Assignment 3

Computational Structural Mech. & Dyn.

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3.1

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)} \qquad \mu = G = \frac{E}{2(1+\nu)}$$

1. Find inverse for E, ν in terms of λ, μ

$$\lambda = \frac{2\mu\nu}{1-2\nu} \rightarrow (1-2\nu)\lambda = 2\mu\nu \rightarrow \lambda - 2\nu\lambda = 2\mu\nu \rightarrow \nu(2\mu + 2\lambda) = \lambda$$

$$\boxed{\nu = \frac{\lambda}{2(\mu+\lambda)}} \qquad E = 2(1+\nu)\mu = 2\left(1 + \frac{\lambda}{2(\mu+\lambda)}\right)\mu = 2\mu \frac{2(\mu+\lambda)+\lambda}{2(\mu+\lambda)} =$$

$$= \frac{2\mu + 3\lambda}{\cancel{2}(\mu+\lambda)} \cancel{2}\mu = \boxed{\mu \frac{3\lambda + 2\mu}{\lambda+\mu}}$$

2. Express the elastic matrix for plane stress and strain states in terms of μ, λ

plane stress
$$\begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{pmatrix} = \frac{E}{1-\nu^2} \underbrace{\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}}_{\mathbf{E}} \begin{pmatrix} e_{xx} \\ e_{yy} \\ e_{xy} \end{pmatrix} = \frac{12\mu\lambda + 8\mu^2}{4\mu^2 + 8\mu\lambda + 3\lambda^2} \begin{bmatrix} 1 & \frac{\lambda}{2(\mu+\lambda)} & 0 \\ \frac{\lambda}{2(\mu+\lambda)} & 1 & 0 \\ 0 & 0 & \frac{1-\lambda}{4(\mu+\lambda)} \end{bmatrix}$$

Plane strain
$$\begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{pmatrix} = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \underbrace{\begin{bmatrix} 1 & \frac{\nu}{1-\nu} & 0 \\ \frac{\nu}{1-\nu} & 1 & 0 \\ 0 & 0 & \frac{1-2\nu}{2(1-\nu)} \end{bmatrix}}_{\mathbf{E}} \begin{pmatrix} e_{xx} \\ e_{yy} \\ e_{xy} \end{pmatrix} = \frac{\frac{3\lambda+2\mu}{\lambda+\mu} - \frac{(3\lambda+2\mu)\lambda}{2(\mu+\lambda)^2}}{\frac{\lambda + 2(\mu+\lambda)}{2(\mu+\lambda)} \cdot \frac{\mu + \cancel{\lambda} - \cancel{\lambda}}{\mu+\lambda}} = \frac{3\lambda^2 + 8\mu\lambda + 4\mu^2}{3\lambda + 2\mu}$$

$$\begin{bmatrix} 1 & \frac{\lambda}{2\mu+\lambda} & 0 \\ \frac{\lambda}{2\mu+\lambda} & \frac{\lambda}{2\mu+\lambda} & 0 \\ 0 & \frac{\mu}{0} & \sim \end{bmatrix}$$

3.2

$a = 1$, $h = 1$

E, ν

ν initally is 0

a)

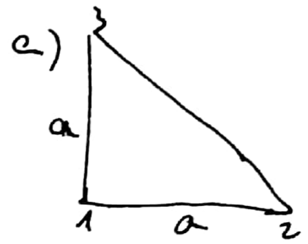
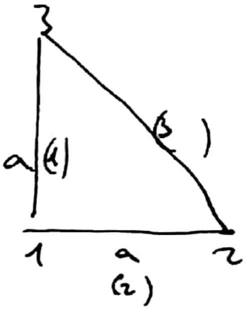


Plate liner triangle

b)



3 bar elements

cross sections $\begin{cases} A_1 = A_2 = A \\ A_3 \end{cases}$

1)

$$K_{tri} = \frac{\overset{1}{\overset{\shortparallel}{h}}}{4A} \begin{bmatrix} y_{23} & 0 & x_{32} \\ 0 & x_{32} & y_{23} \\ y_{31} & 0 & x_{13} \\ 0 & x_{13} & y_{31} \\ y_{12} & 0 & x_{21} \\ 0 & x_{21} & y_{12} \end{bmatrix} \begin{bmatrix} E & E & E \\ E & E & E \\ E & E & E \end{bmatrix} \begin{bmatrix} y_{23} & 0 & y_{31} & 0 & y_{12} & 0 \\ 0 & x_{32} & 0 & x_{13} & 0 & x_{21} \\ x_{32} & y_{23} & x_{13} & y_{31} & x_{21} & y_{12} \end{bmatrix}$$

$A \uparrow \frac{a^2}{2} = \frac{1}{2}$

$\rightarrow E$

1/2

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Assignment 3.2

$$K_{tri} = \frac{E}{2}\begin{bmatrix} y_{23} & 0 & x_{32} \\ 0 & x_{32} & y_{23} \\ y_{31} & 0 & x_{13} \\ 0 & x_{13} & y_{31} \\ y_{12} & 0 & x_{21} \\ 0 & x_{21} & y_{12} \end{bmatrix}\begin{bmatrix} y_{23} & 0 & y_{31} & 0 & y_{12} & 0 \\ 0 & x_{32} & 0 & x_{13} & 0 & x_{21} \\ x_{32} & y_{23} & x_{13} & y_{31} & x_{21} & y_{12} \end{bmatrix} =$$

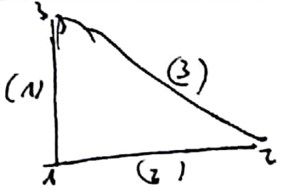
$$\boxed{\begin{matrix} y_{jk} = y_j - y_k \\ x_{jk} = x_j - x_k \end{matrix}}$$

$$= \frac{E}{2}\begin{bmatrix} -a & 0 & -a \\ 0 & -a & -a \\ a & 0 & 0 \\ 0 & 0 & a \\ 0 & 0 & a \\ 0 & a & 0 \end{bmatrix}\begin{bmatrix} -a & 0 & a & 0 & 0 & 0 \\ 0 & -a & 0 & 0 & 0 & a \\ -a & -a & 0 & a & a & 0 \end{bmatrix} = \frac{a^2E}{2}\begin{bmatrix} 2 & 1 & -1 & -1 & -1 & 0 \\ 1 & 2 & 0 & -1 & -1 & -1 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ -1 & -1 & 0 & 1 & 1 & 0 \\ -1 & -1 & 0 & 1 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(Sym)

Given that

$$\begin{cases} x_1 = 0 \\ x_2 = a \\ x_3 = 0 \\ y_1 = 0 \\ y_2 = 0 \\ y_3 = a \end{cases}$$



$$K_{bar} = K^{(1)} + K^{(2)} + K^{(3)}.$$

$$K^{(e)} = \frac{E^{(e)}A^{(e)}}{L^{(e)}}\begin{bmatrix} c^2 & sc & -c^2 & -sc \\ sc & s^2 & -sc & -s^2 \\ -c^2 & -sc & c^2 & sc \\ -sc & -s^2 & sc & s^2 \end{bmatrix}$$

elem 1

K_1 $\varphi = 90°$ $\frac{E^{(1)}A^{(1)}}{L^{(1)}} = \frac{EA}{a}$ $\begin{cases} c = 0 \\ s = 1 \end{cases}$

elem 2

K_2 $\varphi = 0$ $\frac{E^{(2)}A^{(2)}}{L^{(2)}} = \frac{EA}{a}$ $\begin{cases} c = 1 \\ s = 0 \end{cases}$

elem 3

K_3 $\underline{\underline{\varphi = 135°}}$ $\frac{E^{(3)}A^{(3)}}{L^{(3)}} = \frac{EA_3}{\sqrt{2}a}$ $\begin{cases} c = -\sqrt{2}/2 \\ s = \frac{\sqrt{2}}{2} \end{cases}$

$$K^{(1)} = \frac{EA}{a}\begin{array}{c} \begin{matrix} \overbrace{\quad}^{1} & \overbrace{\quad}^{3} \end{matrix} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \end{array}\begin{matrix} \}1 \\ \}3 \end{matrix} \text{ sym}$$

$$K^{(2)} = \frac{EA}{a}\begin{array}{c} \begin{matrix} \overbrace{\quad}^{1} & \overbrace{\quad}^{2} \end{matrix} \\ \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}\begin{matrix} \}1 \\ \}2 \end{matrix} \text{ sym}$$

$$K^{(3)} = \frac{EA_3}{2\sqrt{2}a}\begin{array}{c} \begin{matrix} \overbrace{\quad}^{2} & \overbrace{\quad}^{3} \end{matrix} \\ \begin{bmatrix} 1 & -1 & -1 & 1 \\ -1 & 1 & 1 & -1 \\ -1 & 1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \end{array}\begin{matrix} \}2 \\ \}3 \end{matrix} \text{ sym}$$

$i = 2$
$j = 3$

$$K_{bar} = \frac{E}{a}\begin{array}{c|} \begin{matrix} \overbrace{\qquad}^{1} & \qquad 2 & \qquad 3 \end{matrix} \\ \begin{bmatrix} A & 0 & -A & 0 & 0 & 0 \\ 0 & A & 0 & 0 & 0 & -A \\ -A & 0 & \frac{A_3}{2\sqrt{2}} & -\frac{A_3}{2\sqrt{2}} & -\frac{A_3}{2\sqrt{2}} & \frac{A_3}{2\sqrt{2}} \\ 0 & 0 & -\frac{A_3}{2\sqrt{3}} & \frac{A_3}{2\sqrt{2}} & \frac{A_3}{2\sqrt{2}} & -\frac{A_3}{2\sqrt{2}} \\ 0 & 0 & -A_3/2\sqrt{2} & \frac{A_3}{2\sqrt{2}} & \frac{A_3}{2\sqrt{2}} & -\frac{A_3}{2\sqrt{2}} \\ 0 & -A & \frac{A_3}{2\sqrt{2}} & -\frac{A_3}{2\sqrt{2}} & -\frac{A_3}{2\sqrt{2}} & \frac{A_3}{2\sqrt{2}} \end{bmatrix} \end{array}\begin{matrix} 1 \\ \\ 2 \\ \\ 3 \\ \end{matrix}$$

(Sym)

2 – Any values for $K_{bar} = K_{tri}$?
of $A_1 = A_2$
A_3

There are NOT values which makes them equivalent.

to ~~force~~ them equal: $\begin{cases} A_3 = 0 & \text{1a vals} \\ A_3 = \sqrt{2}a^3 & \text{1 vals} \end{cases}$ $\Big\}$ $\left(\frac{3}{8}\sqrt{2}a^3\right) = A_3$ (aprox)

$A = a^3$ 2 vals
$A = \frac{a^3}{2}$ 4 vals
$\Big\}$ $\boxed{\begin{matrix} A \\ \parallel \\ \frac{2}{3}a^3 \end{matrix}}$

2/2