Computational Structural Mechanics and Dynamics
Assignment 2

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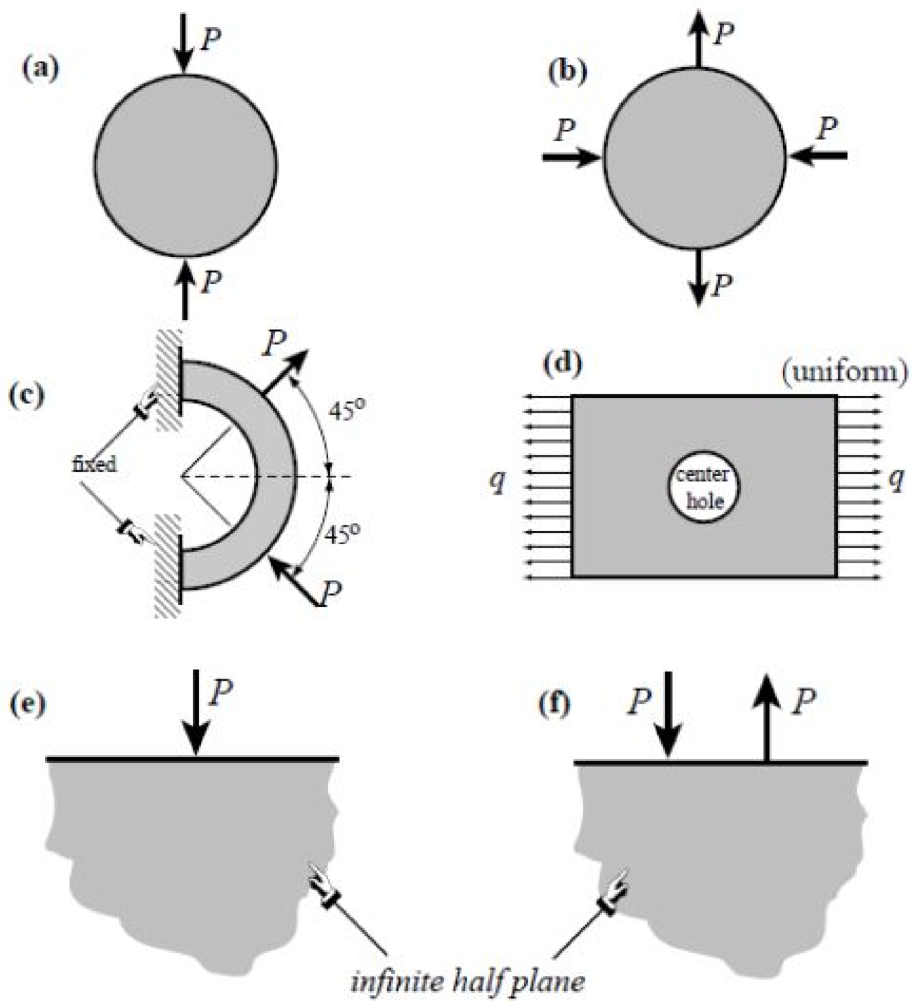
Assignment 2.1

1. Identify the symmetry and ant-isymmetry lines in the two-dimensional problems illustrated in the figure.
2. Having identified those symmetry/antisymmetry lines, state whether it is possible to cut the complete structure to one half or one quarter before laying out a finite element mesh. Then draw a coarse FE mesh indicating, with rollers or fixed supports, which kind of displacement BCs you would specify on the symmetry or antisymmetry lines.



Answer

Sometimes it can be noticed that the problem that is going to be analyzed can be simplified a lot. For example it can be subdivided in smaller equivalent systems in order to being able to study only one of them, then generalize to the whole one. This is possible only if there are some symmetries or anti-symmetries. The symmetry line is the imaginary line where you could "fold" the system and the loads will match exactly. The anti-symmetry line is the imaginary line for which the loads acting on the two halves have the same direction perpendicular to the line, but opposite direction parallel to that line.

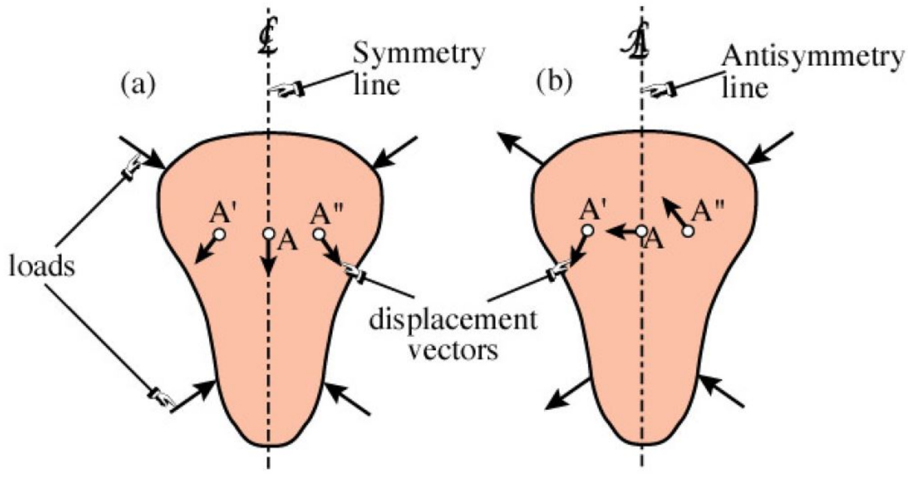


Figure 1: Symmetry and anti-symmetry lines

As shown in the left side of figure 1, the loads are symmetric, if the system is "fold", the load will match exactly. In the right side of the same figure, it is easy to see that the direction perpendicular to the anti-symmetry line is always the same for the respective loads on each side, but the parallel component (in this case the vertical one) is in the opposite direction.

1. Finding the symmetry and anti-symmetry lines for each problem:
   (a) The system has two symmetry lines passing through the centre of the system. One is perpendicular to the loads acting on it, the other one is parallel to them.
   (b) The system presents two lines of symmetry and two of anti-symmetry. The two symmetry lines are the ones passing through the centre and parallel to the loads. The anti-symmetry are the the symmetry ones, but rotated of $\frac{\pi}{4}$.
   (c) This system is going to have one anti-symmetry line. This line is the horizontal one, passing through the centre (the dotted line in the figure).
   (d) This system has two symmetry lines, passing through the center of the system and parallel to the x and y axis (like the ones in the first problem).
   (e) This system has one symmetry line, coinciding with the direction of the load P.
   (f) The last system has one anti-symmetry line, passing through the mid point of the two loads and parallel to them.

2. Representation of the symmetry and anti-symmetry lines, coarse mesh, supports and BCs:
   (a) To study the problem in an easier way, this system can be divided in four parts and study only one of that part, splitting the load in two halves $P^{eq} = \frac{P}{2}$. To have an equivalent simpler system, vertical and horizontal rollers are added in each node lying on the symmetry lines, plus an hinge in the centre. The rollers are going to transmit the component according to their "rolling" direction and to generates an opposite load on the normal direction; this is what is described by the symmetry line, so it is a good simplification. The hinge doesn't let the node moving in any direction (only permits the rotation), that's why it is applied in the center of the system, where the two symmetries axis intersect. That point, by definition, can not move as is the center of symmetry: the hing well describes this behaviour. In figure 2 are shown the coarse mesh and the relative constraints. The displacements BCs prescribed by the

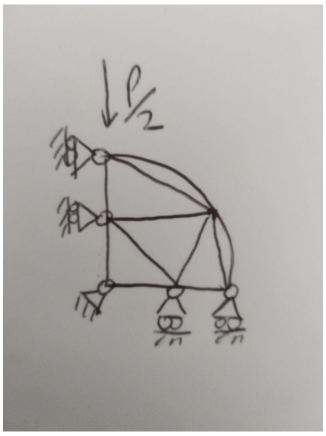


Figure 2: System a) simplified

constraints are: $u_x = 0$ of the nodes on the vertical line, $u_y = 0$ of the nodes on the horizontal line. As can be noticed, the point in the middle is lying on both of the symmetry lines so has $u_u = u_y = 0$.

(b) The problem can be simplified in two ways. One is the same as in figure 2. In this case the vertical load has the opposite direction and there will be an extra load on the horizontal direction (according with the verse in the general problem), divided by two as well: $P_x^{eq} = \frac{P}{2}$. The other way is referring to the anti-symmetry lines, changing all the constraints applied while simplifying. It will be well explained in the next problem. It is not represented here because it will be better and easier to use as referent simplification criteria the symmetry line. The displacements BCs are the same as the system above.

(c) This problem can be simplified splitting the figure in two parts and adding two vertical rollers to the nodes lying on the anti-symmetry line. This because, as it can be noticed, the horizontal component of the load is going to be erased by the opposite direction of the force. The opposite load is acting as a constraint that doesn't transmit the horizontal component of the force on the other side of the line. It generates an opposite load and is exactly what is going to happen in the system (so the simplification is valid). Regarding the vertical components of the loads, they have the same direction and same intensity. So the rollers well describe the system as, by definition, they don't act on the direction in which they are lying (the vertical one in this case), so the vertical component of the load is going to be conserved. The displacements BCs are $u_x = 0$ for both of the nodes.

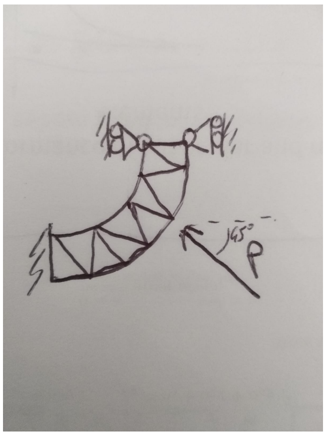


Figure 3: System c) simplified

(d) This problem can be simplified considering only $\frac{1}{4}$ of the system. Due to the symmetry, the nodes on the vertical symmetry line can't have horizontal displacements, as the ones lying on the horizontal symmetry line can't move on the vertical axis. It can be noticed from the problem that the circumference, under this loads, will equally expand in every direction. That's why, in the nodes on the symmetry lines, the simplified system has only rollers. If there wasn't an hole in the center of the plate, the constraint would have been an hinge in the middle. The displacements BCs are $u_x = 0$ on the vertical line, and $u_y = 0$ on the horizontal line.

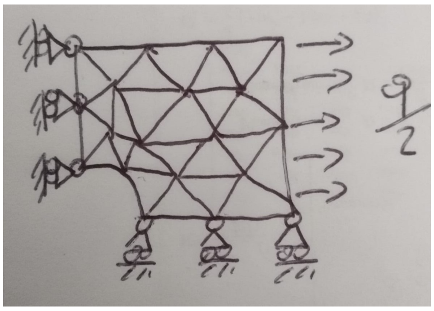


Figure 4: System d) simplified

(e) This problem is a semi-infinite plane with a vertical load, it can be split in two semi-infinite planes with only half of the load acting on it. The displacements BCs are $u_x = 0$.

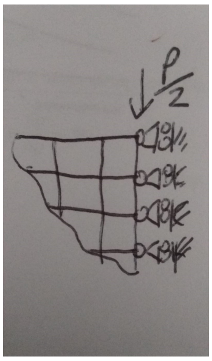


Figure 5: System e) simplified

(f) The last problem has the same geometry of the previous one; a semi-infinite plane with vertical loads. In this case we also have a load acting along the opposite direction. We can find an anti-symmetry line as said in the point $1.f$, lying in the middle between the two opposite-directed loads. This will generate a momentum (a couple of opposite loads not lying on the same line). This momentum will make the system rotating in terms of small displacements, but the nodes on the line will only move in the horizontal direction, having prescribed the vertical one. That displacement is prescribed as the loads have the opposite direction and if the distance is the same, they will generate an equal and opposite force, so the nodes won't move vertically. This behaviour is well-described by the rollers lying on an horizontal plane. The displacements BCs are $u_y = 0$.

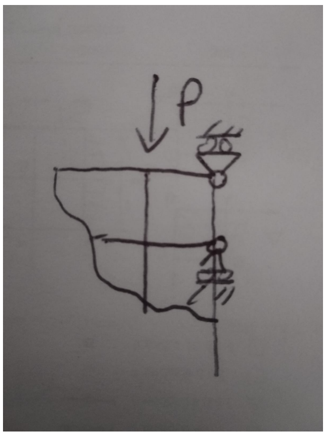


Figure 6: System f) simplified

Assignment 2.2

Explain the difference between "Verification" and "Validation" in the context of the FEM Modelling procedure.

Working in Finite Elements means working with approximations. Working with approximations means that not always the mathematical model matches the real one, and sometimes the results, because of the simplifications done, are not the expected ones. That's why, in a FEM modelling process, the two processes of verification and validation are important. These two processes guarantee that the solution of the computational problem well-describes the real one.

1. Verification: the process of verification checks if we have solved the problem correctly. If the equations have been solved in the right way, if the mesh chosen are well-suit for the problem, if the solution converges or diverges.

2. Validation: The process of validation checks if we have chosen the right model. If our computational model well-represents the physical one, if the boundary conditions considered are the right ones and well-written.

As can be noticed, both of these problems depend on the approximation done. The approximation done solving the governing equation, the size of the mesh etc. These are all simplification done that can make the solution diverge (part of the Verification process). The same can happen, or we can have very strange results, if the boundary condition are bad-imposed (part of the Validation process).

In figure 7, the whole computational process is shown, focusing on the two processes.

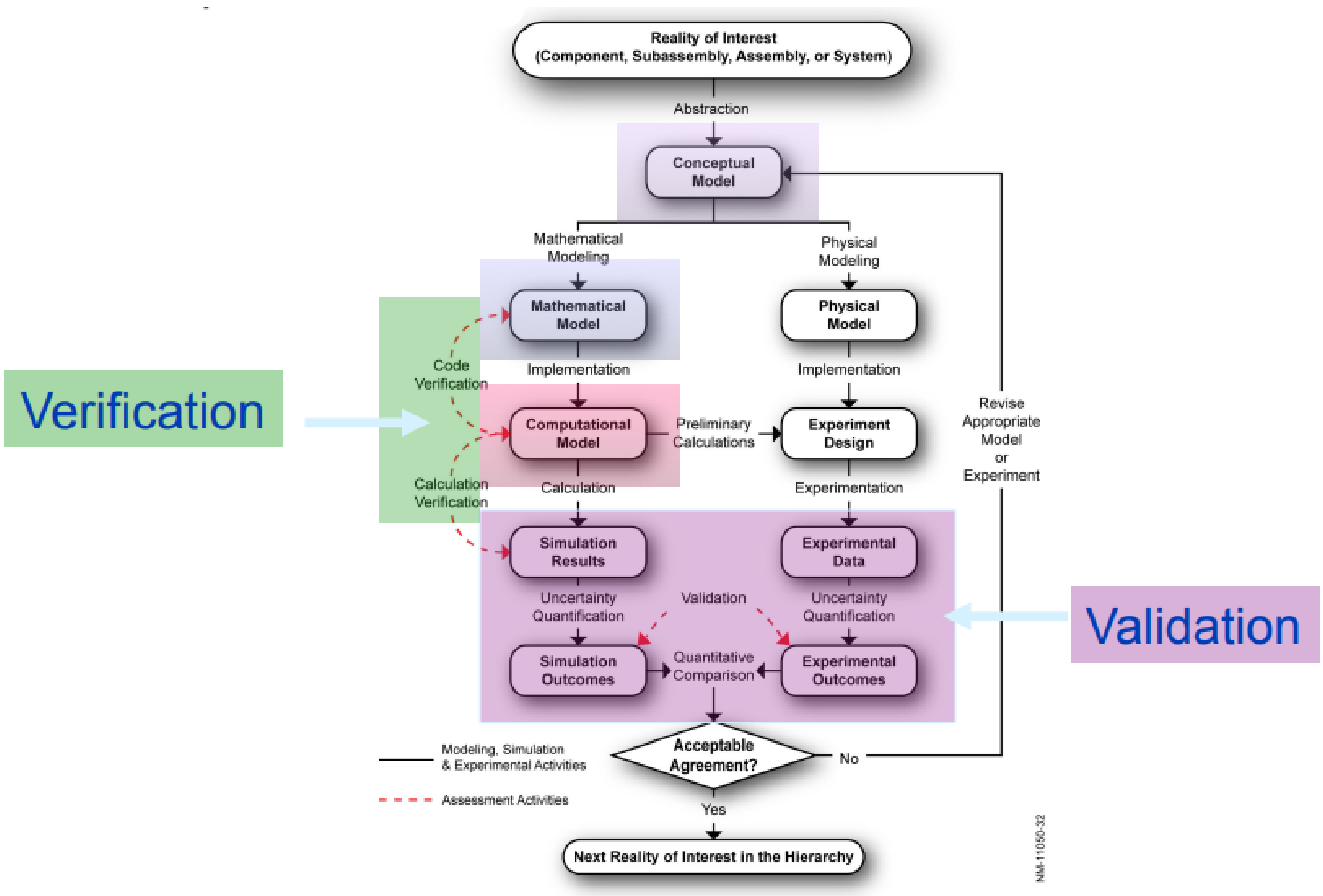


Figure 7:

Assignment 2.3

In this problem it is asked to find the consistent node forces in the two cases: the first one is when the area is changing, the second one when the area is constant and equal to:

$$A = A_i(1-\zeta) + A_j\zeta \tag{1}$$

In which ζ is the local coordinate in the bar; when $\zeta = 0$ the bar area is A_i and when $\zeta = 1$ the area will be A_j.

Here it is asked to solve the problem with a different method, called Minimum Potential Energy Principle (MPE). With this method, is computed the total potential energy of the structure as the difference between the external work and the internal energy. With this principle the stiffness matrix and the nodal forces vector are computed. The final results are the same as the ones obtained with the Direct Stiffness Method (DSM), but here the reference length is not the length of the member anymore, but the length of the finite element. This method provides an easier and faster way than DSM to compute the equations needed. That's why it is considered more efficient when the system gets more complex.

Using the MPE, to find the mentioned forces, we apply the following equation:

$$f_{ext} = \int_0^l q \begin{bmatrix} 1-\zeta \\ \zeta \end{bmatrix} d\zeta \tag{2}$$

As it can be noticed, the 1^{st} component of the vector is referred to the force in $_i$ and the 2^{nd} is referred to the force in $_j$, and $\zeta = \frac{x-x_l}{l}$ in which l is the total length of the bar. Knowing the flux $q(x) = \rho A\omega^2 x$, the consistent nodal forces vector becomes:

$$f = \rho\omega^2 l^{e2} \int_0^l \begin{bmatrix} (A_i(1-\zeta) + A_j\zeta)\zeta(1-\zeta) \\ (A_i(1-\zeta) + A_j\zeta)\zeta^2 \end{bmatrix} d\zeta \tag{3}$$

and solving the integral we obtain:

$$f = \begin{bmatrix} \frac{1}{12}\rho\omega^2 l^2(A_i + A_j) \\ \frac{1}{4}\rho\omega^2 l^2(\frac{A_i}{3} + A_j) \end{bmatrix} \tag{4}$$

Generalizing the solution to a bar with constant area $A = A_i = A_j$, the force vector will be:

$$f = \begin{bmatrix} \frac{1}{6}\rho\omega^2 l^2 A \\ \frac{1}{3}\rho\omega^2 l^2 A \end{bmatrix} \tag{5}$$