International Centre for
Numerical Methods in Engineering

Universitat Politècnica de Catalunya

Master of Science in Computational Mechanics

Computational Structural Mechanics and Dynamics

Assignment 7

Eugenio José Muttio Zavala

March 29th, 2019

Submitted To:
Prof. Francisco Zárate

ASSIGNMENT 7: PLATES

a. *Analyze the shear blocking effect on the Reissner Mindlin element and compare with the MZC element. For the Simple Support Uniform Load square plate. Using 5x5 Mesh*

1. $t = 0,001$
2. $t = 0,010$
3. $t = 0,020$
4. $t = 0,100$
5. $t = 0,400$

Solution:

It is known as theory that the plate formulation given by Kirchoff is restricted to thin plate situations only (thickness/average side ≤ 0.10). Then, a different formulation assuming that the normals to the plate do not remain orthogonal to the mid–plane after deformation, given by Reissner and Mindlin is considered, this allows to obtain transverse shear deformation effects. But, some difficulties arise when Reissner-Mindlin elements are used for thin plate situations due to the excessive influence of the transverse shear deformation terms. The "shear locking" defect is analogous to that found when Timoshenko beam elements are applied to slender beams [Oñate]. In order to compare MZC and Reissner Mindlin formulations, a FEM model is created by using GiD and the plates solver given in class. The figure 0.1 shows the plate dimensions considered.

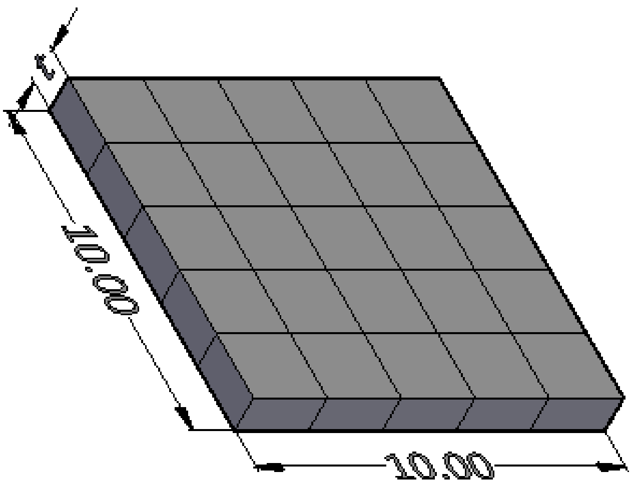
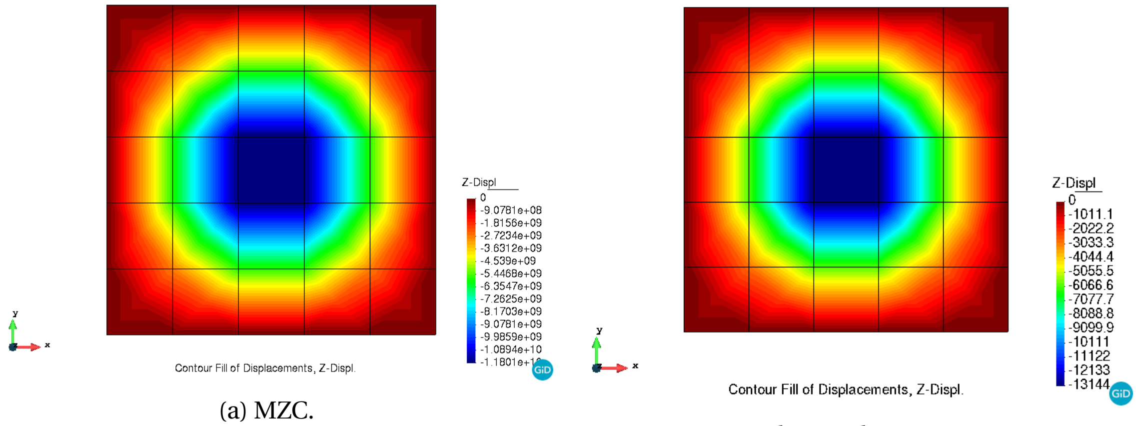


Figure 0.1: Geometry of the plate and mesh considered.

The principal results to verify in order to compared both elements should be the displacement obtained at the center of the plate which is the maximum value reached. Then the figure 0.2 shows the same discretization employed (5x5 elements) with the same thickness, corresponding to $t = 0.001\ m$, which is the test with the minor value of thickness. Then it is expected that Reissner-Mindlin element presents a completely different result, which is a value tending to the order of $1e+04$, in contrast the MZC element presents a displacement of $1E+10$. Even though, those values are not physically admisible[1], they show us a displacement magnitude greater to the MZC plate. This can be explained as the RM formulation has an additional stiffness that tends bigger values than the exact, but only for the thin plates.



(a) MZC.

(b) Quadratic.

Figure 0.2: Displacement field using GiD for a) MZC and b) RM plate elements.

Now, consider the graph depicted in the figure 0.3, it is clearly shown how the maximum displacement of the plates are getting closer if the thickness is bigger, but if the thickness is diminished as the graph shows considering the relation L/t, then the greater values of the horizontal axis corresponds to a greater difference of the Reissner-Mindlin plate. In other words, it is proved that when the thickness $\mathbf{t} \to \mathbf{0}$ then the displacement $\delta \to \infty$. Furthermore, when $\mathbf{t} \to \mathbf{0}$, the bending part of the system of equations $\mathbf{K}_b$ is negligible and the shear part $\mathbf{K}_s$ determines the magnitude of the solution, this effect can be visible in the moment graph shown in figure 0.4.

[1] This values apperead due to the mechanical properties chosen, which corresponds to non-physical behavior fot thin plates.

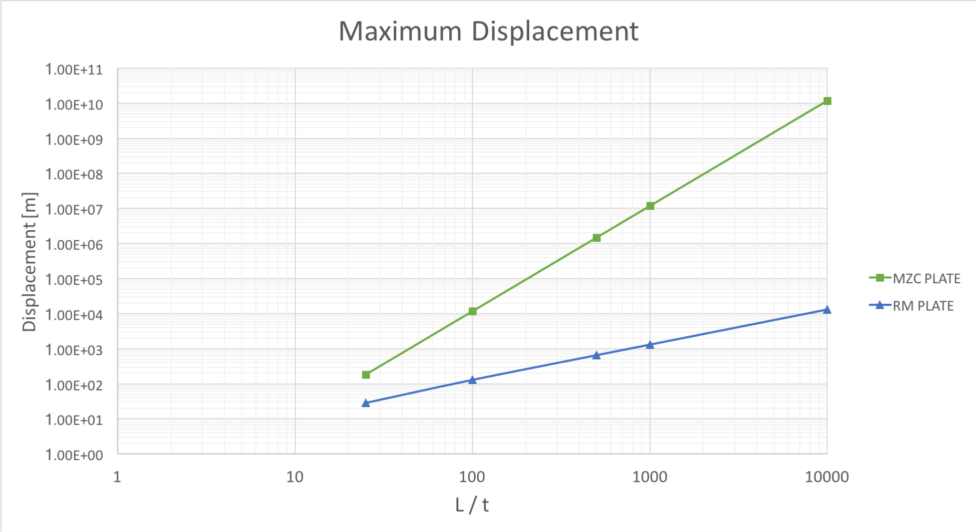


Figure 0.3: Displacement graph of the MZC and RM plates.

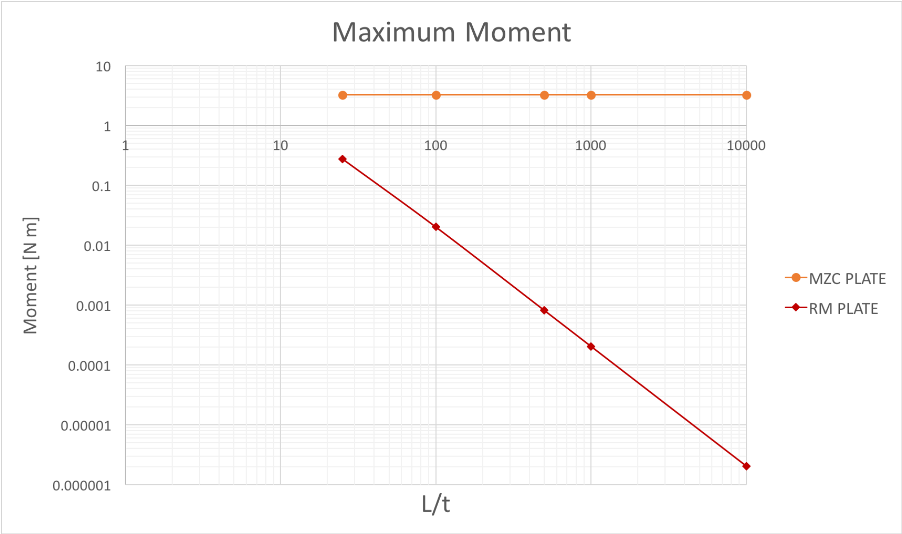


Figure 0.4: Moment graph of the MZC and RM plates.

b.*Define and verify a patch test mesh for the MCZ element.*
Solution:

As the theory explains, there are three modalities to prove the convergence of an element. The figure 0.5 shows the three ways to perform this test. The *test A* requires to prescribe all the nodes of a solid, then it is needed to verify $\mathbf{K}_{ij}\mathbf{a}_j^P - \mathbf{f}_i^P = 0$. The *test B*, only the values of $\mathbf{a}^P$ corresponding to the boundaries of the patch are inserted, and $\mathbf{a}_i$ is found as $\mathbf{a}_i = \mathbf{K}_{ij}^{-1}\left(\mathbf{f}_i^P - \mathbf{K}_{ij}\mathbf{a}_j^P\right)$ for $i \neq j$, and then compare this value with the exact one. Satisfaction of patch test A and B is *necessary condition* for convergence. The *test C* requires to fix the minimum number of displacements necessary to eliminate the rigid body motion solving $\mathbf{Ka} = \mathbf{f}^P$ where $\mathbf{f}^P$ represents prescribed boundary forces. This test helps to detect singularities.

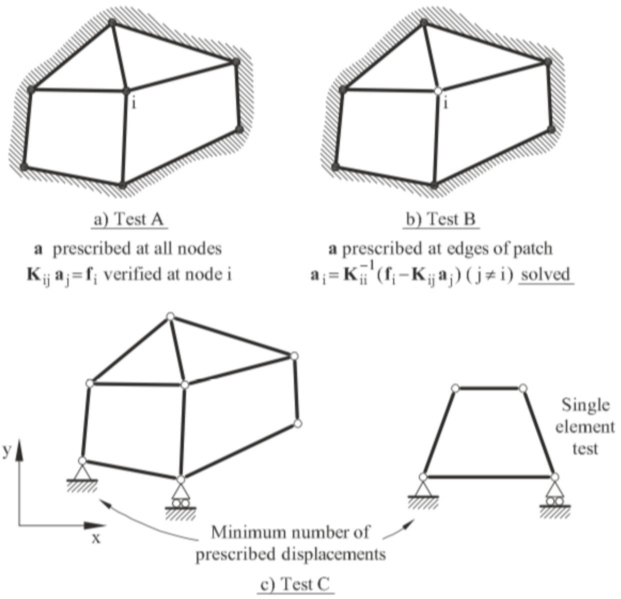


Figure 0.5: Three modalities of Patch Test.

The patch test done in this work is the *Test B*. Then a plate is shown in figure 0.6, considering a coarse structured mesh of 4 MZC elements. Moreover, a prescribed linear displacement field is imposed in the boundaries with the form:

$$w = c - ax - by \tag{0.1}$$

Where a, b and c are arbitrary numbers. If choosing $a = 0.03m$, $b = 0.05m$, and $c = 0.07m$, then the linear field imposed in the boundary nodes, and the *free node 4* displacement, is shown in the table 0.1.

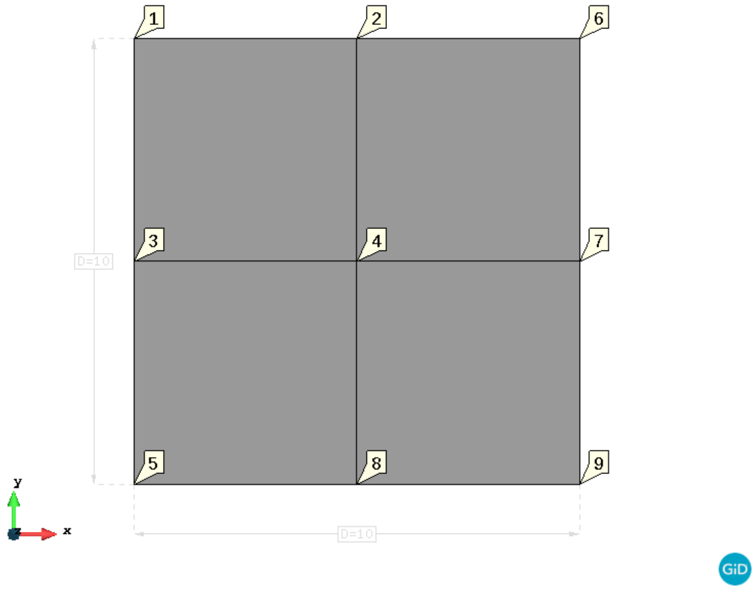


Figure 0.6: Plate with 4 structured MZC elements.

Solving the system of equations, the result of the displacement field in the domain of the element is shown in the figure 0.7a, after the computation, the free node displacement is exactly the same as previously computed in the table 0.1. Then, **the patch tests is passed.**

Now in order to give a counterexample of the MZC plate approximation, a quadratic displacement field can be imposed. As the theory explains, it is expected that the free node can not approximate the exact value. Then a new table (0.2) is computed using:

$$w = c - ax^3 - by^2 + xy \tag{0.2}$$

The results of the FEM analysis is shown in the figure 0.7b, where it is noticeable that there is an approximation error in the central node, which it was supposed to have $-1.93\ m$ of displacement.

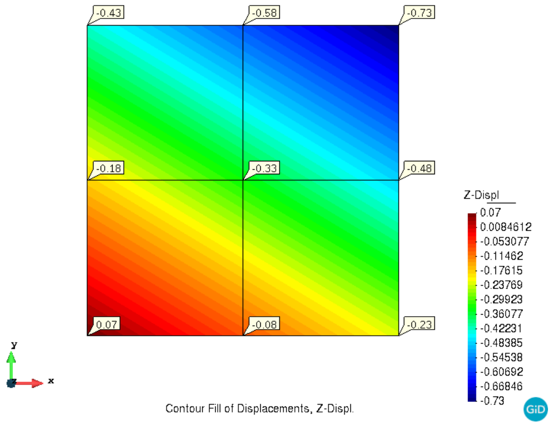
Node	X	Y	W
1	0	10	-0.43
2	5	10	-0.58
3	0	5	-0.18
4	5	5	-0.33
5	0	0	0.07
6	10	10	-0.73
7	5	0	-0.08
8	10	5	-0.48
9	10	0	-0.23

Table 0.1: Linear displacement field imposed in the boundary of a plate, and solution in the 4th node.

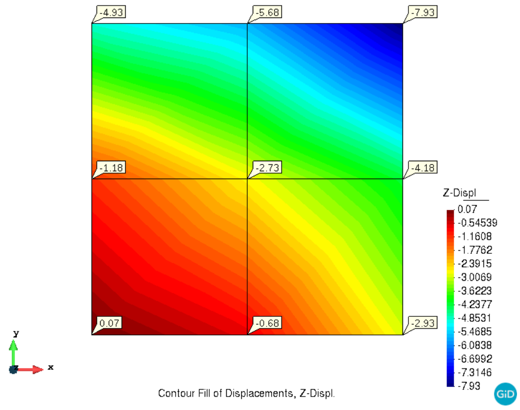
Node	X	Y	W
1	0	10	-4.93
2	5	10	-5.68
3	0	5	-1.18
4	5	5	-1.93
5	0	0	0.07
6	10	10	-7.93
7	5	0	-0.68
8	10	5	-4.18
9	10	0	-2.93

Table 0.2: Nonlinear displacement field imposed in the boundary of a plate, and solution in the 4th node.

Finally, another counterexample is proposed by using the same linear displacement field in the boundary nodes (0.1), but the geometry of the plate is not completely structured as the previous example. The geometry and the results are shown in the figure 0.8. It can be seen that there is a bigger error estimate of the central node by using this non-structured mesh, and it is because of the intrinsic formulation of the MZC element, and also during the running of the code there is a warning message when this type of geometries are intended to solve.

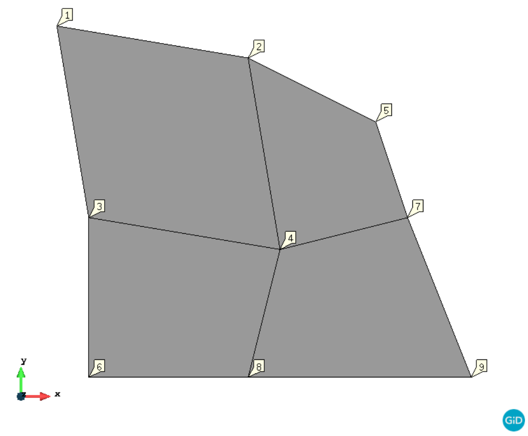


(a) Linear.

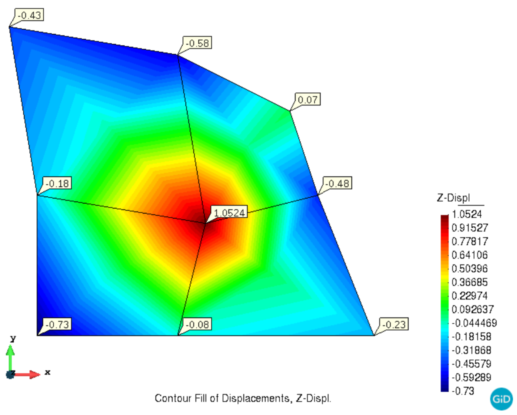


(b) Quadratic.

Figure 0.7: Results with a different displacement field imposed in the boundary



(a) Plate with 4 non structured MZC elements.



(b) Results of a linear displacement field imposed in the boundary .

Figure 0.8: Non-Structured Mesh.