Assignment 5.1:

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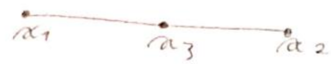
For the given geometric representation of a three nodes bar element



where:

$x_1 = 0$

$x_2 = l$

$x_3 = \frac{1}{2} l + \alpha l$

The shape functions for nodes 1, 2 and 3 are as follows

$$N_1 = -\tfrac{1}{2}\,\xi(1-\xi)$$
$$N_2 = \tfrac{1}{2}\,\xi(1+\xi)$$
$$N_3 = (1-\xi^2)$$

The representation of the domain becomes

$$\begin{aligned} x &= N_1 x_1 + N_2 x_2 + N_3 x_3 \\ &= \tfrac{1}{2}\xi(1-\xi)x_1 + \tfrac{1}{2}\xi(1+\xi)x_2 + (1-\xi^2)x_3 \\ &= \tfrac{1}{2}\xi l + \tfrac{1}{2}\xi^2 l + \tfrac{1}{2} l + \alpha l - \tfrac{1}{2}\xi^2 l - \alpha \xi^2 l \\ &= \tfrac{1}{2}\xi l + \tfrac{1}{2} l + \alpha l - \alpha \xi^2 l \end{aligned}$$

Applying the Jacobian

$$\begin{aligned} J &= \frac{\partial x}{\partial \xi} = 0 \\ &= \tfrac{1}{2} l - 2\alpha \xi l = 0 \\ \alpha &= \frac{1}{4\xi} \end{aligned}$$

Thus the minimum α that makes the Jacobian vanish is $\alpha = \pm 1/4$ occuring at $\xi = \pm 1$

* The strain is calculated as follows:

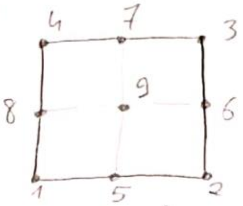
$$\varepsilon = \frac{\partial N}{\partial \xi}\, u\, J^{-1}$$

The Jacobian is zero at the end point hence the strain tends to infinity at the end points which is a singularity

Assignment 5.2:

assignment 5.2

A matlab code was used in this problem. A similar approach to the previous question was implemented where:



$$x = \sum_{i=1}^{9} N_i x_i$$

$$y = \sum_{i=1}^{9} N_i y_i$$

In the case of 2D the Jacobian matrix takes the form:

$$J = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix}$$

The Jacobian determinant is calculated at node 2. The results show that moving node 5 towards node 2 results in the vanishing of the Jacobian when the halfway point is reached ie: $x = 0.75$

MATLAB code

```
syms z i
N1= 0.25*(z-1)*(i-1)* z * i ;
N2= -0.25*z*(1+z)*i*(1-i) ;
N3= 0.25*z*(1+z)*(1+i)*i ;
N4= -0.25*(1-z)*(1+i)*i ;
N5= -0.5*(1+z)*(1-z)*(1-i)*i ;
N6= 0.5*z*(z+1)*(1+i)*(1-i) ;
N7= -0.5*(1-z^2)*(1+ i )*i ;
N8= -0.5*z*(1-z)*(1-i)*i ;
N9= (1-z^2)*(1-i^2) ;
x = N2+N3+0.75*N5+N6+0.5*N7+0.5*N9 ;
y = N3+N4+0.5*N6+N7+0.5*N8+0.5*N9 ;
J=[ diff(x,z) , diff(y,z) ; diff(x,i), diff(y,i) ] ;
subs ( det( J ) , [ z , i ] , [ 1 , -1])
```

Results

```
>> Assi5

ans =

0
```