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# Assignment 7

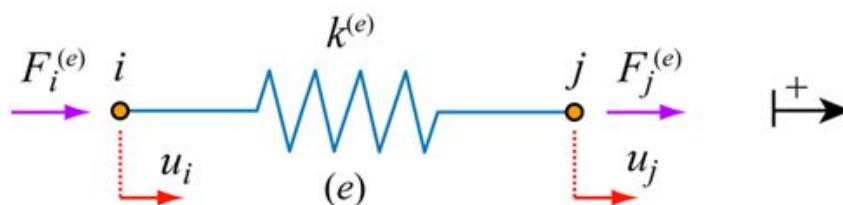
## Computational Structural Mechanics and Dynamics

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# 1 Assignment 7.1

Analyze the shear blocking effect on the Reissner Mindlin element and compare with the MZC element. For the Simple Support Uniform Load square plate. Use the 5x5 Mesh

	t = 0,001
	t = 0,010
E= 10.92	t = 0,020
v= 0,3	t = 0,100
Q = 1.0	t = 0,400

Matlab programs were downloaded through the mat-fem page offered by CIMNE, these programs were installed together with the GiD interface to model the plate and generate an output file with the extension ".m". The plate that was generated has the characteristics mentioned in the problem statement with square dimensions of length 4. The physical characteristics were taken as dimensionless, since the analysis will be of comparison under the same physical criteria, it won't have influence in our analysis.

Once this program was generated, the conditions were modified directly on the file with the extension ".m" and it was run as many times as necessary in the cases of Reissner Mindlin and MZC element. The conditions that were varied during each simulation were the thickness of the plate. Presented below in the following table:

L	t
4	0.001
4	0.01
4	0.02
4	0.1
4	0.4

## 1.1 Results

The results obtained are summarized in the following tables:

- Reissner Mindlin

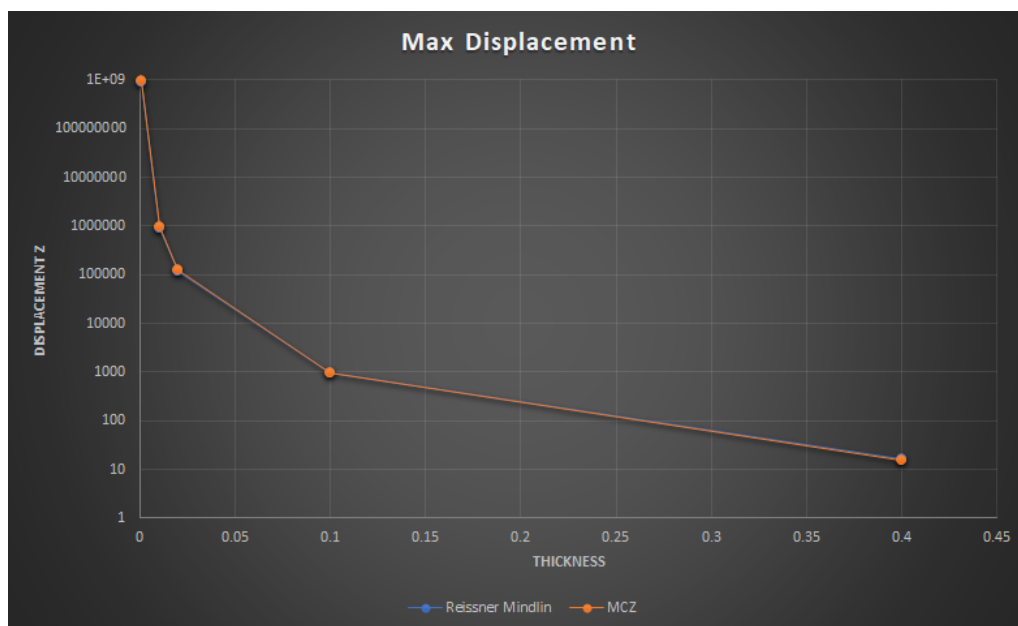
Reissner Mindlin							
t	Displacement z	Z force	Mxy	Mx	My	Rotx	Roty
0.001	-932268000	1.30537	-0.453755	-0.706902	-0.706902	820495000	820495000
0.01	-932397	1.30555	-0.453543	-0.706992	-0.706992	820637	820637
0.02	-116598	1.30607	-0.4529	-0.707263	-0.707263	102633	102633
0.1	-944.744	1.32024	-0.434117	-0.715423	-0.715423	833.864	833.864
0.4	-16.7218	1.36214	-0.306222	-0.780089	-0.780089	14.4912	14.4912

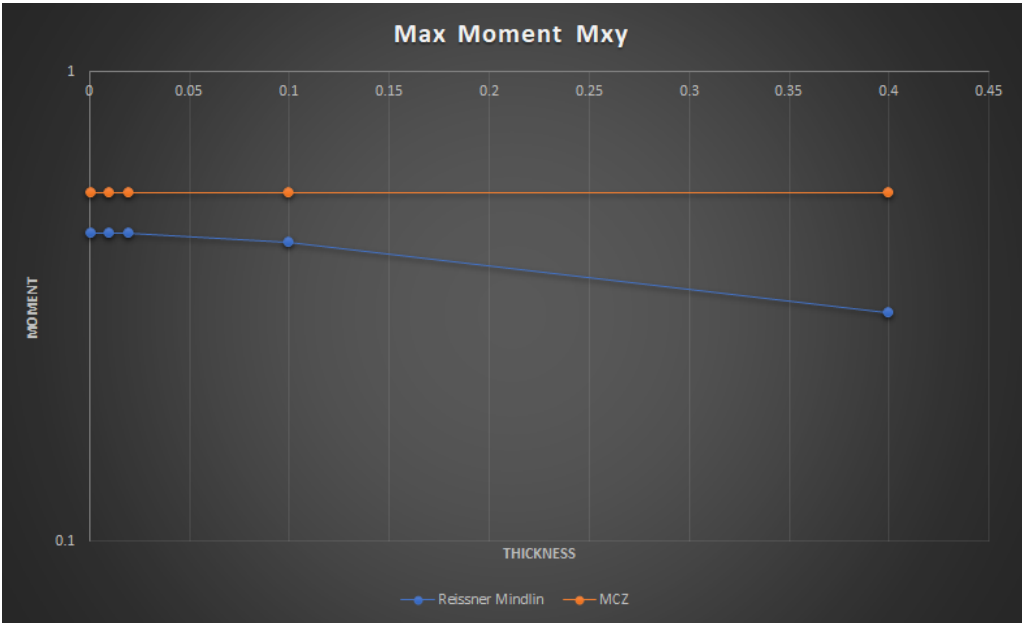
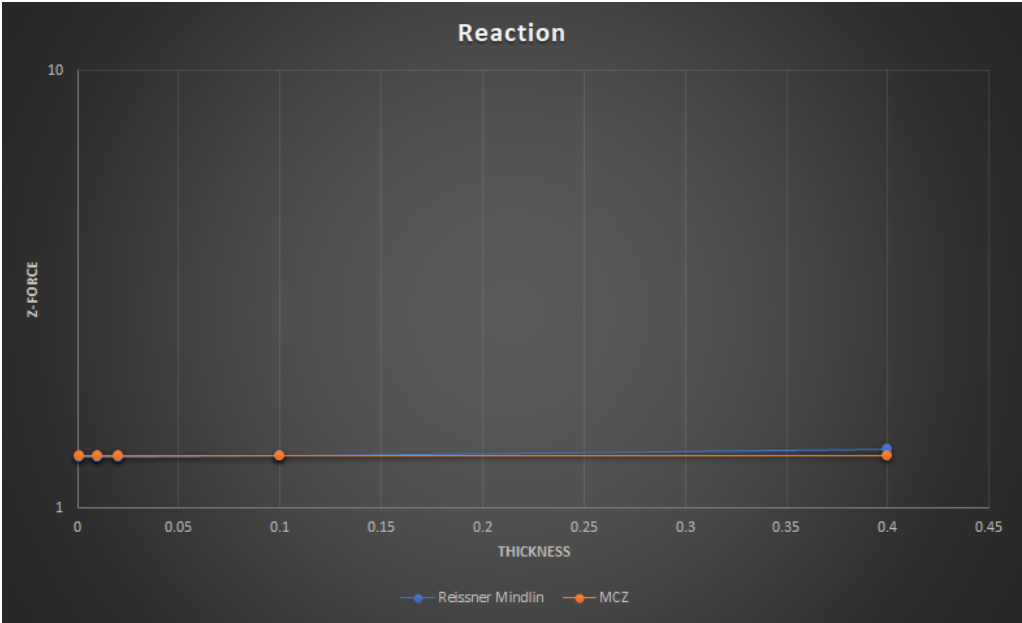
Reissner Mindlin		
t	Q <sub>x</sub>	Q <sub>y</sub>
0.001	0.960002	0.960002
0.01	0.960175	0.960175
0.02	0.960697	0.960697
0.1	0.976121	0.976121
0.4	1.07896	1.07896

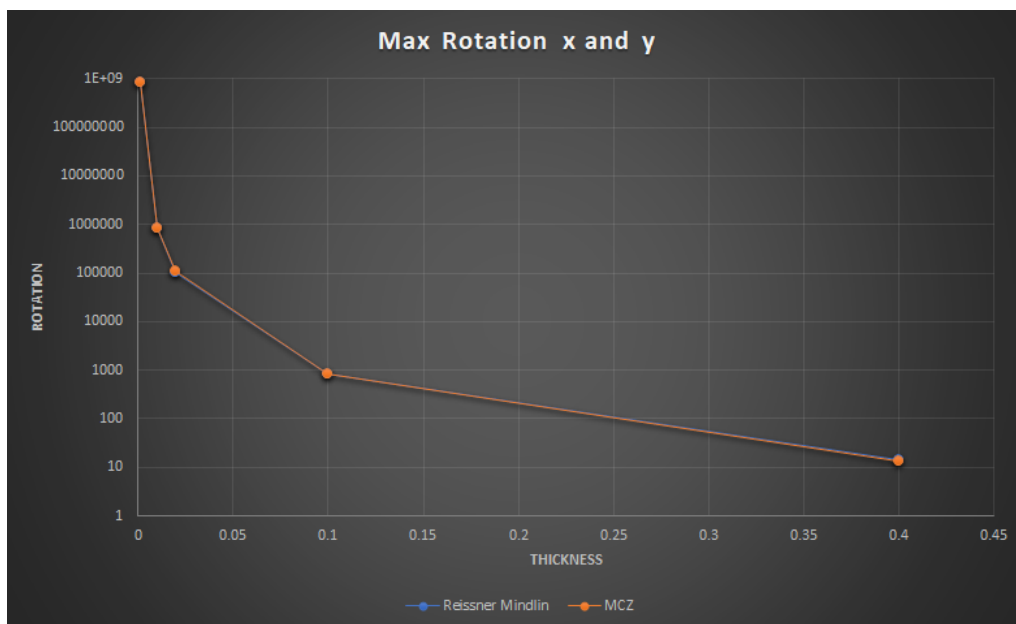
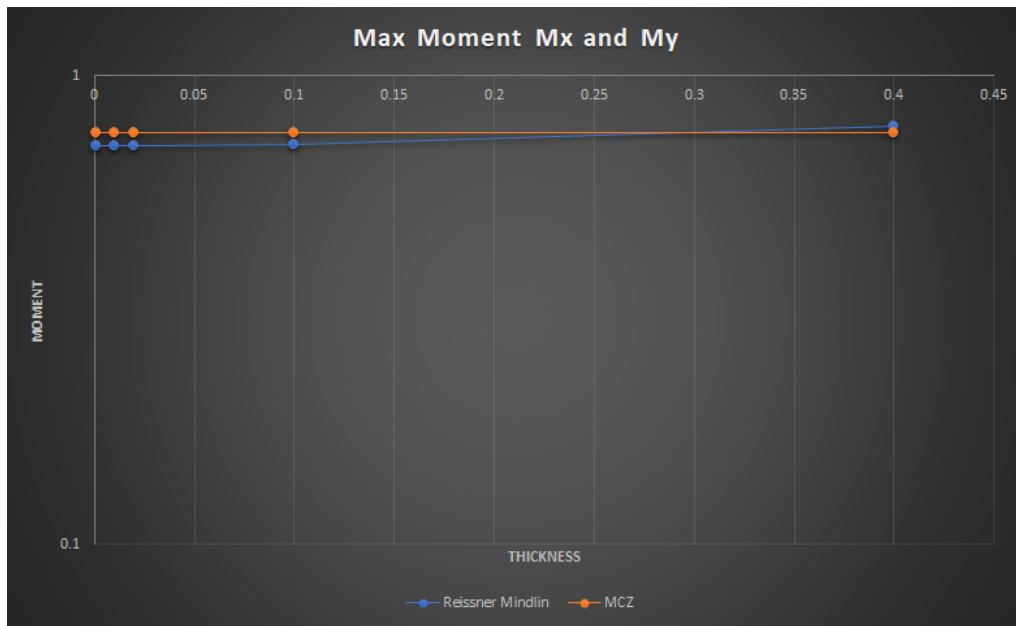
- MZC

MZC							
t	Displacement z	Z force	M <sub>xy</sub>	M <sub>x</sub>	M <sub>y</sub>	Rot <sub>x</sub>	Rot <sub>y</sub>
0.001	-988537000	1.31542	-0.551506	-0.757715	-0.757715	861093000	861093000
0.01	-9.89E+05	1.31542	-0.551506	-0.757715	-0.757715	861093	861093
0.02	-123567	1.31542	-0.551506	-7.58E-01	-0.757715	107637	107637
0.1	-988.537	1.31542	-0.551506	-7.58E-01	-0.757715	861.093	861.093
0.4	-15.4459	1.31542	-0.551506	-7.58E-01	-0.757715	13.4546	13.4546

To facilitate the comparison of these results, the following graphs were created:







## 1.2 Discussion

Concluding from the comparison shown in the tables and graphs above, the following points can be highlighted:

- When it comes to plates with a very small thickness, our displacements have magnitudes that the materials will not allow without causing a break in their structure completely, the reason is that the solution blows as the thickness becomes smaller. This phenomenon can also be noticed in rotations.
- Although a large change in the value of the moment was expected between methods because one considers the shear energy while the other does not, it is possible to

appreciate that the change in the solution of the moments obtained presents an order of approximately 6% only.

- Mxy moments present greater changes between methods, something that was expected, and which also suffer a greater change in percentage as the plate grows in thickness, this is again presented by the shear blocking effect.

Thanks to these observations, it can be concluded again that Reissner-Mindlin as well as Timoshenko in the case of beams, will be a better estimator in the case of displacements, while MZC will be a better estimator of moments.

## 2 Assignment 7.2

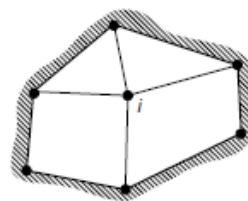
Define and verify a patch test mesh for the MZC element.

### 2.1 Patch test theory

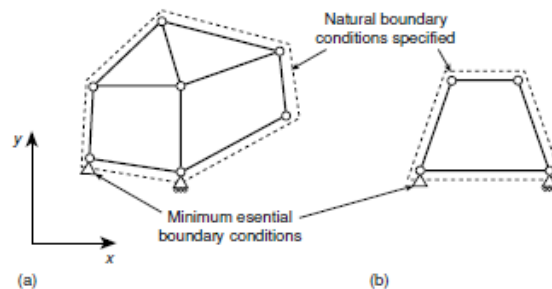
A known linear field  $u^p$  is prescribed at all nodes of a patch of 2D elements, where  $Q^p$  are prescribed temperatures in a heat conduction problem, or prescribed displacements in an elasticity problems, etc. For each internal node  $j$  in the patch we verify that:

$$K_{ij}a_j^p - f_j^p = 0$$

where  $a_j^p$  is the nodal unknown vector corresponding to the known field and  $f_j^p$  is a vector resulting from any external flux (or forces) required to satisfy the governing differential equations for the known solution. Generally, in problems expressed in cartesian coordinates  $f_j^p = 0$ . [1]

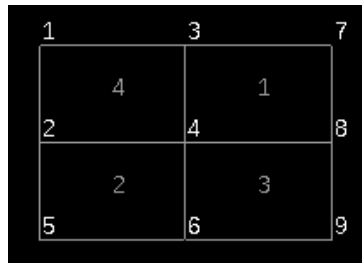


Test A  
a prescribed on all nodes  
 $K_{ij}a_j = f_j$  verified at node  $i$



## 2.2 Implementation

To implement the patch test for the rectangular element of MZC if you considered the following mesh:



Where the coordinates of the nodes were:

Node	x	y
1	0	2
2	0	1
3	1.5	2
4	1.5	1
5	0	0
6	1.5	0
7	3	2
8	3	1
9	3	0

Now a linear constraint field is proposed below:

$$\textit{Displacement} - z = -1 \times 10^{-4}(x)$$

$$\textit{Rotation} - x = 1 \times 10^{-4}(x)$$

$$\textit{Rotation} - y = 1 \times 10^{-4}(y)$$

This constraints were applied on each node in the GiD interface and then create the input file for matlab.

Once it was run, the rows values of the stiffness matrix corresponding to the degrees of freedom of node 4 (midnode) were multiplied for each constraint vector of this node, obtaining the following values:

$$\textit{Displacement} - z = 0.0511$$

$$\textit{Rotation} - x = 0.0365$$

$$\textit{Rotation} - y = 0.0322$$

With this values the patch test passes, therefore, as we refine our mesh, we will be converging to the exact solution.

## References

- [1] EUGENIO OÑATE, PEDRO DÍEZ, FRANCISCO ZÁRATE, ANTONIA LARESE, *INTRODUCTION TO THE FINITE ELEMENT METHOD*, 2008, UPC.