Assignment 4.1

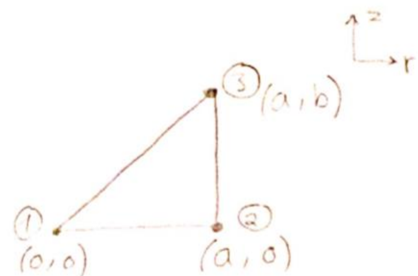
1)

assignment 4.1

1)

Where:

$$E = E\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1/2 \end{bmatrix}$$



$$[k]^e = 2\pi \int B^T E B \, r \, dA$$

B and E are constant over the triangle

r is to be taken as a const. computed at the centroid of the triangle

where $\bar{r} = \dfrac{r_1 + r_2 + r_3}{3}$

The solution was obtained using the following Matlab code

MATLAB code:

```
%Material properties
syms E;
nu = 0;
syms a b;

%Coordinates of the triangle
r1 = 0; z1 = 0;
r2 = a; z2 = 0;
r3 = a; z3 = b;

%calculate the moduli matrix
e = ((E*(1-nu))/((1+nu)*(1-2*nu))) * [1 nu/(1-nu) nu/(1-nu) 0 ;...
    nu/(1-nu) 1 nu/(1-nu) 0 ; nu/(1-nu) nu/(1-nu) 1 0 ;...
    0 0 0 (1-2*nu)/(2*(1-nu)) ];

%Calculate the area of the triangle
A = 0.5 * ( r1*(z2-z3) + r2*(z3-z1) + r3*(z1-z2) );

%Calculate beta ,omega (required for the B matrix
%and the shape functions)

bei = z2 - z3;
bej = z3 - z1;
bek = z1 - z2;

omi = r3 - r2;
omj = r1 - r3;
omk = r2 - r1;

%calculate the radius at the centroid
ro = (r1 + r2 + r3)/3;

%Calculate the B matrix
B = (1/(2*A)) * [ bei 0 bej 0 bek 0 ; 0 omi 0 omj 0 omk ;...
    omi bei omj bej omk bek ; (2*A)/(3*ro) 0 (2*A)/(3*ro) 0 (2*A)/(3*ro) 0];

%calculate the k matrix
k = B.' * e * B * (2*pi*ro*A) ;
```

The resultant K matrix is as follows

Columns 1-3

```
k =

[    (3*pi*E*b)/4,             0,                       -(7*pi*E*b)/12,
[               0,   (2*pi*E*b)/3,                       (2*pi*E*a)/3,
[ -(7*pi*E*b)/12,   (2*pi*E*a)/3, (2*a^2*b*pi*((9*E)/(8*a^2) + E/b^2))/3,
[               0, -(2*pi*E*b)/3,                       -(2*pi*E*a)/3,
[     (pi*E*b)/12, -(2*pi*E*a)/3,     (2*a^2*b*pi*(E/(8*a^2) - E/b^2))/3,
[               0,             0,                                    0,
```

Columns 4-6

```
                                0,                       (pi*E*b)/12,                   0]
                    -(2*pi*E*b)/3,                     -(2*pi*E*a)/3,                   0]
                    -(2*pi*E*a)/3, (2*a^2*b*pi*(E/(8*a^2) - E/b^2))/3,                   0]
(2*a^2*b*pi*(E/a^2 + E/b^2))/3,                       (2*pi*E*a)/3, -(2*pi*E*a^2)/(3*b)]
                     (2*pi*E*a)/3, (2*a^2*b*pi*(E/(8*a^2) + E/b^2))/3,                   0]
          -(2*pi*E*a^2)/(3*b),                                  0,  (2*pi*E*a^2)/(3*b)]
```

2)

The summation of the column or rows is done by the following code section:

```
%Substract the the elements of the specified rows or columns
[nrow,ncol] = size(k);
for i = [2 4 6]
    sum = sym(zeros(ncol));
    for j = 1 :  ncol
        sum(i) = sum(i) + k(i,j);
    end
    disp(simplify(sum(i)))
end
```

```
>> tri_axysim_41
0

0

0
```

The results are as expected: the summation of rows (columns) 2,4 and 6 is null. These terms vanish due to the nature of the problem in hand. For nu equal to zero, any radial load does not cause any deformation in the z direction. This is translated to the constitutive matrix where the terms related to the stiffness or motion in the z direction when subjected to a radial load. Moreover, the summation of rows (columns) 1,3 and 5 is not null because these are the terms related to the radial motion.

3)

3)

$$f_b^e = \int N^T b \bar{r} \, dA$$

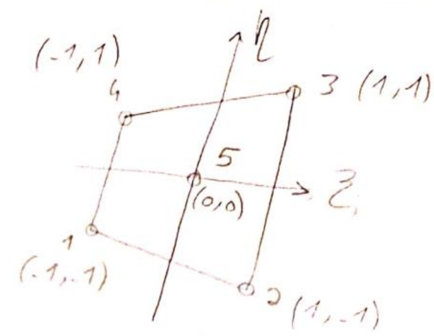
~~Let~~ $\bar{r} = \frac{r_1 + r_2 + r_3}{3} = \frac{a}{3}$ $\quad \| \quad A = \frac{1}{2} ab$

For a linear triangular element $\int_\Omega \xi_1 d\Omega = \int \xi_2 d\Omega = \int \xi_3 d\Omega = \frac{1}{3} A$

$$f_b^e = \left(\frac{a^2 b}{18}\right) \begin{bmatrix} 0 \\ -g \\ 0 \\ -g \\ 0 \\ -g \end{bmatrix}$$

Assignment 4.2:

assignment 4.2



* $N_5^e = c_5 L_{1\text{-}2} L_{2\text{-}3} L_{3\text{-}4} L_{4\text{-}1}$

$= c_5(\eta+1)(\xi-1)(\eta-1)(\xi+1) = c_5(1-\eta^2)(1-\xi^2)$

For node $5 \Rightarrow \xi = \eta = 0$

$N_5^e(0,0) = c_5 = 1$

hence $N_5^e = (1-\eta^2)(1-\xi^2)$

The continuity check is valid for all sides where: on $L_{1\text{-}2} \Rightarrow \eta = -1$

$L_{2\text{-}3} \Rightarrow \xi = 1$

$L_{4\text{-}3} \Rightarrow \eta = 1$

$L_{4\text{-}1} \Rightarrow \xi = -1$

hence N_i $i = 1 \to 4$ are linear functions and the polynomial variation is of order 1

* $\bar{N}_i^e = 1/4\,(1+\xi_i\xi)(1+\eta_i\eta)$; $i = 1,2,3,4$

$N_1^e = 1/4\,(1-\xi)(1-\eta)$

$N_2^e = 1/4\,(1+\xi)(1-\eta)$

$N_3^e = 1/4\,(1+\xi)(1+\eta)$

$N_4^e = 1/4\,(1-\xi)(1+\eta)$

* $N_i = \bar{N}_i + \alpha N_5$ $\quad 1 \to 4$

$= 1/4\,(1+\xi_i\xi)(1+\eta_i\eta) + \alpha(1-\eta^2)(1-\xi^2)$

at node $5 \Rightarrow \xi = \eta = 0$

$N_i = 1 + \alpha = 0 \quad \Rightarrow \alpha = -1/4$

* $\sum_{i=1}^{5} N_i^e$

$= 1/4(1-\xi-\eta+\xi\eta) + 1/4(1+\xi-\eta-\xi\eta) + 1/4(1+\xi+\eta+\xi\eta) + 1/4(1-\xi+\eta-\xi\eta)$

$- 4\left[1/4(1-\eta^2)(1-\xi^2)\right] + (1-\eta^2)(1-\xi^2)$

$= 1$