Course:
Computational Structural Mechanics and Dynamics

Assignment-6

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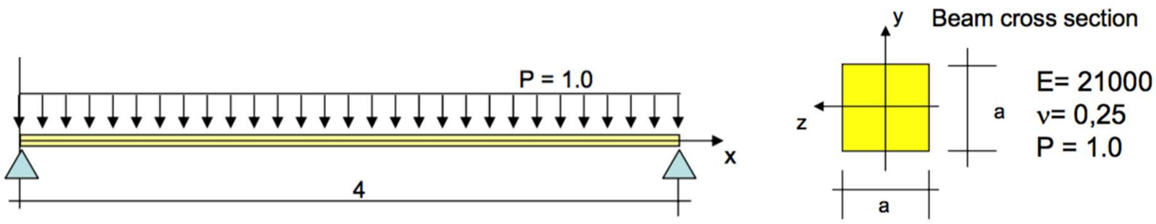
a) Program in Matlab the Timoshenko 2 Nodes Beam element with reduce integration for the shear stiffness matrix.
b) Solve the following problem with a 64 element mesh with:
   - the 2 nodes Euler Bernulli element;
   - the 2 nodes Timoshenko Full Integrate element
   - the 2 nodes Timoshenko Reduce Integration element.
   Compare maximum displacements, moments and shear for the 3 elements against a/L relationship.



a = 0,001
a = 0,005
a = 0,010
a = 0,020
a = 0,050
a = 0,100
a = 0,200
a = 0,400

Results of the MatLab implementation

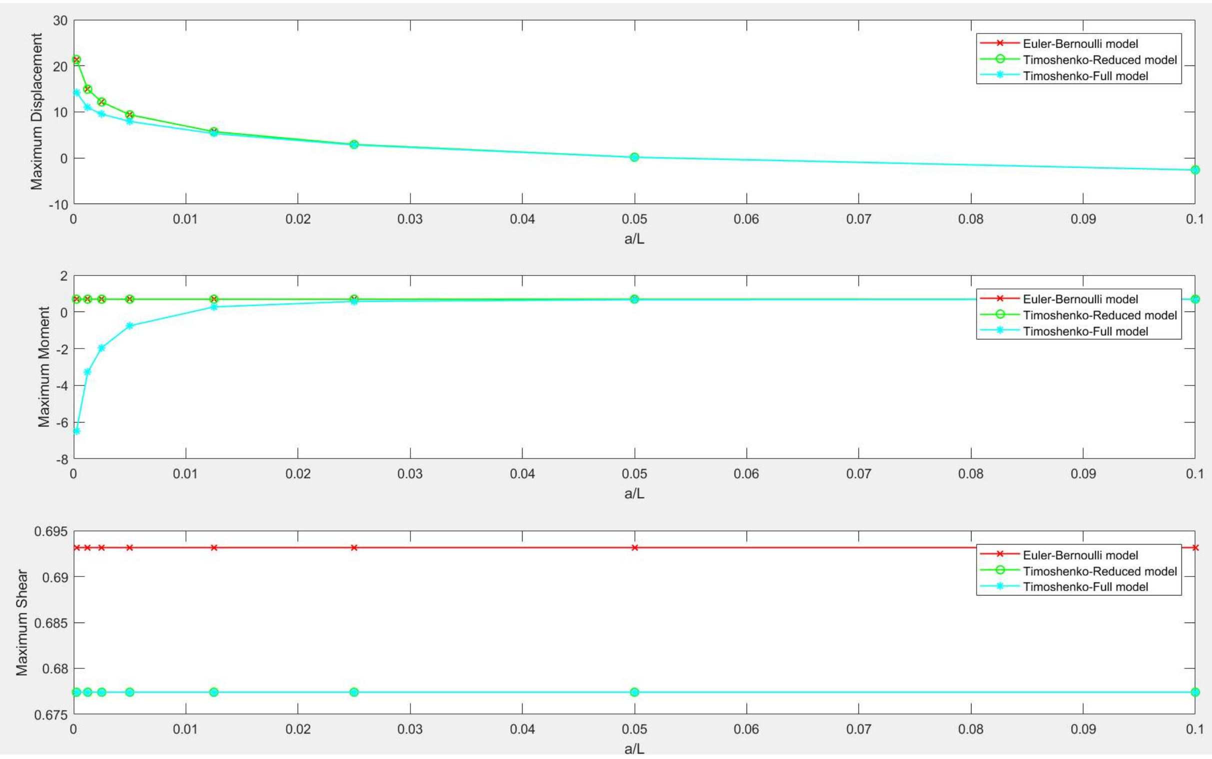
Using the *Viga_Timoshenko* and *Viga_Euler_Bernoulli* MatLab code, we provided the following problem using 64 elements, increasing the size of the cross section. Here there are the results reported in tables: each table is referred to a different model of the beam.

Euler-Bernoulli model					
a	**L**	**a/L**	**Maximum Disp.**	**Maximum Shear**	**Maximum Moment**
0.001	4	0.00025	-1.90E+09	2	1.999913
0.005	4	0.00125	-3.05E+06	2	1.999913
0.02	4	0.005	-1.19E+04	2	1.999913
0.05	4	0.0125	-3.05E+02	2	1.999913
0.1	4	0.025	-19.04761905	2	1.999913
0.2	4	0.05	-1.190476191	2	1.999913
0.4	4	0.1	-0.074404762	2	1.999913

Timoshenko-reduced integration model					
a	**L**	**a/L**	**Maximum Disp.**	**Maximum Shear**	**Maximum Moment**
0.001	4	0.00025	-1.90E+09	1.9687501	1.999024
0.005	4	0.00125	-3.05E+06	1.96875	1.999023

0.02	4	0.005	-1.19E+04	1.96875	1.999023
0.05	4	0.0125	-3.05E+02	1.96875	1.999023
0.1	4	0.025	-19.06875	1.96875	1.999023
0.2	4	0.05	-1.197154018	1.96875	1.999023
0.4	4	0.1	-0.076161412	1.96875	1.999023

Timoshenko-full integration model					
a	**L**	**a/L**	**Maximum Disp.**	**Maximum Shear**	**Maximum Moment**
0.001	4	0.00025	-1.46E+06	1.96875	0.001534
0.005	4	0.00125	-5.74E+04	1.96875	0.037658
0.02	4	0.005	-2.80E+03	1.96875	0.469783
0.05	4	0.0125	-2.00E+02	1.96875	1.314426
0.1	4	0.025	-16.87518104	1.96875	1.768721
0.2	4	0.05	-1.159637903	1.96875	1.936003
0.4	4	0.1	-0.075561027	1.96875	1.982887



Conclusions and comments

As we see according to that the ratio a/L<0.1 we have a slender beam. And as we know from the theory, the Euler-Bernoulli element shows good results for slender beams.
All the models show big changes for the maximum value of the vertical displacement: the Euler Bernoulli

in this set of slender ratio values works very well, and the same can be said for the reduced integration version of the Timoshenko's model, which has the same displacements. On the other hand for the full integration version of the Timoshenko model we can see smaller values of the displacements, due to the so called "sheak-locking" effect, which brings smaller values of the strains and increases the stiffness of the beam.
While the models of Euler-Bernoulli and reduced-Timoshenko show the same values for the bending moment and the shear (there is a small difference in the shear values due to the hypothesis in the Timoshenko model), the full integrated Timoshenko shows an increasing value of the maximum moment of the beam, due to an increasing value of the cross section thickness and stiffness, that affects the integration of the global stiffness for the bending problem (the shear locking effect appears and affects the rotational DOFs).
We can say that the Euler-Bernoulli's describes this problem in a very good way, in particular it's very precise for the bending problem. The Timoshenko's is better in the shear problem, but it's not so necessary for these slender beams. In any case, the fully integration Timoshenko's model should be avoided, since it brings trouble in the bending problem.