{bmatrix} = 40 \begin{bmatrix} 0.5 & 0.5 & -0.5 & -0.5 \\ & 0.5 & -0.5 & -0.5 \\ & & 0.5 & 0.5 \\ symm & & & 0.5 \end{bmatrix} \begin{bmatrix} u_{x3} \\ u_{y3} \\ u_{x4} \\ u_{y4} \end{bmatrix}$$

in which $EA = 200\sqrt{2}$ and $L = 5\sqrt{2}$, so $\frac{EA}{L} = 40$.

Element 4:

$$\begin{bmatrix} f_{x4} \\ f_{y4} \\ f_{x1} \\ f_{y1} \end{bmatrix} = 40 \begin{bmatrix} 0.5 & 0.5 & -0.5 & -0.5 \\ & 0.5 & -0.5 & -0.5 \\ & & 0.5 & 0.5 \\ symm & & & 0.5 \end{bmatrix} \begin{bmatrix} u_{x4} \\ u_{y4} \\ 0 \\ 0 \end{bmatrix}$$

in which $EA = 200\sqrt{2}$ and $L = 5\sqrt{2}$, so $\frac{EA}{L} = 40$.

Assembling the global system:

$$\begin{bmatrix} f_{x1} \\ f_{y1} \\ f_{x2} \\ f_{y2} \\ f_{x3} \\ f_{y3} \\ f_{x4} \\ f_{y4} \end{bmatrix} = \begin{bmatrix} 30 & 20 & -10 & 0 & 0 & 0 & -20 & -20 \\ & 20 & 0 & 0 & 0 & 0 & -20 & -20 \\ & & 10 & 0 & 0 & 0 & 0 & 0 \\ & & & 5 & 0 & -5 & 0 & 0 \\ & & & & 40 & 40 & -20 & -20 \\ & & & & & 40 & -20 & -20 \\ & & & & & & 40 & 40 \\ symm & & & & & & & 40 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ u_{x2} \\ 0 \\ u_{x3} \\ u_{y3} \\ u_{x4} \\ u_{y4} \end{bmatrix}$$

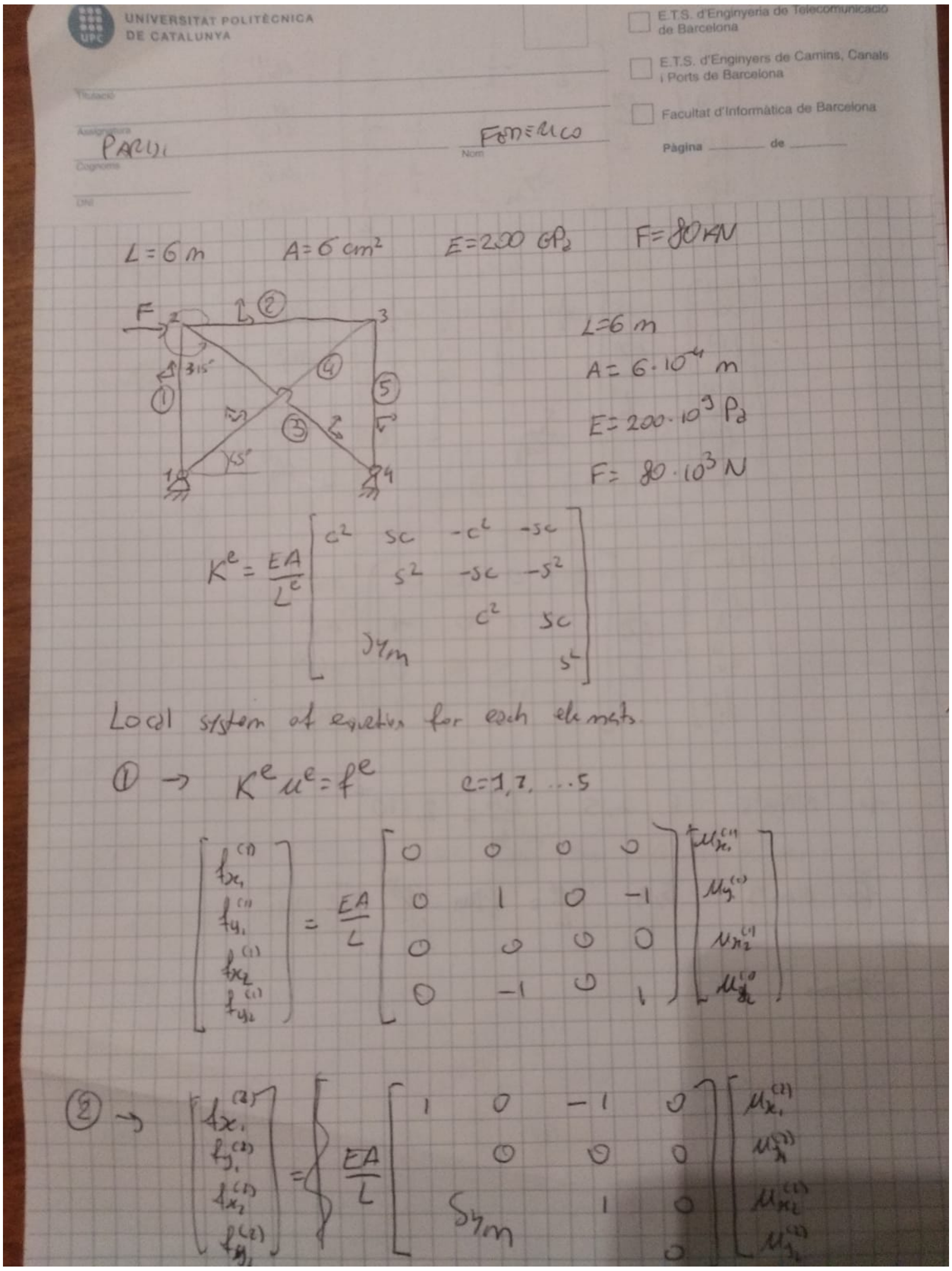
Reducing the matrix:

$$\begin{bmatrix} 0 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 10 & 0 & 0 & 0 & 0 \\ & 40 & 40 & -20 & -20 \\ & & 40 & -20 & -20 \\ & & & 40 & 40 \\ symm & & & & 40 \end{bmatrix} \begin{bmatrix} u_{x2} \\ u_{x3} \\ u_{y3} \\ u_{x4} \\ u_{y4} \end{bmatrix}$$

As said before, the system is singular. In a matrix, when a system is singular, the determinant is 0 and when this happens, it means that some rows and columns are linearly dependent. This is easy to see (without computing the determinant of the $5x5$ matrix as the second and third rows are the same, as the last two columns.

This means that the system doesn't have a unique solution but it can assume infinite configurations. Physically it means that there is not only one way in which the structure can move, and this comes from, as said before, to the fact that we added too many DOF when we split the third element in two smaller ones.

Classwork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Pàgina de
L = 6 m A = 6 cm² E = 200 GPa F = 80 kN
L = 6 m
A = 6·10⁻⁴ m
E = 200·10⁹ Pa
F = 80·10³ N
Local system of equations for each elements
K^e u^e = f^e e = 1, 2, ... 5

③ →
$$\begin{bmatrix} f_{x_1}^{(3)} \\ f_{y_1}^{(3)} \\ f_{x_2}^{(3)} \\ f_{y_2}^{(3)} \end{bmatrix} = \frac{EA}{L^{(3)}} \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ \text{Sym} & & \frac{1}{2} & -\frac{1}{2} \\ & & & \frac{1}{2} \end{bmatrix} \begin{bmatrix} u_{x_1}^{(3)} \\ u_{y_1}^{(3)} \\ u_{x_2}^{(3)} \\ u_{y_2}^{(3)} \end{bmatrix}$$

④ →
$$\begin{bmatrix} f_{x_1}^{(4)} \\ f_{y_1}^{(4)} \\ f_{x_2}^{(4)} \\ f_{y_2}^{(4)} \end{bmatrix} = \frac{EA}{L^{(4)}} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \text{Sym} & & \frac{1}{2} & \frac{1}{2} \\ & & & \frac{1}{2} \end{bmatrix} \begin{bmatrix} u_{x_1}^{(4)} \\ u_{y_1}^{(4)} \\ u_{x_2}^{(4)} \\ u_{y_2}^{(4)} \end{bmatrix}$$

⑤ →
$$\begin{bmatrix} f_{x_1}^{(5)} \\ f_{y_1}^{(5)} \\ f_{x_2}^{(5)} \\ f_{y_2}^{(5)} \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} 0 & 0 & 0 & 0 \\ & 1 & 0 & -1 \\ & & 0 & 0 \\ & & & 1 \end{bmatrix} \begin{bmatrix} u_{x_1}^{(5)} \\ u_{y_1}^{(5)} \\ u_{x_2}^{(5)} \\ u_{y_2}^{(5)} \end{bmatrix}$$

for elements 1, 2, 5 $\frac{EA}{L} = 2 \cdot 10^7$

for elements 3, 4 $\frac{EA}{L} = \sqrt{2} \cdot 10^7$

~~Assemb~~

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Titulació

Assignatura

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Assembling the global matrix, taking into account the compatibility condition the global stiffness matrix is:

$$K_{g_1} = \sqrt{2} \cdot 10^{7} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 & -\frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ & \frac{1}{2}+\sqrt{2} & 0 & -\sqrt{2} & -\frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ & & \frac{1}{2}+\sqrt{2} & -\frac{1}{2} & -\sqrt{2} & 0 & -\frac{1}{2} & \frac{1}{2} \\ & & & \frac{1}{2}+\sqrt{2} & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ & & & & \frac{1}{2}+\sqrt{2} & \frac{1}{2} & 0 & 0 \\ & \text{Sym} & & & & \frac{1}{2}+\sqrt{2} & 0 & -\sqrt{2} \\ & & & & & & \frac{1}{2} & -\frac{1}{2} \\ & & & & & & & \frac{1}{2}+\sqrt{2} \end{bmatrix}$$

Applying the BCs imposed by the constraints: hinges:

$u_{x_1} = u_{y_1} = 0 = u_{x_4} = u_{y_4}$

f_{x_1}

f_{y_1}

$80 \cdot 10^3$

→ global system of equation to be solved

$$\begin{bmatrix} f_{x_1} \\ f_{y_1} \\ 80 \cdot 10^3 \\ 0 \\ 0 \\ 0 \\ f_{x_4} \\ f_{y_4} \end{bmatrix} = \sqrt{2} \cdot 10^7 \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 & -\frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ & \frac{1}{2}+\sqrt{2} & 0 & -\sqrt{2} & -\frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ & & \frac{1}{2}+\sqrt{2} & -\frac{1}{2} & -\sqrt{2} & 0 & -\frac{1}{2} & +\frac{1}{2} \\ & & & \frac{1}{2}+\sqrt{2} & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ & & & & \frac{1}{2}+\sqrt{2} & \frac{1}{2} & 0 & 0 \\ & & & & & \frac{1}{2}+\sqrt{2} & 0 & -\sqrt{2} \\ & & & & & & \frac{1}{2} & \frac{1}{2} \\ & & & & & & & \frac{1}{2}+\sqrt{2} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ u_{x_2} \\ u_{y_2} \\ u_{x_3} \\ u_{y_3} \\ 0 \\ 0 \end{bmatrix}$$

I can remove column and rows ~~data~~ corresponding to the prescribed displacements.

$$\begin{bmatrix} 80 \cdot 10^3 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \sqrt{2} \cdot 10^7 \begin{bmatrix} \frac{1}{2}+\sqrt{2} & -\frac{1}{2} & -\sqrt{2} & 0 \\ -\frac{1}{2} & \frac{1}{2}+\sqrt{2} & 0 & 0 \\ -\sqrt{2} & 0 & \frac{1}{2}+\sqrt{2} & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2}+\sqrt{2} \end{bmatrix} \begin{bmatrix} u_{x_2} \\ u_{y_2} \\ u_{x_3} \\ u_{y_3} \end{bmatrix} \rightarrow$$

$$\rightarrow \begin{cases} 80 \cdot 10^3 = \sqrt{2} \cdot 10^7 \left[\left(\frac{1}{2}+\sqrt{2}\right) u_{x_2} - \frac{u_{y_2}}{2} - \sqrt{2}\, u_{x_3} \right] \\ 0 = \left[-\frac{u_{x_2}}{2} + \left(\frac{1}{2}+\sqrt{2}\right) u_{y_2} \right] \sqrt{2} \cdot 10^7 \\ 0 = \sqrt{2} \cdot 10^7 \left[-\sqrt{2}\, u_{x_2} + \left(\frac{1}{2}+\sqrt{2}\right) u_{x_3} + \frac{1}{2} u_{y_3} \right] \\ 0 = \sqrt{2} \cdot 10^7 \left[\frac{u_{x_3}}{2} + \left(\frac{1}{2}+\sqrt{2}\right) u_{y_3} \right] \end{cases} \rightarrow \begin{cases} u_{x_2} = 8{,}54 \cdot 10^{-3}\,m \\ u_{y_2} = 2{,}23 \cdot 10^{-3}\,m \\ u_{x_3} = 6{,}77 \cdot 10^{-3}\,m \\ u_{y_3} = -1{,}76 \cdot 10^{-3}\,m \end{cases}$$