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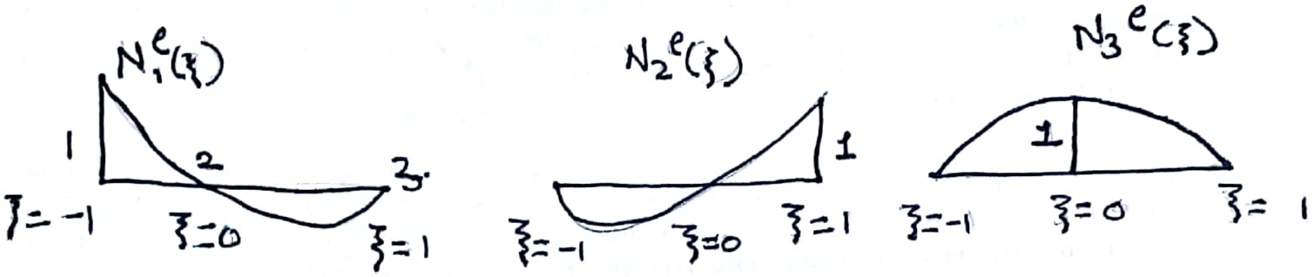
Assignment 5

5.1

I)



$$N_1^{(e)}(\xi) = a_0 + a_1\xi + a_2\xi^2$$

Shape function for 3 ($\xi=0$) $N_2^e=0$

$$N_2^e(\xi) = a_0 + a_1(0) + a_2(0)^2$$

$$\boxed{a_0 = 0}$$

<u>Node 1</u> $\xi=-1$ $N_2=0$

$$N_1^2 = a_1(1) + a_2(1)^2$$

$$\boxed{a_1 + a_2 = 0}$$

$$a_1 = -a_2 = -\tfrac{1}{2}$$

$$a_0 = 0 \quad a_1 = -\tfrac{1}{2} \quad a_2 = \tfrac{1}{2}$$

$$\boxed{N_1^e(\xi) = -\tfrac{1}{2}\xi + \tfrac{1}{2}\xi^2}$$

II)

$$N_2^e(\xi) = b_0 + b_1\xi + b_2(\xi)^2.$$

@ Node 3 $\quad \xi = 0, \quad N_2^e = 0.$

$$\therefore N_2^e = b_0 + b_1(0) + b_2(0)^2$$

$$\boxed{b_0 = 0}$$

@ Node 1.

$\xi = -1 \quad N_2^e = 0.$

$$\cancel{\therefore N_2^e = b_0 + b_1(0) + b_2(0)^2}$$

$$N_2^e = b_1(-1) + b_2(-1)^2$$

$$b_2 - b_1 = 0$$

@ Node 2

$\xi = 1 \quad N_2^e = 1$

$$N_2^e = b_2(1) + b_2(1)^2.$$

$$b_1 + b_2 = 1$$

$\therefore b_2 = b_1 = 1/2 \qquad b_0 = 0 \qquad b_1 = 1/2$

$b_2 = 1/2$

$$\boxed{N_2^e(\xi) = \tfrac{1}{2}\xi + \tfrac{1}{2}\xi^2.}$$

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III)

$$N_{3(\xi)}^{e} = C_0 \cancel{\xi} + C_1 \xi + C_2 \xi^2 .$$

@ Node 3 $\quad \xi = 0 \quad N_3^e = 1$

$$N_3^e = C_0 + C_1(0) + C_2(0)^2$$

$$C_0 = 1$$

@ Node 1 $\quad \xi = -1 \quad N_3^{(e)} = 0$

$$N_3^e = 1 + C_1(-1) + C_2(-1)^2$$

$$0 = 1 - C_1 + C_2$$

$$C_1 + C_2 = 1$$

@ Node 2

$\xi = 1 \quad N_3^e = 0.$

$$N_3^e = 1 + C_1(1) + C_2(1)^2$$

$$C_1 + C_2 = -1$$

$$\therefore C_1 = 0 \qquad C_2 = -1$$

$$\boxed{N_{3(\xi)}^{e} = 1 - \xi^2}$$

b) Adding all nodes.

$N_1 + N_2 + N_3$

$= -\frac{1}{2}\xi + \frac{1}{2}\xi^2 + \frac{1}{2}\xi + \frac{1}{2}\xi^2 + 1 - \xi^2$

$= 1.$

c) Shape function with respect to natural co-ordinates

$N_1^e(\xi) = \frac{-1}{2}\xi + \frac{1}{2}\xi^2$

$$\boxed{\frac{dN_1^e}{d\xi} = -\frac{1}{2} + \xi}$$

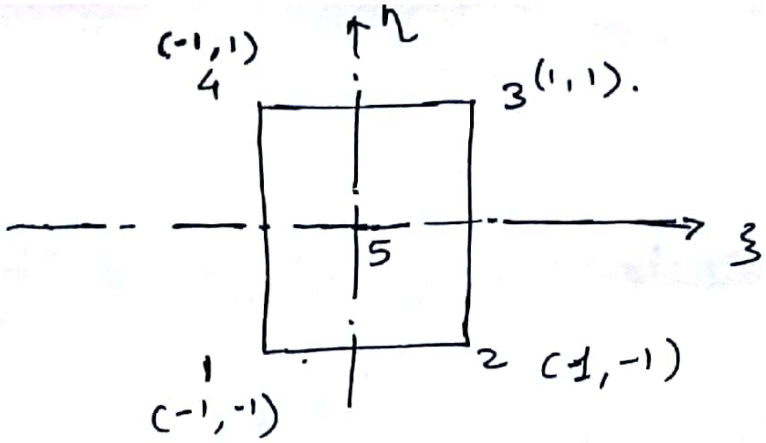
$N_2^e(\xi) = \frac{1}{2}\xi + \frac{1}{2}\xi^2$

$$\boxed{\frac{dN_2^e}{d\xi} = \frac{1}{2} + \xi}$$

$N_3^e(\xi) = 1 - \xi^2$

$$\boxed{\frac{dN_3^e}{d\xi} = -2\xi}$$

5.2



Line product Method ⇒

$$N_5(\xi,\eta) = C_5(\xi^2-1)(\eta^2-1).$$

$$N_5(0,0) = C_5$$

$$= 1$$

$$\therefore N_5(\xi,\eta) = (\xi^2-1)(\eta^2-1)$$

4 Noded quadreture using Line Product method.

$$N_1^e = \frac{1}{4}(1-\eta)(1-\xi)$$

$$N_2^e = \frac{1}{4}(1+\xi)(1-\eta).$$

$$N_3^e = \frac{1}{4}(1+\xi)(1+\eta)$$

$$N_4^{(e)} = \frac{1}{4}(1-\xi)(1+\eta).$$

Now, Considering 5 Noded quadrilateral.

$$N_i^e = N_i^e + \alpha N_5$$

Node 1

$$N_1 = \frac{1}{4}(1-\xi)(1-\eta) + \alpha(\eta^2-1)(\xi^2-1).$$

$$N_1(0,0) = 0 = \frac{1}{4} + \alpha$$

$$\therefore \boxed{\alpha = -\frac{1}{4}}.$$

Node 2

$$N_2 = \frac{1}{4}(1+\xi)(1-\eta) + \alpha(\eta^2-1)(\xi^2-1).$$

$$N_2(0,0) = \frac{1}{4} + \alpha.$$

$$\boxed{\alpha = -\frac{1}{4}}.$$

Node 3.

$$N_3 = \frac{1}{4}(1+\xi)(1+\eta) + \alpha(\eta^2-1)(\xi^2-1)$$

$$N_3(0,0) = \frac{1}{4} + \alpha$$

$$\boxed{\alpha = -\frac{1}{4}}$$

Node 4.

$$N_4 = \frac{1}{4}(1-\xi)(1+\eta) + \alpha(\eta^2-1)(\xi^2-1)$$

$$N_4(0,0) = \frac{1}{4} + \alpha.$$

$$\boxed{\alpha = -\frac{1}{4}}$$

∴ **Shape function values**

$$N_1 = \frac{1}{4}\left[(1-\xi)(1-\eta) - (\eta^2-1)(\xi^2-1)\right]$$

$$N_2 = \frac{1}{4}\left[(1+\xi)(1-\eta) - (\eta^2-1)(\xi^2-1)\right]$$

$$N_3 = \frac{1}{4}\left[(1+\xi)(1+\eta) - (\eta^2-1)(\xi^2-1)\right].$$

$$N_4 = \frac{1}{4}\left[(1-\xi)(1+\eta) - (\eta^2-1)(\xi^2-1)\right]$$

$$N_5 = (\eta^2-1)(\xi^2-1).$$

Sum of the shape function.

$N_1+N_2+N_3+N_4+N_5.$

$$= \frac{1}{4}(1-\xi-\eta+\xi\eta) - \frac{1}{4}(\eta^2-1)(\xi^2-1) + \frac{1}{4}(1+\xi-\eta+\xi\eta)$$

$$-\frac{1}{4}(\eta^2-1)(\xi^2-1) + \frac{1}{4}(\eta^2-1)(\xi^2-1)$$

$$+\frac{1}{4}\left[(1+\xi)(1-\eta) - (\eta^2-1)(\xi^2-1)\right]$$

$$+\frac{1}{4}\left[(1-\xi)(1+\eta) - (\eta^2-1)(\xi^2-1)\right].$$

$$+(\eta^2-1)(\xi^2-1)$$

$$= 1.$$

5.3

~~R~~ rank k $\quad r = \min(n_f - n_r, n_E n_g)$

rank effi. $\quad d = (n_f - n_R) - r.$

n_f - No. of element DoF

n_R - No. of independant rigid body nodes.

n_E order of E stress- strain Matrix

n_g number of Gauss points in integration rule for k

r actual rank of stiffness matrix.

$n_E n_g \geq n_f - n_R$

For hexahedron $n_E = 6 \quad n_R = 6.$

$n_F = n \times 3.$

$\therefore n_g \geq \frac{n_F - 6}{6}$

$\min n_g \left(\frac{n_p - 6}{6}\right)$

1) For 8 node.

$n = 8$

$n_f = n \times 3$

$= 8 \times 3 = 24$

$n_f - n_R = 24 - 6 = 18 = n_p$

$\text{Min } n_g = \frac{n_p - 6}{6} = \underline{3}$

$\therefore$ Recomended rule $2 \times 2 \times 2$

2) For 20 node hexahedron.

$n = 20.$

$n_f = 60$

$n_f - n_R = n_p$

$= 54$

$\min n_G = \underline{9}$

$\underline{\text{Recomended rule} = 3 \times 3 \times 3.}$

3) For 27 node.

$n = 2\cancel{0}7$

$n_f = 81$

$n_f - n_R = n_p$

$= 75.$

$\min n_G = \underline{12.5}$

$\underline{\text{Recomended rule} = 3 \times 3 \times 3.}$

4) <u>64 node.</u>

$n = 64$

$n_f = \cancel{\ } 192$

$n_f - n_R = n_p$

$= 186$

min $n_g = 31$

<u>Recomended rule. $4 \times 4 \times 4$</u>