UPC - BARCELONA TECH
MSc Computational Mechanics
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Computational Structural Mechanics and Dynamics

Assignment 7 Plate Theory

Due 2/04/2018

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a1) What kind of strategy (theory, elements, integration rule, boundary conditions, etc) will you use for solving the following problem?

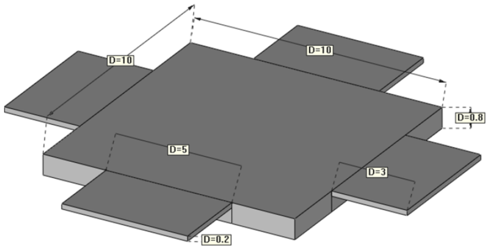

Figure 0.1: Problem # 1

For this first case of analysis, it is easy to realize that the mid-plane line of the lateral parts is not aligned with that of the central part. Thus, a complete 2D abstraction of the problem is not possible as we cannot "cut" the whole body by its mean plane. In other words, this case cannot be considered a properly defined plate with the corresponding bending state.

Therefore, a priori, neither the Kirchhoff nor the Reissner-Mindlin plate theories could be applied to model this problem and get a solution. Instead we could go and analyze the entire body as a 3D problem. Since the geometry is not complicated at all, i.e. straight sides, a structured mesh using hexahedral elements can be considered which usually tend to perform better over tetrahedral elements in bending dominated problems. Also, the symmetry of the structure allows to simplify the computations.

a2) What kind of strategy (theory, elements, integration rule, boundary conditions, etc) will you use for solving the following problem?

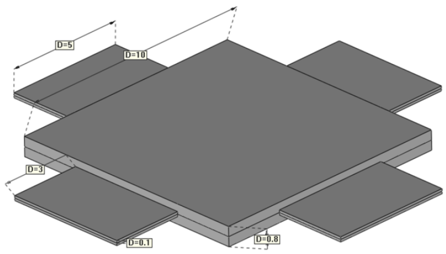

Figure 0.2: Problem # 2

For this second problem, we see that the middle plane of the whole body is equidistant from both upper and lower faces. Also, as we are only considering an uniform distributed load applied on the upper face, there is no in-middle plane loading and axial strains are zero. Thus, this is in fact a plate and we can model this case as a 2D abstraction. Now we can check the ratio thickness/average side to see if we are dealing with a thick or thin plate. Note that we have different lengths on central and lateral parts.

$$\frac{t}{L} = \frac{0.8}{10} = 0.08 > 0.05 \Rightarrow \text{moderately thick plate (central part)}$$

$$\frac{t}{L} = \frac{0.1}{4} = 0.025 < 0.05 \Rightarrow \text{thin plate (lateral parts)}$$

Therefore, the so called Reissner-Mindlin plate theory seems to be more suitable to solve this problem, which accounts for transverse shear deformation effects which becomes more important for plates in which $t/L > 0.05$. This theory also allows to get rid of the continuity requirements of the C^1 Kirchhoff elements. But the presence of the thin lateral parts of the plate may lead to the socalled shear locking effect discussed in class. Thus, a technique needs to be used in order to overcome this issue. A general technique consist in imposing "a priori" transverse shear strain field, vanishing in the thin limit.

When meshing the plate to solve this problem, we can consider the 4-noded plate quadrilateral element QLLL (for Quadrilateral, biLinear deflection, biLinear rotations and Linear transverse shear strain fields). This QLLL element it is proved to satisfy the conditions to be used with the transverse shear strain technique discussed previously and it is also proved to be robust in practical applicatons. Moreover, the computation of the stiffness matrix requires a full 2 x 2 quadrature for all terms, and this preserves the element from spurious oscillations in the solution.

Finally, it is interesting to note the complete symmetry of the plate, both geometrically and loading. Thus, it seems smart to just analyze one quarter of it and impose symmetry boundary conditions on the mirror axes. These conditions would read,

$$\theta_x = 0 \text{ on the y axis} \qquad ; \qquad \theta_y = 0 \text{ on the x axis}$$

but still considering the same uniform distributed load q.

b) Define and verify a patch test mesh for the MCZ element. Discuss the results observed.

The patch test is based on selecting an arbitrary patch of elements and imposing upon it a displacement field. This test is nothing but an assesment of the stability of the finite element solution and hence provides not only a neccesary but sufficient condition for convergence. The procedure is as follows: first the displacements of the nodes at the boundary of the patch are prescribed. Then, the values of the displacements at the internal nodes are computed using the FE code and lately compared with the exact ones provided by the expresion of the imposed displacement field.

Consider the patch of a mesh in Figure 0.3 down below,

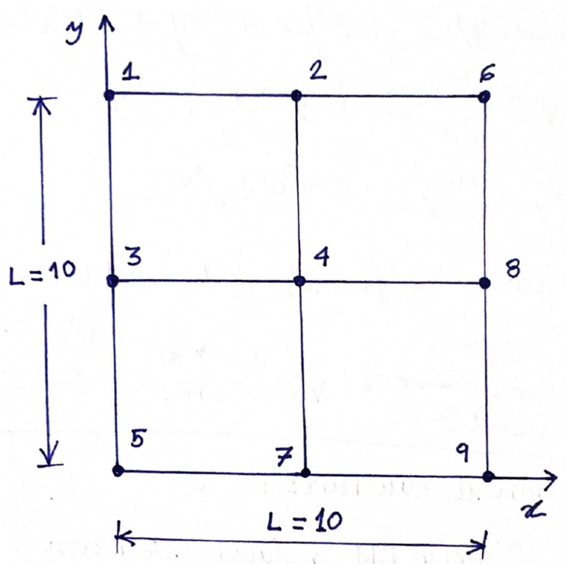

Figure 0.3: Considered patch

with coordinates

	Coordinates	
Node #	x_i	y_i
1	0	10
2	5	10
3	0	5
4	5	5
5	0	0
6	10	10
7	5	0
8	10	5
9	10	0

Table 0.1: Coordinates of the nodes on the patch

as they are in the file *Clamp_UL_1.m* provided with the *Matlab* code.

A simple patch test can be applied to the MCZ thin plate element in order to asses a good representation of rigid body displacements and the absense of spurious modes. Let us define a simple linear displacement field of the form,

$$w = a + bx + cy \tag{1}$$

with a, b and c arbitrary and also

$$\frac{\partial w}{\partial x} = \theta_x = b \qquad ; \qquad \frac{\partial w}{\partial y} = \theta_y = c \tag{2}$$

being θ_x and θ_y the rotations about the x and y axis respectively.

Particularly, let's now impose on this patch

$$w = 1 + 2x + 3y \qquad ; \qquad \theta_x = 2 \qquad ; \qquad \theta_y = 3$$

which translates into the following values on the nodes on the boundary,

Node	w	θ_x	θ_y
1	31	2	3
2	41	2	3
3	16	2	3
5	1	2	3
6	51	2	3
7	11	2	3
8	36	2	3
9	21	2	3

Table 0.2: Values of displacement and rotations imposed on the boundary nodes of the patch.

Now it remains to obtain the displacement and rotations at the interior node, number 4. Values of $w = 26$ and $\theta_x = 2$, $\theta_y = 3$ are expected. If we now run the *Plate_MCZ.m* file, providing a input file with the proper information, we can check that we get

```
val =

   (1,1)       31.0000
   (2,1)        2.0000
   (3,1)        3.0000
   (4,1)       41.0000
   (5,1)        2.0000
   (6,1)        3.0000
   (7,1)       16.0000
   (8,1)        2.0000
   (9,1)        3.0000
  (10,1)       26.0000
  (11,1)        2.0000
  (12,1)        3.0000
```

Figure 0.4: Solution obtained in Matlab for a linear displacement field after executing the file *Plate_MCZ.m*.

As we can see, the values in Figure 0.4 of the 10, 11 and 12 entries, corresponding to the information of node 4, match the expected values. Thus, the MCZ element has passed the patch test for a prescribed displacement field. This means that this element is able to reproduce a linear behaviour on a given mesh.

Now, we can repeat this proccess for a quadratic displacement field. Let us consider then,

$$w(x, y) = 1 + x + y + xy + x^2 + y^2$$

and rotations

$$\theta_x = \frac{\partial w(x,y)}{\partial x} = 1 + 2x + y \qquad ; \qquad \theta_y = \frac{\partial w(x,y)}{\partial y} = 1 + 2y + x$$

Therefore, for the boundary nodes of the considered patch in Figure 0.3, with corresponding coordinates in Table 0.1, this imposed displacement field gives the following values

Node #	$w(x, y)$	θ_x	θ_y
1	111	11	21
2	191	21	26
3	31	6	11
5	1	1	1
6	321	31	31
7	32	11	6
8	191	26	21
9	111	21	11

Table 0.3: Values of displacements and rotations of the boundary nodes for a quadratic displacement field.

Now, we need to compute the displacement and rotations for central node. The values for nodes # 4, taking into account the choosen field are expected to be $w = 86$, $\theta_x = 16$ and $\theta_y = 16$. For this purpose, we run the provided code *Plate_MCZ.m* imposing the previous values on the boundary nodes. We obtain,

```
val =

   (1,1)     111.0000
   (2,1)      11.0000
   (3,1)      21.0000
   (4,1)     191.0000
   (5,1)      21.0000
   (6,1)      26.0000
   (7,1)      31.0000
   (8,1)       6.0000
   (9,1)      11.0000
  (10,1)      86.0000
  (11,1)      16.0000
  (12,1)      16.0000
```

Figure 0.5: Solution obtained in Matlab for a quadratic displacement field after executing the file *Plate_MCZ.m*.

As one can tell, the three last values in Figure 0.5 are precisely the displacement and rotations of the central node # 4 and they match with the analytical ones. Thus, the socalled MCZ element is able to catch a quadratic solution.