Assignment 2

Computational Structural Mechanics and Dynamics

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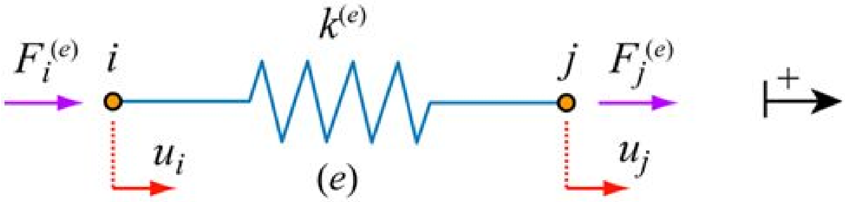


Contents

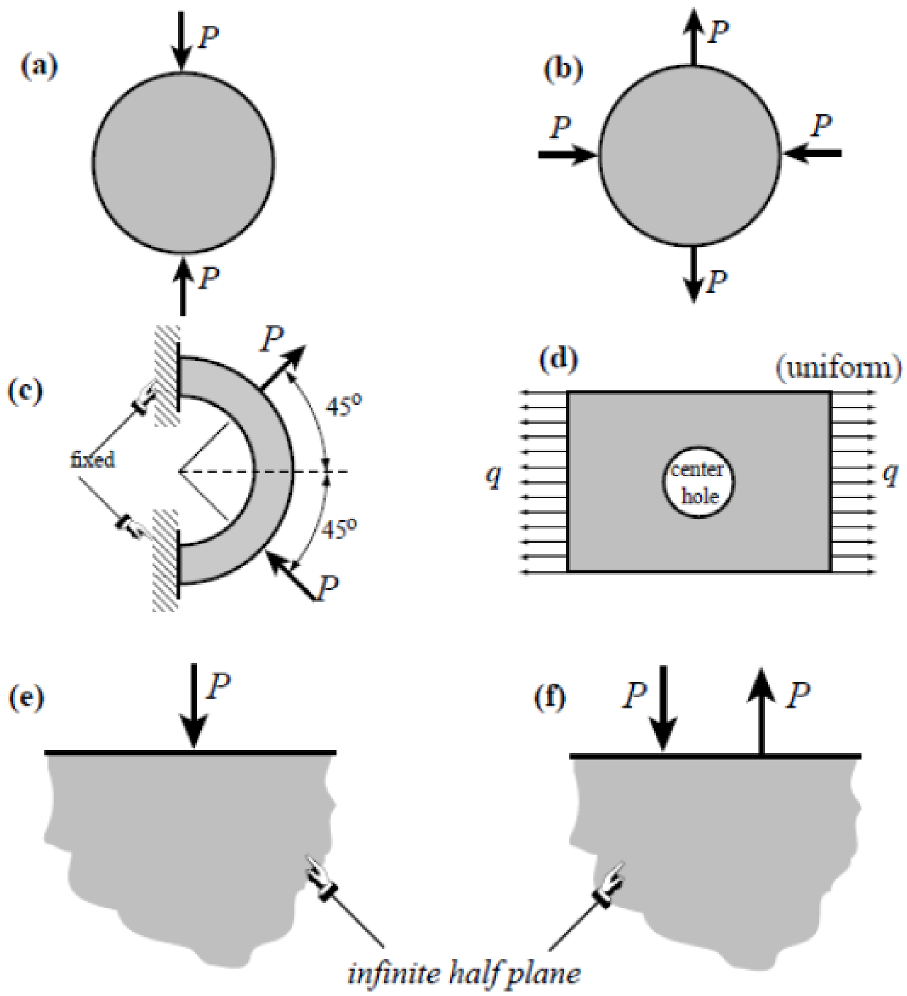
1 Assignment 2.1

On "FEM Modelling: Introduction":

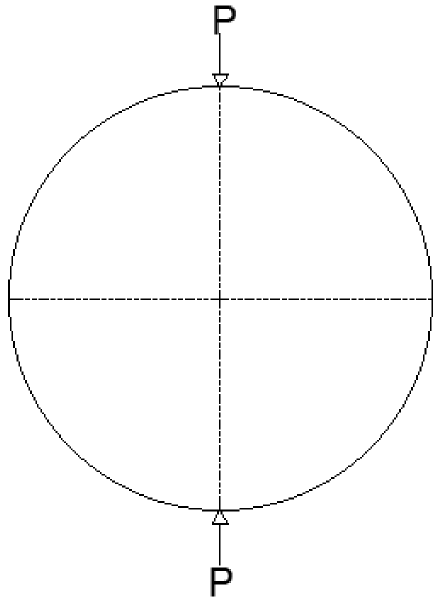
1. Identify the symmetry and antisymmetry lines in the two-dimensional problems illustrated in the figure. They are:

   (a) a circular disk under two diametrically opposite point forces (the famous "Brazilian test" for concrete)
   (b) the same disk under two diametrically opposite force pairs
   (c) a clamped semiannulus under a force pair oriented as shown
   (d) a stretched rectangular plate with a central circular hole.
   (e) and (f) are half-planes under concentrated loads.

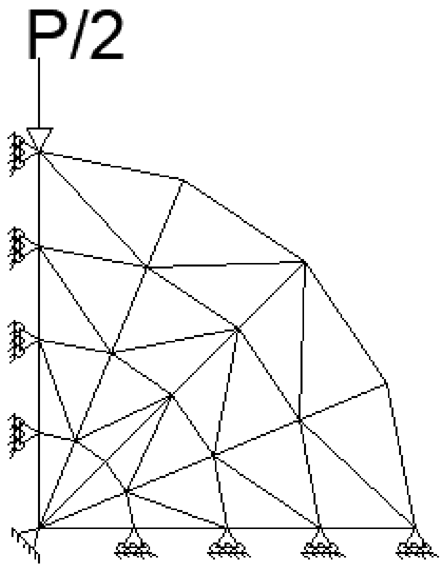
2. Having identified those symmetry/antisymmetry lines, state whether it is possible to cut the complete structure to one half or one quarter before laying out a finite element mesh. Then draw a coarse FE mesh indicating, with rollers or fixed supports, which kind of displacement BCs you would specify on the symmetry or antisymmetry lines.



1.1 Figure (a)

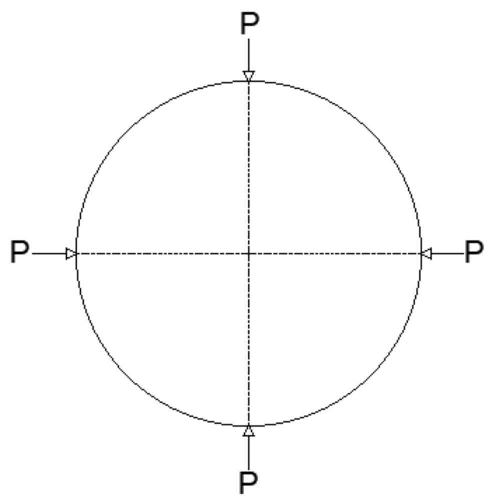


(a) Symmetry Axis

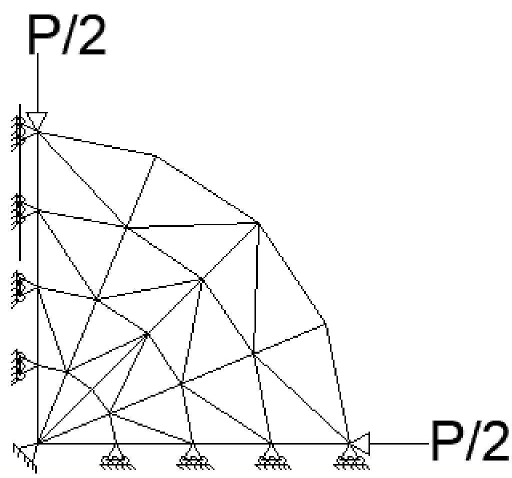


(b) FE Mesh and BCs

1.2 Figure (b)

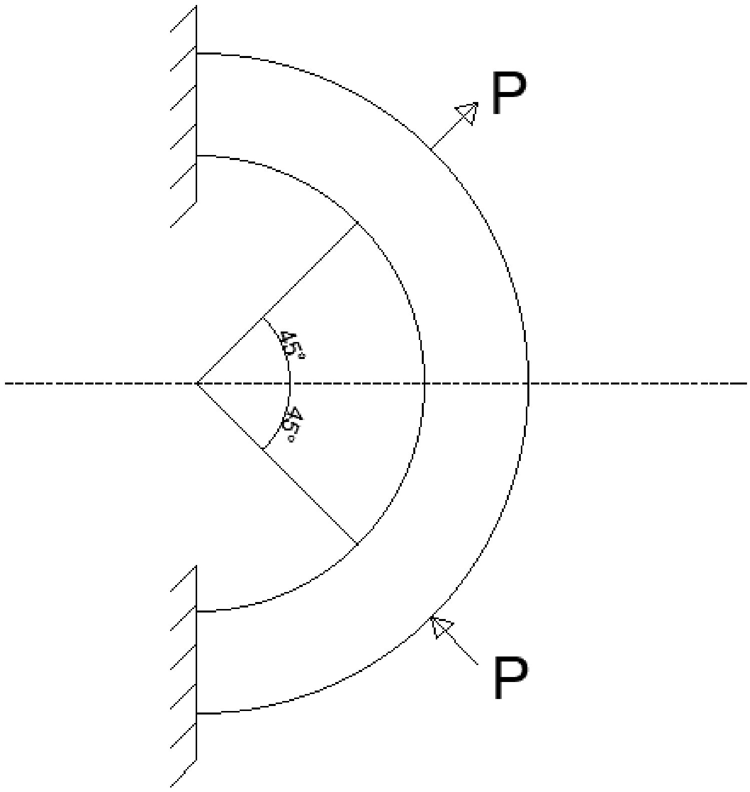


(a) Symmetry Axis

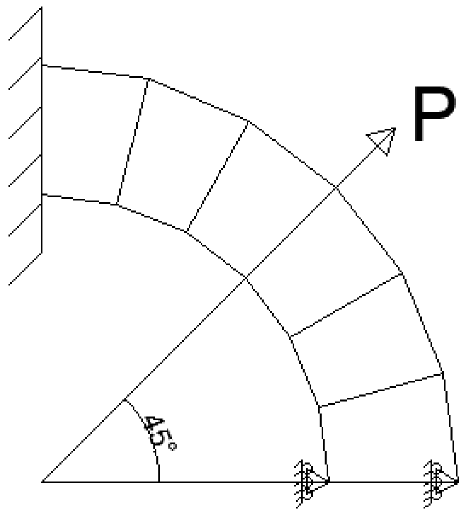


(b) FE Mesh and BCs

1.3 Figure (c)

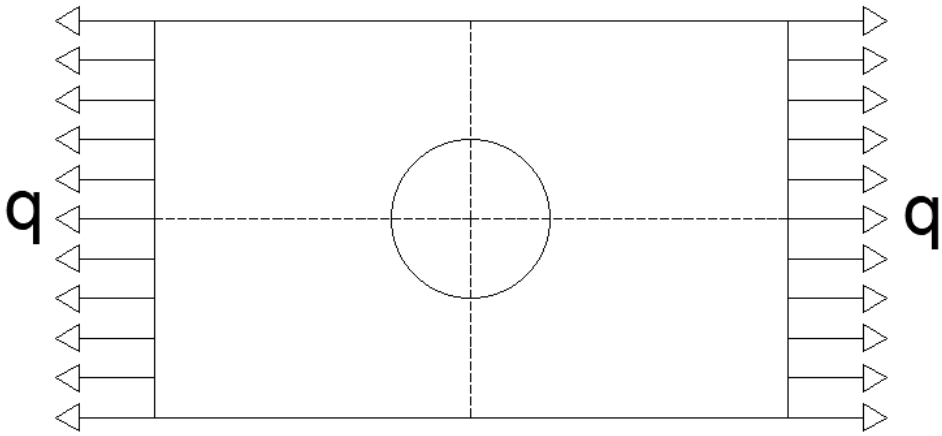


(a) Antisymmetry Axis

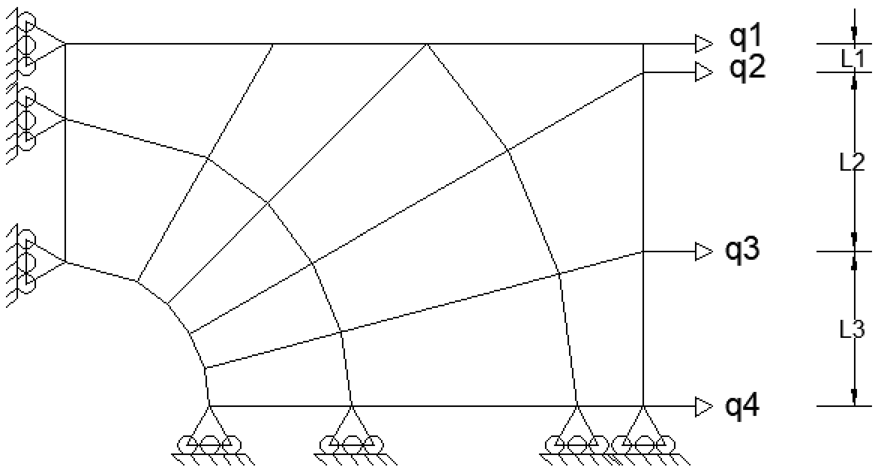


(b) FE Mesh and BCs

1.4 Figure (d)



(a) Symmetry Axis



(b) FE Mesh and BCs

Where:

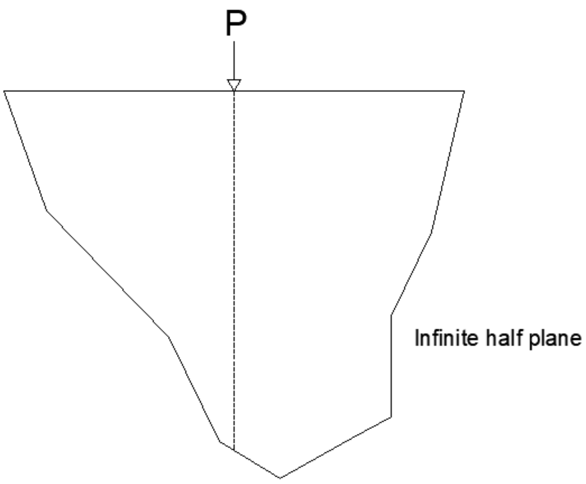
$$q_1 = \frac{qL_1}{2}$$

$$q_2 = \frac{q(L_1 + L_2)}{2}$$

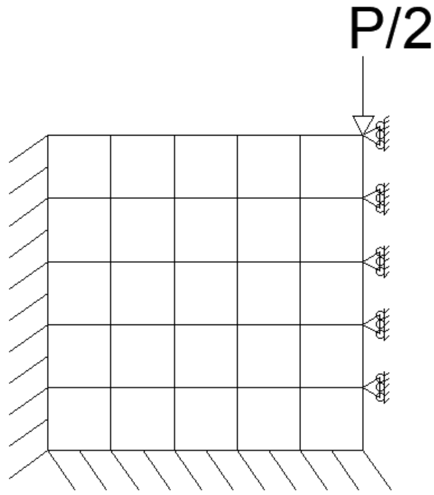
$$q_3 = \frac{q(L_2 + L_3)}{2}$$

$$q_4 = \frac{qL_3}{2}$$

1.5 Figure (e)

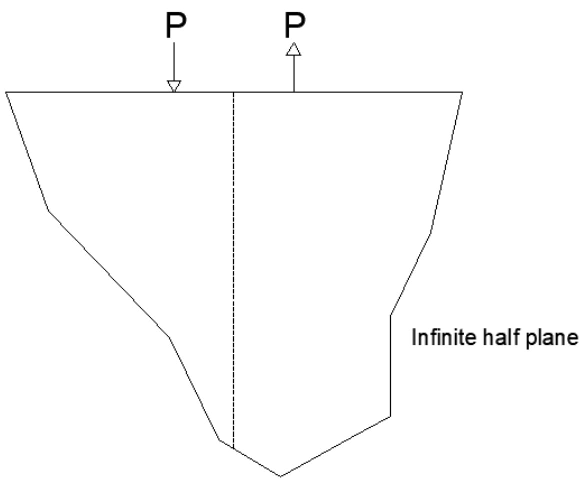


(a) Symmetry Axis

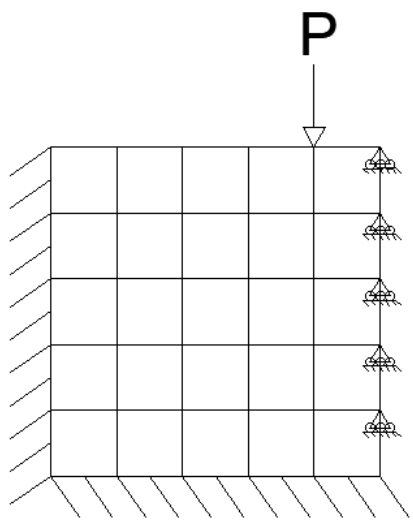


(b) FE Mesh and BCs

1.6 Figure (e)



(a) Symmetry Axis



(b) FE Mesh and BCs

2 Assignment 2.2

Explain the difference between *"Verification"* and *"Validation"* in the context of the FEM-Modelling procedure. Validation: Is the evaluation of the structural and computational models compared to experimental data. The experimental data can be obtained trough scaled models or prototypes.

Verification: Is the comparison between the numerical results and the analytical solutions or more accurate numerical solutions. This process is mostly implemented to reduce the possible numerical errors, by using error estimation techniques.

Both verification and validation help us measure the model's errors, but the difference between them can be summarized by saying that the verification tell us if we are solving the problems accurately while validation tell us if we are solving the right problem.

3 Assignment 2.3

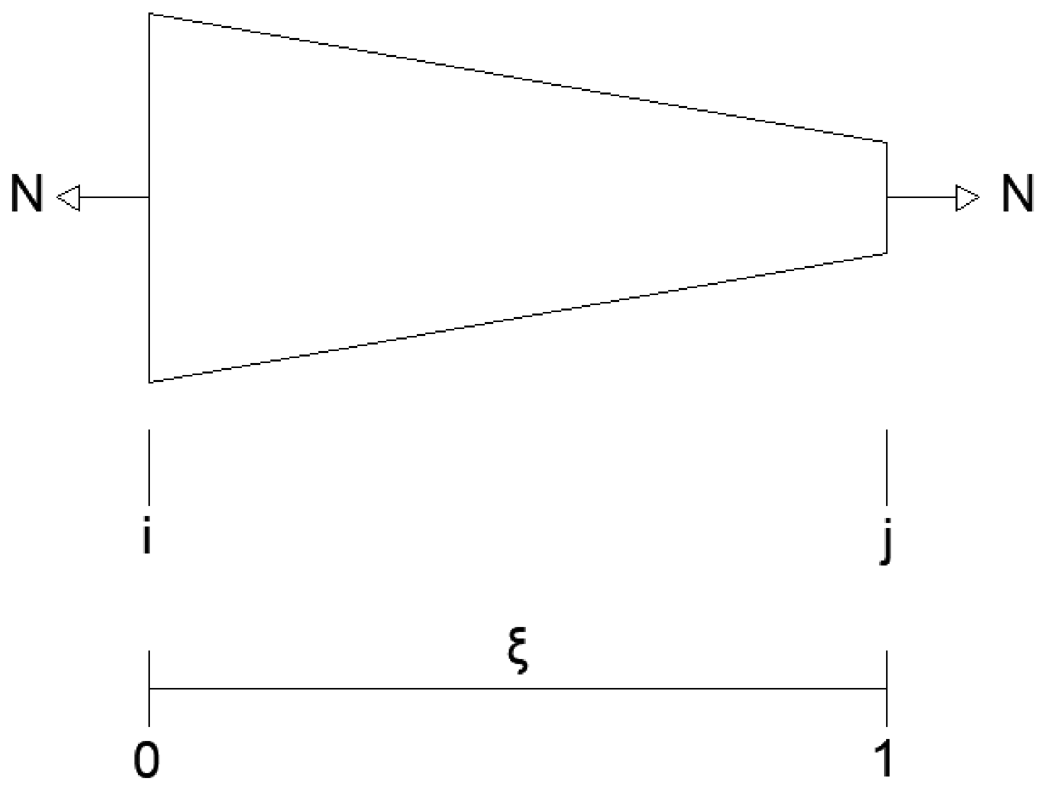
On "Variational Formulation":

1. A tapered bar element of length l and areas A_i and A_j with A interpolated as

$$A = A_i(1-\xi) + A_j\xi$$

and constant density ρ rotates on a plane at uniform angular velocity ω (rad/sec) about node i. Taking axis x along the rotating bar with origin at node i, the centrifugal axial force is $q(x) = \rho A\omega^2 x$ along the length in which x is the longitudinal coordinate $x = x^e$.

Find the consistent node forces as functions of ρ, A_i, A_j, ω and l, and specialize the result to the prismatic bar $A = A_i = A_j$.



3.1 Solution

$$f_{ext} = \int_0^1 q \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} l d\xi$$

Where:

$$\xi = \frac{x - x_1}{l}$$

For this case $x_1 = 0$:

$$\xi = \frac{x}{l} \quad => \quad x = \xi l$$

Substituting in f_ext:

$$f_{ext} = \rho\omega^2 l^2 \int_0^1 [A_i(1-\xi) + A_j\xi]\xi \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} d\xi$$

Therefore:

$$f_i = f_{ext} = \rho\omega^2 l^2 \int_0^1 [A_i(1-\xi) + A_j\xi]\xi(1-\xi)d\xi = \frac{\rho\omega^2 l^2(A_i + A_j)}{12}$$

$$f_j = f_{ext} = \rho\omega^2 l^2 \int_0^1 [A_i(1-\xi) + A_j\xi]\xi(\xi)d\xi = \frac{\rho\omega^2 l^2(A_i + 3A_j)}{12}$$

For $A = A_i = A_j$:

$$f_i = \frac{\rho\omega^2 l^2 A}{6}$$

$$f_j = \frac{\rho\omega^2 l^2 A}{3}$$

4 Discussion

- Assignment 2.1
  The symmetry and antisymmetry lines will help us reduce the computational cost of our model, as well as facilitate its modeling, reducing the work of the analysis linearly proportional to the number of symmetry and antisymmetry planes that are presented.

- Assignment 2.2
  Verification and validation of a model will always be necessary to verify that the desired performance is being obtained, as well as to improve the method and know if it meets the established permissible errors. These errors can be improved by making a finer discretization.

- Assignment 2.3
  It can be calculated straight forward by implementing the energetic principle, making it easier and faster to obtain the node forces vector.