Master of Science in Computational Mechanics

Computational Structural Mechanics and Dynamics

Assignment 7: Thin and thick plates

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Shear locking effect: comparison of plate elements

For the comparison of the MZC and Reissner-Mindlin plate elements, a sample problem will be solved: a plate with an acting uniform load and simply supported along the boundaries. Figure (1) shows the geometry, boundary conditions and material properties of the problem. As depicted in the figure, a 5x5 mesh will be used.

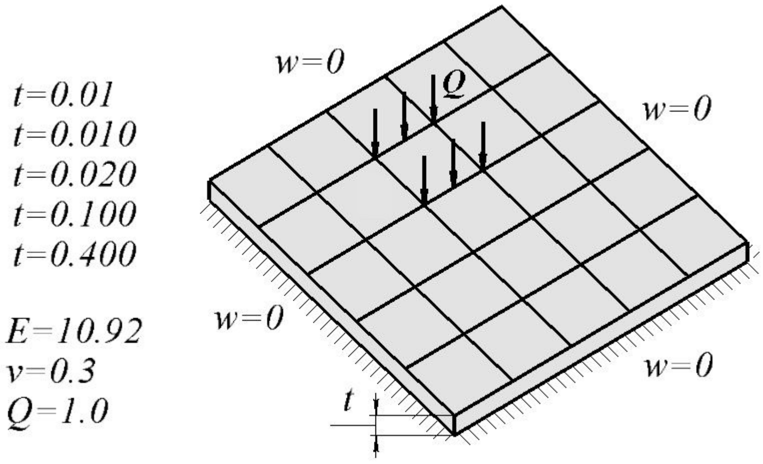

Fig. 1 – Boundary conditions and material properties

As seen in Figure (1), the value t is varied to study the influence of thickness on the results obtained with the different plate elements based on Kirchoff and Reissner-Midlin theory of plate bending. To analyze the results, MATFEM was used as based computing program and GiD for pre- and post-processing of the data.

Tables (1) and (2) show the results of numerical computations using the MZC and Reissner-Mindlin for different thickness values.

	MZC Elements			
Ratio L/t	**max** w **[m]**	**max** X**-Mom [N · m]**	**max** Y**-Mom [N · m]**	**max** XY**-Mom [N · m]**
10000	1.1802E+10	3.247	3.247	0.827
1000	1.1802E+07	3.247	3.247	0.827
500	1.4752E+06	3.247	3.247	0.827
100	1.1802E+04	3.247	3.247	0.827
25	1.8440E+02	3.247	3.247	0.827

Table 1 – Results using MZC plate elements

	Reissner-Mindlin Elements					
Ratio L/t	**max w [m]**	**max X-Mom [N·m]**	**max Y-Mom [N·m]**	**max XY-Mom [N·m]**	**max X-Shear [N]**	**Max Y-shear [N]**
10000	1.314E+04	2.029E-06	2.029E-06	6.465E-07	6.835	6.835
1000	1.314E+03	2.029E-04	2.029E-04	6.465E-05	6.834	6.834
500	6.570E+02	8.113E-04	8.113E-04	8.113E-04	6.833	6.833
100	1.303E+02	2.010E-02	2.010E-02	6.400E-03	6.792	6.792
25	2.883E+01	2.769E-01	2.769E-01	8.810E-02	6.248	6.248

Table 2 – Results using Reissner-Mindlin (fully integrated) plate elements

Figure (2) shows the logarithmic pattern of the ratio of the vertical displacement predicted by the MZC element w_K and the Reissner-Mindlin element w_{RM}. It can be seen that as the thickness of the plate decreases, i.e. the ratio L/t increases, the Reissner-Mindlin plate predicts a smaller vertical displacement resulting in a prediction of an over-stiff element. Moreover, as the thickness increases, i.e. the ratio L/t decreases, the value computed using both elements tends to convergency and the ratio of w_K and w_{RM} approaches 1. This effect, known as shear locking effect, results in over-stiff predictions by the Reissner-Mindlin plate elements.

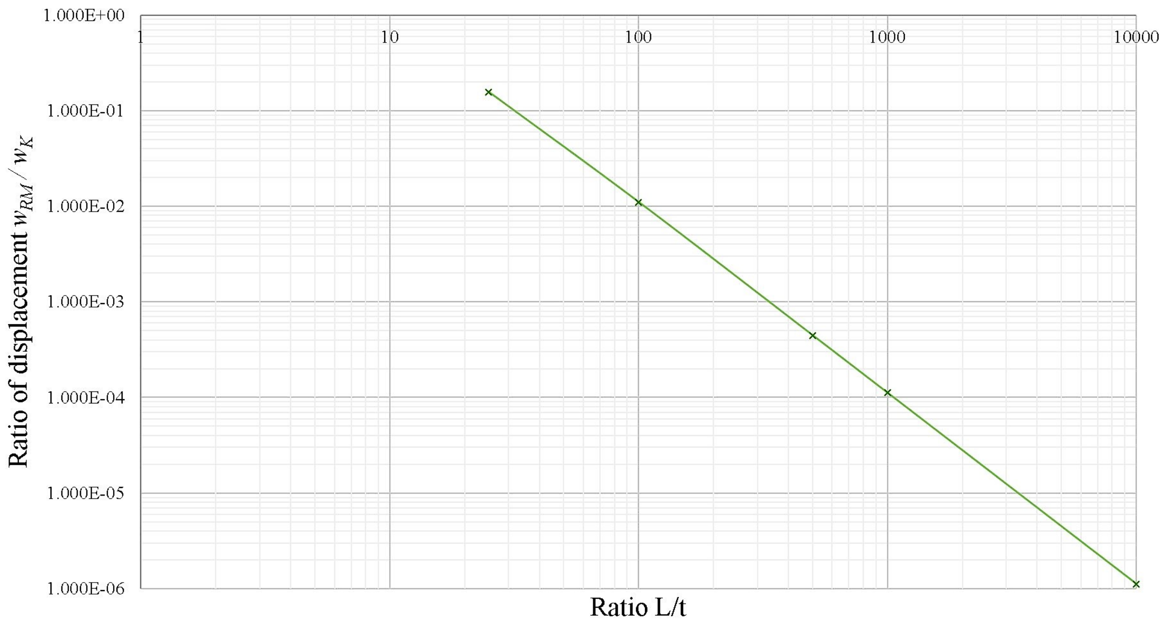

Fig. 2 – Comparison of maximum deflection predicted by MZC and Reissner-Mindlin plate elements

To overcome this effect of the Reissner-Mindlin plate element, several solutions have been proposed including entirely new elements or selective/partial integration of the stiffness matrices associated with the given element.

Patch test of the MZC plate element

The patch test is a technique used for the development of non-conforming finite elements. The test is based on the assumption that, physically speaking, a *good element* should solve simple problems exactly whether individually or as a component of a mesh.

For the MZC plate element, the central node of an structured computational domain of quadrilateral elements will be used as the patch node and a displacement patch test will be carried out (see Figure (3)). For the given patch, having the origin at node 1, a displacement of the form $w(x, y) = 0.1x - 0.4y + 0.05$ will be imposed at the boundaries (see Table (3))

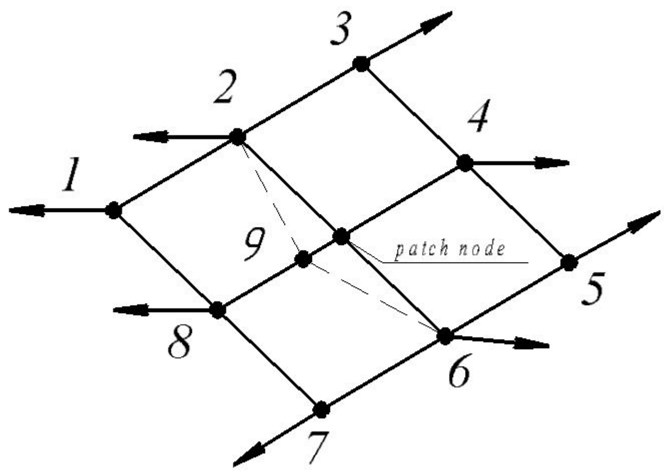

Fig. 3 – Patch node and boundaries (Please note that the arrows play only an illustrative role

	x-coordinate	y-coordinate	Imposed displacement, [mm]
Node 1	0	0	0.05
Node 2	0	1	-0.35
Node 3	0	2	-0.75
Node 4	1	2	-0.65
Node 5	2	2	-0.55
Node 6	2	1	-0.15
Node 7	2	0	0.25
Node 8	1	0	0.15
Node 9	1	1	-

Table 3 – Coordinates and imposed vertical displacement of the nodes

Using the mathematical expression for the displacement field previously defined, the value in node 9 should be equal to -0.25 mm, which perfectly coincides with the result obtained using MAT-Fem. Thus, for the given boundary and imposed displacement field, the MZC element satisfies the patch test.

Now, we will consider a distorted mesh with node 9 translated 0.2 mm vertically below its original position (see Figure (3)). For the given node, the mathematical expression for the displacement field yields a value of -0.17 mm whereas the value computed by MAT-Fem equals -0.21 mm. Hence, for the given distorted unstructured mesh the MZC element fails to satisfy the patch test. Similarly, it can be proven that imposing a displacement field of the form $w(x, y) = ax^2 + by^2 + cx^2y + dxy^2 + exy + f$, results in the MZC being incapable of satisfying the patch test, which is directly related to the shape functions of the element's definition.