Computational Structural Mechanics and Dynamics

Assignment 6

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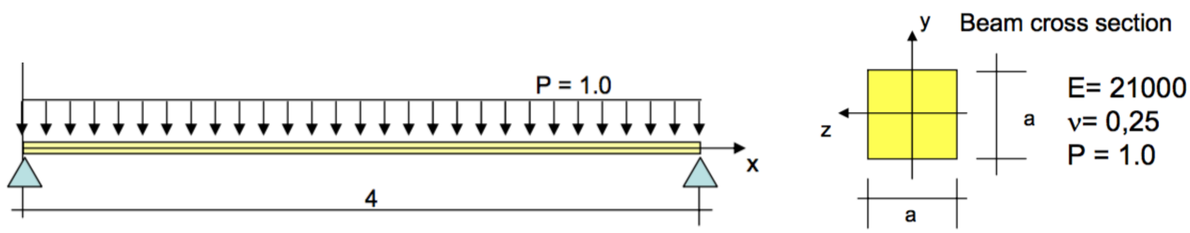
a) Program in Matlab the Timoshenko 2 Nodes Beam element with reduce integration for the shear stiffness matrix.
b) Solve the following problem with a 64 element mesh with:
   - the 2 nodes Euler Bernulli element;
   - the 2 nodes Timoshenko Full Integrate element
   - the 2 nodes Timoshenko Reduce Integration element.
   Compare maximum displacements, moments and shear for the 3 elements against a/L relationship.



a = 0,001
a = 0,005
a = 0,010
a = 0,020
a = 0,050
a = 0,100
a = 0,200
a = 0,400

The cross section of the beam for each case can be changed through changing the variable "a" in the corresponding code "SimpleSupUL_Beam_64.m" for each of the methods used.

The results are shown in tables below regarding the values asked for with respect to the slender ratio.

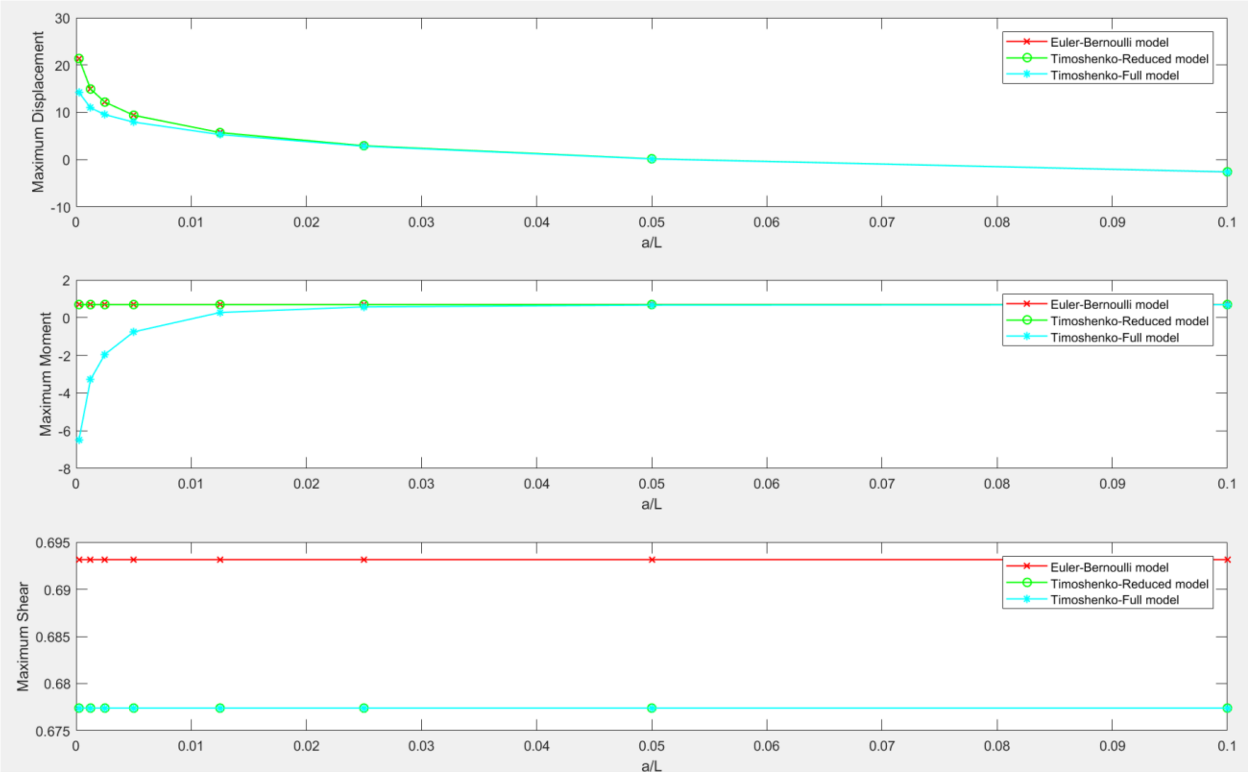
Euler-Bernoulli model					
a	**L**	**a/L**	**Maximum Disp.**	**Maximum Shear**	**Maximum Moment**
0.001	4	0.00025	-1.90E+09	2	1.999913
0.005	4	0.00125	-3.05E+06	2	1.999913
0.02	4	0.005	-1.19E+04	2	1.999913
0.05	4	0.0125	-3.05E+02	2	1.999913
0.1	4	0.025	-19.04761905	2	1.999913
0.2	4	0.05	-1.190476191	2	1.999913
0.4	4	0.1	-0.074404762	2	1.999913

Timoshenko-reduced integration model					
a	**L**	**a/L**	**Maximum Disp.**	**Maximum Shear**	**Maximum Moment**
0.001	4	0.00025	-1.90E+09	1.9687501	1.999024

0.005	4	0.00125	-3.05E+06	1.96875	1.999023
0.02	4	0.005	-1.19E+04	1.96875	1.999023
0.05	4	0.0125	-3.05E+02	1.96875	1.999023
0.1	4	0.025	-19.06875	1.96875	1.999023
0.2	4	0.05	-1.197154018	1.96875	1.999023
0.4	4	0.1	-0.076161412	1.96875	1.999023

Timoshenko-full integration model					
a	**L**	**a/L**	**Maximum Disp.**	**Maximum Shear**	**Maximum Moment**
0.001	4	0.00025	-1.46E+06	1.96875	0.001534
0.005	4	0.00125	-5.74E+04	1.96875	0.037658
0.02	4	0.005	-2.80E+03	1.96875	0.469783
0.05	4	0.0125	-2.00E+02	1.96875	1.314426
0.1	4	0.025	-16.87518104	1.96875	1.768721
0.2	4	0.05	-1.159637903	1.96875	1.936003
0.4	4	0.1	-0.075561027	1.96875	1.982887

The following graph is plotted using MATLAB to be able to compare the results in a better way.



As we see according to that the ratio a/L is less than 0.1 we have a slender beam. And as we know from the theory, the Euler-Bernoulli element shows good results for slender beams. This method works very well for this problem especially for bending stress. The Timoshenko's works better in the shear which is not considered significant for this problem. In bending moment stress, the fully integration Timoshenko doesn't behave well.

We have compared the maximum displacement, shear and bending stresses for each method. Among all, Euler Bernoulli shows better results because of the interval that a/L lies in. After that, reduced Timoshenko has good results regarding displacement. However, when it comes to full Timoshenko, we can say because of the shear locking effect, resulting in increase of stiffness matrix and decrease of strain, it shows smaller values of displacement.

Regarding the bending and shear stress, Euler-Bernoulli and reduced-Timoshenko have very small difference, which is because of the hypothesis used in Timoshenko. But at the end they almost appear to have same results. As the slender ration becomes higher, the full integrated Timoshenko shows higher values of the maximum moment of the beam which is because of increase in stiffness which also affects the integration of the global stiffness for the bending problem by affecting rotational degrees of freedom due to shear locking.