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Assignment 3.1

Plane Stress $\frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$

Plane Strain $\frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix}$

$$\frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix} = \frac{E^*}{1-\nu^{*2}}\begin{bmatrix} 1 & \nu^* & 0 \\ \nu^* & 1 & 0 \\ 0 & 0 & \frac{1-\nu^*}{2} \end{bmatrix}$$

It must be true component by component. Let's focus in two components (K_{33} and K_{11})

Let's start with component 33 and using $(1-\nu^{*2}) = (1+\nu^*)(1-\nu^*)$

$$\frac{E}{(1+\nu)2} = \frac{E^*(1-\nu^*)}{(1-\nu^*)(1+\nu^*)2} \longrightarrow \frac{E^*}{(1+\nu^*)} = \frac{E}{(1+\nu)} \rightarrow \boxed{E^* = \frac{(1+\nu^*)}{(1+\nu)}E}$$

component 11:

$$\frac{E(1-\nu)}{(1+\nu)(1-2\nu)} = \frac{E^*}{1-\nu^{*2}} = \frac{E(1+\nu^*)}{(1+\nu)(1-\nu^*)(1+\nu^*)} \Longrightarrow$$

$$\frac{1-\nu}{(1+\nu)(1-2\nu)} = \frac{1}{(1+\nu)(1-\nu^*)} \Longrightarrow 1-\nu^* = \frac{1-2\nu}{1-\nu} \Longrightarrow$$

$$\nu^* = 1 - \frac{1-2\nu}{1-\nu} = \frac{1-\nu-1+2\nu}{1-\nu} = \boxed{\frac{\nu}{1-\nu} = \nu^*}$$

then:

$$E^* = \frac{1+\frac{\nu}{1-\nu}}{1+\nu} E = \frac{1-\nu+\nu}{(1-\nu)(1+\nu)} E = \frac{E}{(1-\nu)(1+\nu)} = \boxed{\frac{E}{1-\nu^2} = E^*}$$

b) If we repeat the same procedure:

$$\frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} = \frac{E^*}{(1+\nu^*)(1-2\nu^*)}\begin{bmatrix} 1-\nu^* & \nu^* & 0 \\ \nu^* & 1-\nu^* & 0 \\ 0 & 0 & \frac{1-2\nu^*}{2} \end{bmatrix}$$

3-3 component:

$$\frac{E\,(1-\nu)}{2(1+\nu)(1-\nu)} = \frac{E^*\,(1-2\nu^*)}{2(1+\nu^*)(1-2\nu^*)} \longrightarrow \frac{E}{(1+\nu)} = \frac{E^*}{(1+\nu^*)} \longrightarrow$$

$$\longrightarrow \boxed{E^* = \frac{E\,(1+\nu^*)}{(1+\nu)}}$$

1-1 component.

$$\frac{E}{(1-\nu)(1+\nu)} = \frac{E^*\,(1-\nu^*)}{(1+\nu^*)(1-2\nu^*)} = \frac{E\,(1-\nu^*)}{(1+\nu)(1-2\nu^*)} \longrightarrow \frac{1}{(1-\nu)} = \frac{1-\nu^*}{1-2\nu^*}$$

$$(1-\nu) = \frac{(1-2\nu^*)}{1-\nu^*} \rightarrow \nu = \frac{1-\nu^* - 1 + 2\nu^*}{1-\nu^*} = \frac{\nu^*}{1-\nu^*} \rightarrow$$

$$\nu(1-\nu^*) - \nu^* = 0 \rightarrow \nu - \nu\nu^* - \nu^* = 0 \rightarrow$$

$$\rightarrow \nu - (1+\nu)\nu^* = 0 \rightarrow \boxed{\nu^* = \frac{\nu}{1+\nu}}$$

then

$$E^* = E \, \frac{1 + \frac{\nu}{1+\nu}}{1+\nu} = E \, \frac{1+\nu+\nu}{(1+\nu)^2} = \boxed{E \, \frac{(1+2\nu)}{(1+\nu)^2} = E^*}$$

Assignment 3.2

a)

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)} \quad (1) \qquad \mu = \frac{E}{2(1+\nu)} \quad (2)$$

From equation (1):

$$E = \frac{\lambda(1+\nu)(1-2\nu)}{\nu}$$ plugging it into equation (2)

$$\mu = \frac{\lambda(1-2\nu)}{2\nu}$$ isolating: ν:

$$2\nu\mu - \lambda + 2\nu\lambda = 0 \rightarrow 2\nu(\mu+\lambda) = \lambda \rightarrow$$

$$\boxed{\nu = \frac{\lambda}{2(\mu+\lambda)}}$$

Now from equation (2) we know:

$$E = 2\mu(1+\nu) = 2\mu\left(1+\frac{\lambda}{2(\mu+\lambda)}\right) = 2\mu\left(\frac{2\mu+3\lambda}{2(\mu+\lambda)}\right) = \boxed{\mu\,\frac{2\mu+3\lambda}{(\mu+\lambda)} = E}$$

b)

Plane Stress

$$\frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$$

from the previous part we know:

$$\nu = \frac{\lambda}{2(\mu+\lambda)} \qquad E = 2\mu(1+\nu)$$

Term multiplying the matrix can be written:

$$\frac{E}{1-\nu^2} = \frac{E}{(1-\nu)(1+\nu)} = \frac{2\mu(1+\nu)}{(1-\nu)(1+\nu)} = \boxed{\frac{2\mu}{1-\nu}}$$

Component 3-3

$$\frac{2\mu}{(1-\nu)}\,\frac{1-\nu}{2} = \boxed{\mu}$$

Component 1-1

$$\frac{2\mu}{1-\nu} = \frac{2\mu}{1-\frac{\lambda}{2(\mu+\lambda)}} = \frac{2\mu}{\frac{2\mu+2\lambda-\lambda}{2(\mu+\lambda)}} = \boxed{\frac{4\mu(\mu+\lambda)}{2\mu+\lambda}}$$

Component 1-2

$$\frac{2\mu\nu}{1-\nu} = \frac{2\mu\lambda}{2(\mu+\lambda)\left(1-\frac{\lambda}{2(\mu+\lambda)}\right)} = \frac{2\mu\lambda}{2(\mu+\lambda)\left(\frac{2\mu+2\lambda-\lambda}{2(\mu+\lambda)}\right)} = \boxed{\frac{2\mu\lambda}{2\mu+\lambda}}$$

So at the end:

Plane Stress

$$\begin{pmatrix} \frac{4\mu(\mu+\lambda)}{2\mu+\lambda} & \frac{2\mu\lambda}{2\mu+\lambda} & 0 \\ \frac{2\mu\lambda}{2\mu+\lambda} & \frac{4\mu(\mu+\lambda)}{2\mu+\lambda} & 0 \\ 0 & 0 & \mu \end{pmatrix}$$

Plane Strain:

$$\frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix}$$

$$E = 2\mu(1+\nu)$$

~~element~~ component 3-3

$$\frac{E}{(1+\nu)(1-2\nu)}\,\frac{1-2\nu}{2} = \frac{2\mu(1+\nu)}{(1+\nu)(1-2\nu)}\,\frac{1-2\nu}{2} = \boxed{\mu}$$

component 1-1

$$\frac{2\mu(1-\nu)}{(1-2\nu)} = \frac{2\mu\left(1-\frac{\lambda}{2(\mu+\lambda)}\right)}{\left(1-\frac{2\lambda}{2(\mu+\lambda)}\right)} = \frac{2\mu(2\mu-\lambda)}{2\mu} = \boxed{2\mu-\lambda}$$

component 1-2

$$\frac{2\mu\nu}{(1-2\nu)} = \frac{2\mu\frac{\lambda}{2(\mu+\lambda)}}{(1-2\nu)} = \frac{2\mu\lambda}{2(\mu+\lambda)\left(1-\frac{2\lambda}{2(\mu+\lambda)}\right)} = \frac{2\mu\lambda}{2(\mu+\lambda)\frac{2\mu}{2(\mu+\lambda)}} = \boxed{\lambda}$$

then. plane Strain:

$$\begin{pmatrix} 2\mu - \lambda & \lambda & 0 \\ \lambda & 2\mu - \lambda & 0 \\ 0 & 0 & \mu \end{pmatrix}$$

b)

$$E = E_\lambda + E_\mu$$

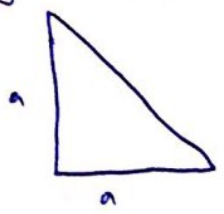
$$\boxed{E_\lambda = \lambda \begin{pmatrix} -1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}} \quad \text{and} \quad \boxed{E_\mu = \mu \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}}$$

$$\boxed{E = \lambda \begin{pmatrix} -1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \mu \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}}$$

d)

$$\boxed{\begin{aligned} E_\lambda(E, \nu) &= \frac{E\nu}{(1+\nu)(1-2\nu)} \begin{pmatrix} -1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ E_\mu(E, \nu) &= \frac{E}{2(1+\nu)} \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{aligned}}$$

Assignment 3.3



$$\boxed{A = \frac{a^2}{2}}$$

a)

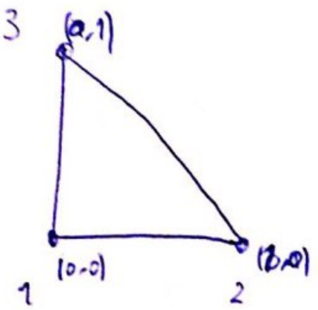
$$K^{(0)} = \frac{hA}{4A^2}\begin{bmatrix} y_{23} & 0 & x_{32} \\ 0 & x_{32} & y_{23} \\ y_{31} & 0 & x_{13} \\ 0 & x_{13} & y_{31} \\ y_{12} & 0 & x_{21} \\ 0 & x_{21} & y_{12} \end{bmatrix} \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}\begin{bmatrix} y_{23} & 0 & y_{31} & 0 & y_{12} & 0 \\ 0 & x_{32} & 0 & x_{13} & 0 & x_{21} \\ x_{32} & y_{23} & x_{13} & y_{31} & x_{21} & y_{12} \end{bmatrix} =$$

$$= \frac{hA}{4A^2}\begin{bmatrix} y_{23} & 0 & x_{32} \\ 0 & x_{32} & y_{23} \\ y_{31} & 0 & x_{13} \\ 0 & x_{13} & y_{31} \\ y_{12} & 0 & x_{21} \\ 0 & x_{21} & y_{12} \end{bmatrix} \frac{E}{1-\nu^2}\begin{bmatrix} y_{23} & \nu x_{32} & y_{31} & \nu x_{13} & y_{12} & \nu x_{21} \\ \nu y_{23} & x_{32} & \nu y_{31} & x_{13} & \nu y_{12} & x_{21} \\ \frac{1-\nu}{2} x_{32} & \frac{1-\nu}{2} y_{23} & \frac{1-\nu}{2} x_{13} & \frac{1-\nu}{2} y_{31} & \frac{1-\nu}{2} x_{21} & \frac{1-\nu}{2} y_{12} \end{bmatrix}$$

$$= \frac{hE}{4A(1-\nu^2)}\begin{bmatrix} y_{23}^2 + \frac{1-\nu}{2} x_{32} x_{32} & \nu y_{23} x_{32} + \frac{1-\nu}{2} y_{23} x_{32} & y_{23} y_{31} + \frac{1-\nu}{2} x_{32} x_{13} & \cdots \\ \nu x_{32} y_{23} + \frac{1-\nu}{2} x_{32} y_{23} & x_{32}^2 + \frac{1-\nu}{2} y_{23}^2 & \nu y_{31} x_{32} + y_{23} \frac{1-\nu}{2} x_{13} & \cdots \\ y_{31} y_{23} + \frac{1-\nu}{2} x_{32} x_{13} & \nu x_{32} y_{31} + \frac{1-\nu}{2} y_{23} x_{13} & y_{31}^2 + \frac{1-\nu}{2} x_{13}^2 & \cdots \\ \nu x_{13} y_{23} + \frac{1-\nu}{2} x_{32} y_{31} & x_{32} x_{13} + \frac{1-\nu}{2} y_{23} y_{31} & \nu y_{31} x_{13} + \frac{1-\nu}{2} x_{13} y_{31} & \cdots \\ y_{12} y_{23} + \frac{1-\nu}{2} x_{21} x_{32} & \nu x_{32} y_{12} + \frac{1-\nu}{2} y_{23} x_{21} & y_{31} y_{12} + \frac{1-\nu}{2} x_{13} x_{21} & \cdots \\ \nu x_{21} y_{23} + \frac{1-\nu}{2} x_{32} y_{12} & x_{21} x_{32} + \frac{1-\nu}{2} y_{23} y_{12} & \nu y_{31} x_{21} + \frac{1-\nu}{2} y_{12} x_{13} & \end{bmatrix}$$

$$
\cdots \begin{bmatrix}
\nu x_{13} y_{23} + \frac{1-\nu}{2} y_{31} x_{32} & y_{12} y_{23} + x_{32} \frac{1-\nu}{2} x_{21} & \nu x_{21} y_{23} + \frac{1-\nu}{2} y_{12} x_{32} \\
\frac{1-\nu}{2} y_{31} y_{23} + x_{32} x_{13} & \nu y_{12} x_{32} + \frac{1-\nu}{2} y_{23} x_{21} & x_{21} x_{32} + \frac{1-\nu}{2} y_{23} y_{12} \\
\frac{1-\nu}{2} x_{13} y_{31} + \nu x_{13} y_{31} & y_{12} y_{31} + \frac{1-\nu}{2} x_{13} x_{21} & \nu x_{21} y_{31} + \frac{1-\nu}{2} x_{13} y_{12} \\
\frac{1-\nu}{2} y_{31}^2 + x_{13}^2 & \frac{1-\nu}{2} x_{21} y_{31} + \nu x_{13} y_{12} & \frac{1-\nu}{2} y_{12} y_{31} + x_{13} x_{21} \\
\frac{1-\nu}{2} x_{21} y_{31} + \nu x_{13} y_{12} & \frac{1-\nu}{2} x_{21}^2 + y_{12}^2 & \frac{1-\nu}{2} y_{12} x_{21} + \nu y_{12} x_{21} \\
x_{13} x_{21} + \frac{1-\nu}{2} y_{31} y_{12} & \frac{1-\nu}{2} y_{12} x_{21} + \nu y_{12} x_{21} & x_{21}^2 + \frac{1-\nu}{2} y_{12}^2
\end{bmatrix}
$$

K_1



Node 1: $(0,0)$
2: $(1,0)$
3: $(0,1)$

$$
\begin{bmatrix}
x_{12} = 0-1 = -1 \\
x_{13} = 0 \\
x_{21} = 1-0 = 1 \\
x_{31} = 0 \\
x_{32} = 0-1 = -1 \\
x_{23} = 1-0 = 1
\end{bmatrix}
\qquad
\begin{bmatrix}
y_{12} = 0-0 = 0 \\
y_{21} = 0 \\
y_{13} = 0-1 = -1 \\
y_{31} = 1 \\
y_{32} = 1-0 = 1 \\
y_{23} = -1
\end{bmatrix}
$$

then:

$$k^{(e)} = \frac{hE}{4A(1-\nu^2)} \begin{bmatrix} \frac{1-\nu}{2}+1 & \frac{1-\nu}{2}+\nu & -1 & -\frac{1-\nu}{2} & -\frac{1-\nu}{2} & -\nu \\ \frac{1-\nu}{2}+\nu & \frac{1-\nu}{2}+1 & -\nu & -\frac{1-\nu}{2} & -\frac{1-\nu}{2} & -1 \\ -1 & -\nu & 1 & 0 & 0 & \nu \\ \frac{-1+\nu}{2} & -\frac{1-\nu}{2} & 0 & \frac{1-\nu}{2} & \frac{1-\nu}{2} & 0 \\ -\frac{1-\nu}{2} & -\frac{1-\nu}{2} & 0 & \frac{1-\nu}{2} & \frac{1-\nu}{2} & 0 \\ -\nu & -1 & \nu & 0 & 0 & 1 \end{bmatrix}$$

when ~~A~~ $\nu = 0$

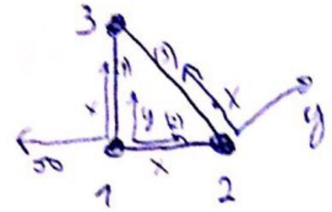
$$\boxed{k^{(e)} = \frac{hE}{4A} \begin{bmatrix} \frac{3}{2} & \frac{1}{2} & -1 & -\frac{1}{2} & -\frac{1}{2} & 0 \\ 1/2 & 3/2 & 0 & -1/2 & -1/2 & -1 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ -1/2 & -1/2 & 0 & 1/2 & 1/2 & 0 \\ -1/2 & -1/2 & 0 & 1/2 & 1/2 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \end{bmatrix}}$$

with $h = 1$

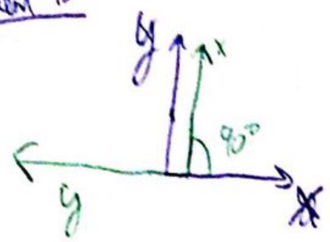
$A = \frac{1}{2}$

$$\boxed{k^{(e)} = \frac{E}{2} \begin{pmatrix} 3/2 & 1/2 & -1 & -1/2 & -1/2 & 0 \\ 1/2 & 3/2 & 0 & -1/2 & -1/2 & -1 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ -1/2 & -1/2 & 0 & 1/2 & 1/2 & 0 \\ -1/2 & -1/2 & 0 & 1/2 & 1/2 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \end{pmatrix}}$$

If we consider the second model



element 1:



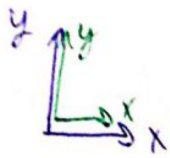
$c = \cos 90° = 0$

$S = \sin 90° = 1$

$$K^{(1)} = \frac{EA_1}{1}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

element 2:

$\cancel{K^{(2)} = \frac{EA_2}{1}}$

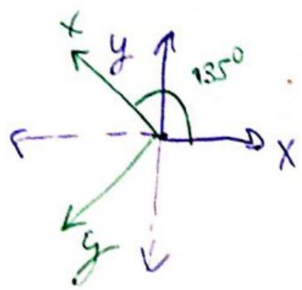


$c = \cos 0° = 1$

$S = \sin 0° = 0$

$$K^{(2)} = \frac{EA_2}{1}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

element 3: $h = L^{(3)} = \sqrt{1^2+1^2} = \sqrt{2}$



$c = \cos(135°) = -\frac{1}{\sqrt{2}}$

$S = \sin(135°) = \frac{1}{\sqrt{2}}$

$$K^{(3)} = \frac{EA_3}{\sqrt{2}}\begin{pmatrix} \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

Now we globalize the stiffness matrices

$$K^{(1)}_{global} = EA_1 \cancel{\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}} = EA_1 \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$K^{(2)}_{global} = EA_2 \begin{pmatrix} 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$K^{(3)}_{global} = \frac{EA_3}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & +\frac{1}{2} \\ 0 & 0 & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

<u>then the total stiffness matrix</u> $\quad K^{(0)} = K_g^{(1)} + K_g^{(2)} + K_g^{(3)}$

$$\boxed{K_g = E \begin{pmatrix} A_2 & 0 & -A_2 & 0 & 0 & 0 \\ 0 & A_1 & 0 & 0 & 0 & -A_1 \\ -A_2 & 0 & A_2 + \frac{A_3}{2\sqrt{2}} & \frac{-A_3}{2\sqrt{2}} & \frac{-A_3}{2\sqrt{2}} & +\frac{A_3}{2\sqrt{2}} \\ 0 & 0 & \frac{-A_3}{2\sqrt{2}} & \frac{A_3}{2\sqrt{2}} & \frac{A_3}{2\sqrt{2}} & -\frac{A_3}{2\sqrt{2}} \\ 0 & 0 & -\frac{A_3}{2\sqrt{2}} & \frac{A_3}{2\sqrt{2}} & \frac{A_3}{2\sqrt{2}} & \frac{-A_3}{2\sqrt{2}} \\ 0 & A_1 & \frac{A_3}{2\sqrt{2}} & -\frac{A_3}{2\sqrt{2}} & -\frac{A_3}{2\sqrt{2}} & A_1 + \frac{A_3}{2\sqrt{2}} \end{pmatrix}}$$

b) There is no values for which $k_{bar} = k_{triangle}$

There are no values for which both matrices look the same.

c) The reason is that physically the triangle is considered as a continuum body (solid body) and the bar model is considered as an empty space except in the edges. In the bar model, elements can only produce forces in the longitudinal direction and not in the transversal direction. That's why in the stiffness matrix appear terms which are zero in one model and not in the other one.

d) Given that we considered $\nu \neq 0$ at the beginning we have:

$$K = \frac{E}{2}\begin{pmatrix} \frac{3-\nu}{2} & \frac{1+\nu}{2} & -1 & \frac{\nu-1}{2} & \frac{\nu-1}{2} & -\nu \\ \frac{1+\nu}{2} & \frac{3-\nu}{2} & -\nu & \frac{\nu-1}{2} & \frac{\nu-1}{2} & -1 \\ -1 & -\nu & 1 & 0 & 0 & \nu \\ \frac{\nu-1}{2} & \frac{\nu-1}{2} & 0 & \frac{1-\nu}{2} & \frac{1-\nu}{2} & 0 \\ \frac{\nu-1}{2} & \frac{\nu-1}{2} & 0 & \frac{1-\nu}{2} & \frac{1-\nu}{2} & 0 \\ -\nu & -1 & \nu & 0 & 0 & 1 \end{pmatrix}$$

compared with the matrix "K" when $\nu = 0$ it can be seen that poisson modulus introduces a non-zero value in the stiffness matrix in positions where there was a 0 when ~~Poisson~~ Poisson effects ~~where~~ were neglected. It can also be seen that ν "balances" most of the terms. For example, all positive terms when $\nu = 0$ now are the same value minus ν

1-2: $\frac{5}{2} \longrightarrow \frac{5-\nu}{2}$

4-6: $\frac{1}{2} \longrightarrow \frac{1-\nu}{2}$

or negative term now are 'corrected' so the effect is not as big as before.

$-\frac{1}{2} \longrightarrow \frac{\nu-1}{2}$

To summarize, given that ν introduces deformation in the "transverse direction" the effect is that most of the terms include this correction.