Computational Structural Mechanics and Dynamics

As6 Beams

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Beams

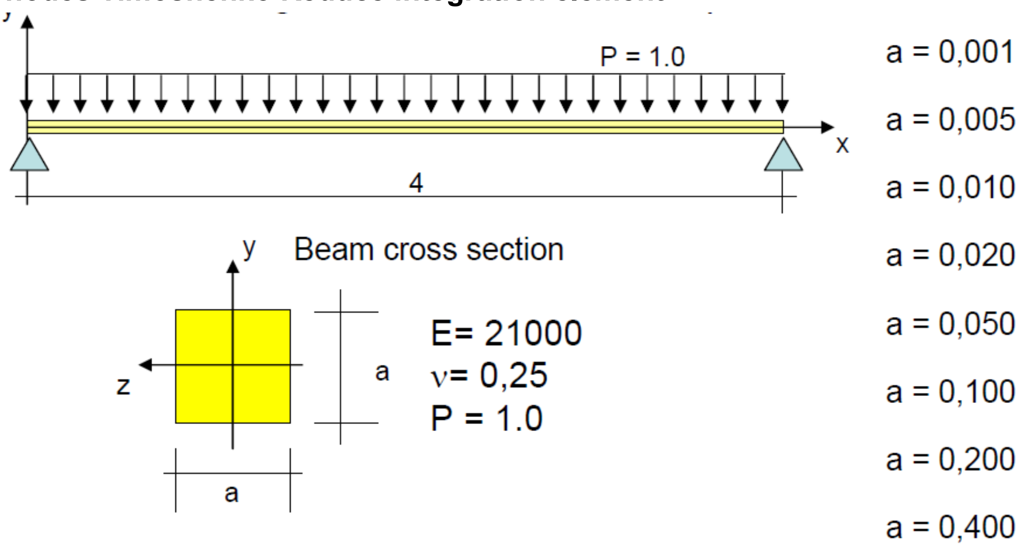
a) **Program in Matlab the Timosheko 2 nodes beam element with reduce integration for the shear stiffness matrix**

[Answer]

In order to implement the reduced integration, the elemental stiffness matrix has been changed as follows:

```
% reduced integration
    K_s = [    1     ,     len/2 ,     -1     ,     len/2 ;
             len/2 , len^2/4 , -len/2 , len^2/4 ;
              -1     ,   -len/2 ,      1     ,   -len/2 ;
             len/2 , len^2/4 , -len/2 , len^2/4 ];
```

b) **Solve the following problem with a 64 element mesh with the**
2 nodes Euler Bernulli element
2 nodes Timoshenko Full integrate element
2 nodes Timoshenko Reduce integration element

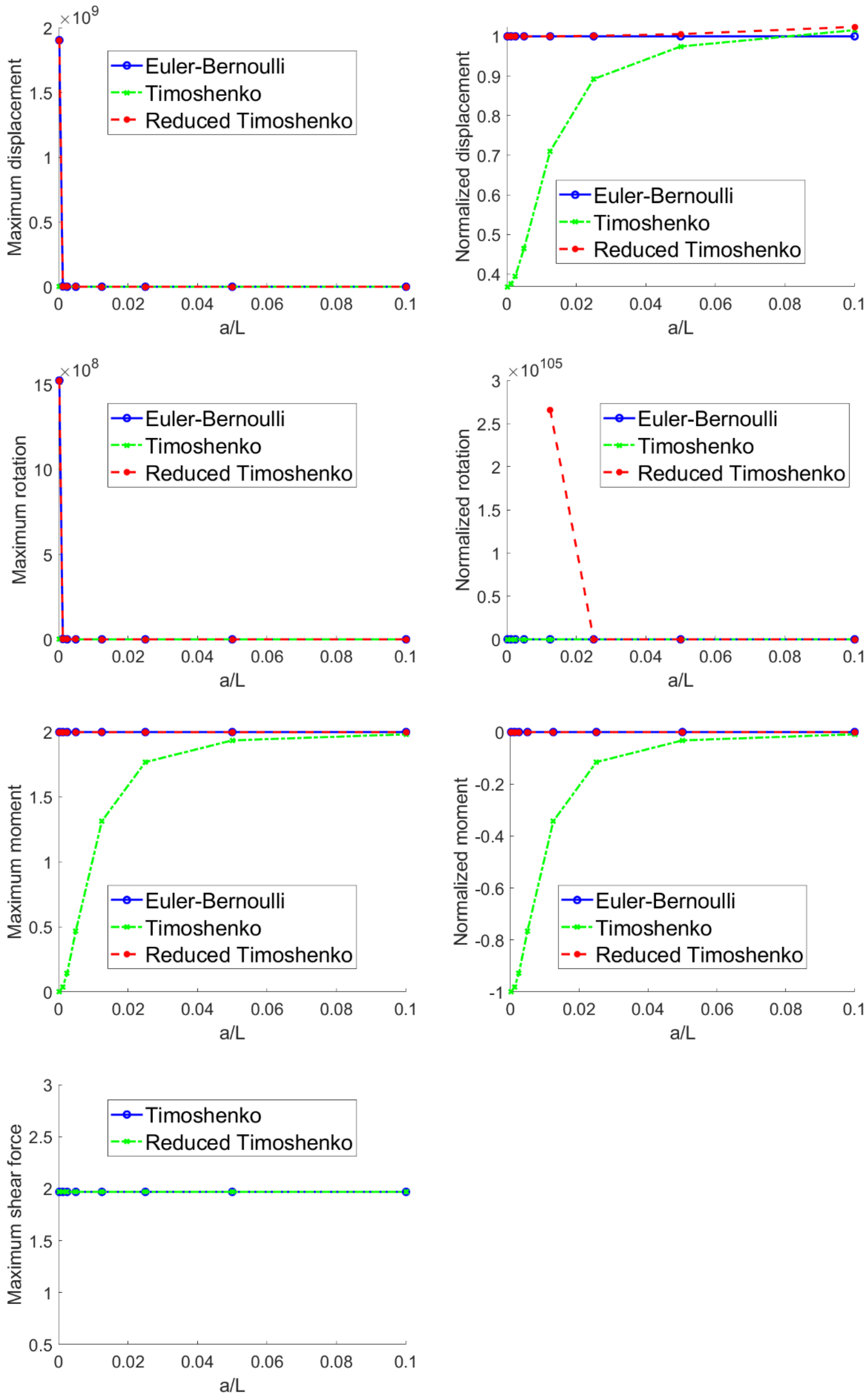


Compare maximum displacements, moments and shear for the 3 elements against the a/L relationship.

[Answer]

Eight different relations of a/L have been computed with three types of element. To compare the results, the Timoshenko Reduced Integration element has been chosen as reference:

The maximum deflection, bending moment and share force of the beam for different a/L relations are plotted in the following figures:



From these plots, it is seen that for low value of a/L, the Euler Bernoulli and the reduced integrated Timoshenko beam yield to the same result while the fully integrated Timoshenko is completely stiff due to shear locking. Because of this phenomenon, the bending moment is highly underestimated. As the relation a/L

increase, the shear effect become more significant. For this reason, the Timoshenko beam is more accurate. The reduced integrated element convergences to the same result than the fully integrated element. Meanwhile, the displacement of Euler Bernoulli beam is higher than what it should due to no consideration of the shear effects. However, even in this case, the Euler Bernoulli beam leads to correct values of the bending moment.

For the shearing fore, both Timoshenko and reduced integrated Timoshenko keep the same result which is accurate.

Generally, Euler-Bernoulli beam theory is the oldest, the simplest classical theory for beam bending. It is used in typical hand calculations of beam deflection. It assumes that the cross-section of the beam is always perpendicular to the neutral axis (also after the deformation). Deflection is calculated only using bending moment, without taking shear forces into account. It can be used to determine shear stresses but doesn't account for their influence on deflection. On the other hand, Timoshenko beam theory is extended version of Euler-Bernoulli theory that takes into consideration deformations caused by shear. It assumes that the cross-section after deformation doesn't have to be perpendicular to the neutral axis.

In engineering, Euler-Bernoulli theory is used for thin beam, while Timoshenko theory is used for thick beam.