Assignment 3.1

1. $K = \int_{\Omega^e} h \cdot B^T E B \, d\Omega$

Differentiating the linear shape functions with respect to x and y returns constant values to shapes, though B is constant.

$K^e = A h B^T E B$

$x_{jk} = x_j - x_k \qquad y_{jk} = y_j - y_k$

$x_{21} = 3 - 0 = 3$

$x_{32} = 2 - 3 = -1$

$x_{13} = 0 - 2 = -2$

$y_{31} = 2 - 0 = 2$

$y_{12} = 0 - 1 = -1$

$y_{23} = 1 - 2 = -1$

$$K = Ah \cdot \frac{1}{2A} \cdot \frac{1}{2A} \begin{bmatrix} -1 & 0 & -1 \\ 0 & -1 & -1 \\ 2 & 0 & -2 \\ 0 & -2 & 2 \\ -1 & 0 & 3 \\ 0 & 3 & -1 \end{bmatrix} \begin{bmatrix} 100 & 25 & 0 \\ 25 & 100 & 0 \\ 0 & 0 & 50 \end{bmatrix} \begin{bmatrix} -1 & 0 & 2 & 0 & -1 & 0 \\ 0 & -1 & 0 & -2 & 0 & 3 \\ -1 & -1 & -2 & 2 & 3 & -1 \end{bmatrix}$$

$$K = \frac{h}{4A} \begin{bmatrix} -100 & -25 & -50 \\ -25 & -100 & -50 \\ 200 & 50 & -100 \\ -50 & -200 & 100 \\ -100 & -25 & 150 \\ 75 & 300 & -50 \end{bmatrix} \begin{bmatrix} -1 & 0 & 2 & 0 & -1 & 0 \\ 0 & -1 & 0 & -2 & 0 & 3 \\ -1 & -1 & -2 & 2 & 3 & -1 \end{bmatrix} = \Big[$$

$$2A = (x_2 y_3 - x_3 y_2) + (x_3 y_1 - x_1 y_3) + (x_1 y_2 - x_2 y_1)$$
$$= (3 \times 2 - 2 \times 1) + (2 \times 0 - 0 \times 2) + (0 \times 1 - 3 \times 0)$$
$$2A = 6 - 2 = 4$$
$$\frac{h}{4A} = \frac{1}{8}$$

$$= \frac{1}{8} \times \begin{bmatrix} 150 & 75 & -100 & -50 & -50 & -25 \\ 75 & 150 & 50 & 100 & -125 & -250 \\ -100 & 50 & 600 & -300 & -500 & 250 \\ -50 & 100 & -300 & 600 & 350 & -700 \\ -50 & -125 & -500 & 350 & 550 & -225 \\ -25 & -250 & 250 & -700 & -225 & 850 \end{bmatrix}$$

$$= \begin{bmatrix} 18{,}75 & 9{,}375 & -12{,}5 & -6{,}25 & -6{,}25 & -3{,}125 \\ 9{,}375 & 18{,}75 & 6{,}25 & 12{,}5 & -15{,}625 & -31{,}25 \\ -12{,}5 & 6{,}25 & 75 & -37{,}5 & -62{,}5 & 31{,}25 \\ -6{,}25 & 12{,}5 & 37{,}5 & 75 & 43{,}75 & -87{,}5 \\ -6{,}25 & -15{,}625 & -62{,}5 & 43{,}75 & 68{,}75 & -28{,}125 \\ -3{,}125 & -31{,}25 & 31{,}25 & -87{,}5 & -21{,}25 & 112{,}75 \end{bmatrix}$$

2.)

- Sum of rows 1, 3 and 5

row 1 : $18{,}7 + 9{,}375 - 12{,}5 - 6{,}25 - 6{,}25 + (-3{,}125) = 0$

row 3 : $-12{,}5 + 6{,}25 + 75 - 37{,}5 - 62{,}5 + 31{,}25 = 0$

row 5 : $-6{,}25 - 15{,}625 - 62{,}5 + 43{,}75 + 68{,}75 - 28{,}125 = 0$

column 1 : $\frac{1}{8} \times (0) = 0$

column 3 : $\frac{1}{8}(-0) = 0$

column 5 : $\frac{1}{8}(0) = 0$

spiral

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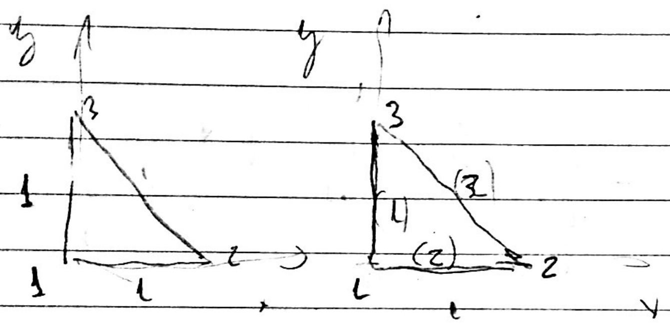
row 2: $\frac{1}{8}(-0) = 0$
row 4: 0
row 6: 0

column 2: $\frac{1}{8}(-0) = 0$
column 4: $\frac{1}{8}(0) = 0$
column 6: 0

3, 2



$x_1 = 0 \quad x_2 = 1 \quad x_3 = 0$

$y_1 = 0 \quad y_2 = 0 \quad y_3 = 1$

$$E = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$$

$\nu = 0$

$x_{21} = 1 \quad y_{31} = 1$

$x_{13} = 0 \quad y_{23} = -1$

$x_{32} = -1 \quad y_{12} = 0$

$$B = \begin{bmatrix} -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \\ -1 & -1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

$$E = \begin{bmatrix} E & 0 & 0 \\ 0 & E & 0 \\ 0 & 0 & \frac{E}{2} \end{bmatrix}$$

$$n^* = \begin{bmatrix} -1 & 0 & -1 \\ 0 & -1 & -1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} E & 0 & 0 \\ 0 & E & 0 \\ 0 & 0 & \frac{E}{2} \end{bmatrix} \cdot B$$

$$n^* = \begin{bmatrix} -E & 0 & -\frac{E}{2} \\ 0 & -E & -\frac{E}{2} \\ E & 0 & 0 \\ 0 & 0 & E/2 \\ 0 & 0 & E/2 \\ 0 & E & 0 \end{bmatrix} B$$

$K = \frac{h}{4A} \cdot B^T \cdot E \cdot B$ → "h" is uniform

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$$K^* = \begin{bmatrix} -E & 0 & -E/2 \\ 0 & -E & -E/2 \\ E & 0 & 0 \\ 0 & 0 & E/2 \\ 0 & 0 & E/2 \\ 0 & E & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \\ -1 & -1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

$A = \frac{1}{2}$, $h = 1$

$$K = \frac{h}{4A} \cdot K^* \rightarrow K = \frac{1}{4\left(\frac{1}{2}\right)} \cdot K^* = \frac{1}{2} K^*$$

$$K = \frac{1}{2} \begin{bmatrix} 3/2\,E & E/2 & -E & -E/2 & -E/2 & 0 \\ E/2 & 3/2\,E & 0 & -E/2 & -E/2 & -E \\ -E & 0 & E & 0 & 0 & 0 \\ -E/2 & -E/2 & 0 & E/2 & E/2 & 0 \\ -E/2 & -E/2 & 0 & E/2 & E/2 & 0 \\ 0 & -E & 0 & 0 & 0 & E \end{bmatrix}$$

For the thrum problem

$$K^{1} = \frac{EA}{L} \begin{bmatrix} c^2 & sc & -c^2 & -sc \\ sc & s^2 & -sc & -s^2 \\ -c^2 & -sc & c^2 & sc \\ -sc & -s^2 & sc & s^2 \end{bmatrix}$$

$$K^{(1)} = EA \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{matrix} M_{x1} \\ M_{y1} \\ M_{x3} \\ M_{y3} \end{matrix}$$

$\alpha = 90°$

$1 \text{ and } 2$

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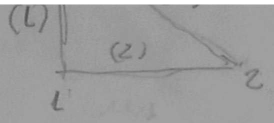

$\sin\varphi - 30 = \cos\varphi$ ___/___/___

$$k^{2} = EA \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} u_{x_1} \\ u_{y_1} \\ u_{x_2} \\ u_{y_2} \end{matrix}$$

2 and 3

$$k^{3} = EA'\frac{\sqrt{2}}{2} \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{matrix} u_{x_3} \\ u_{y_3} \\ u_{x_1} \\ u_{y_1} \end{matrix} = EA' \begin{bmatrix} \frac{\sqrt{2}}{4} & -\frac{\sqrt{2}}{4} & -\frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{4} \\ -\frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{4} & -\frac{\sqrt{2}}{4} \\ -\frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{4} & -\frac{\sqrt{2}}{4} \\ \frac{\sqrt{2}}{4} & -\frac{\sqrt{2}}{4} & -\frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{4} \end{bmatrix}$$

3 and 1

$k = k^{(1)} + k^{(2)} + k^{(3)}$ (augmented k)

$$k^{(1)} = E \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & A & 0 & 0 & 0 & -A \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -A & 0 & 0 & 0 & A \end{bmatrix} \qquad k^{(2)} = E \begin{bmatrix} A & 0 & -A & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -A & 0 & A & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$k^{(3)} = E \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\sqrt{2}}{4}A' & -\frac{\sqrt{2}}{4}A' & -\frac{\sqrt{2}}{4}A' & \frac{\sqrt{2}}{4}A' \\ 0 & 0 & -\frac{\sqrt{2}}{4}A' & \frac{\sqrt{2}}{4}A' & \frac{\sqrt{2}}{4}A' & -\frac{\sqrt{2}}{4}A' \\ 0 & 0 & -\frac{\sqrt{2}}{4}A' & \frac{\sqrt{2}}{4}A' & \frac{\sqrt{2}}{4}A' & -\frac{\sqrt{2}}{4}A' \\ 0 & 0 & \frac{\sqrt{2}}{4}A' & -\frac{\sqrt{2}}{4}A' & -\frac{\sqrt{2}}{4}A' & \frac{\sqrt{2}}{4}A' \end{bmatrix} \Rightarrow K = E \begin{bmatrix} A & 0 & -A & 0 & 0 & 0 \\ 0 & A & 0 & 0 & 0 & -A \\ -A & 0 & \frac{\sqrt{2}}{4}A'+A & -\frac{\sqrt{2}}{4}A' & -\frac{\sqrt{2}}{4}A' & \frac{\sqrt{2}}{4}A' \\ 0 & 0 & -\frac{\sqrt{2}}{4}A' & \frac{\sqrt{2}}{4}A' & \frac{\sqrt{2}}{4}A' & -\frac{\sqrt{2}}{4}A' \\ 0 & 0 & -\frac{\sqrt{2}}{4}A' & \frac{\sqrt{2}}{4}A' & \frac{\sqrt{2}}{4}A' & -\frac{\sqrt{2}}{4}A' \\ 0 & -A & \frac{\sqrt{2}}{4}A' & -\frac{\sqrt{2}}{4}A' & -\frac{\sqrt{2}}{4}A' & \frac{\sqrt{2}}{4}A'+A \end{bmatrix}$$