Computational Structural Mechanics and Dynamics
Assignment 7 - Plates

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March 30th, 2020

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1 Comparison Between Reissner Mindlin and MZC Elements

The comparison between the two element types was performed using a 5x5 square plate, discretized into a mesh of 25 square first order elements, with the following configuration:

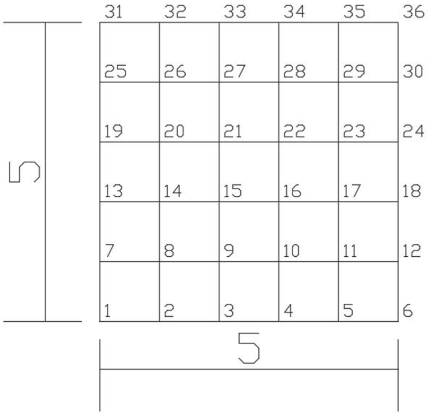

Figure 1: Mesh

The boundary conditions consist of fixing the vertical displacement degrees of freedom of all nodes at the boundary of the plate, while allowing free rotation. This BC configuration intends to simulate a simply supported plate. As for the loads, a uniform unitary load was applied to all elements.

Analysis for both types of plates was performed for all suggested plate thickness using the MatFEM plates Matlab script available at the CIMNE website. Since the mesh does not have a node at the center of the plate (where the largest displacements and moments should be), both displacements and moments were measured in **node 15** and taken as the maximum values. The following results were obtained.

1.1 Displacements

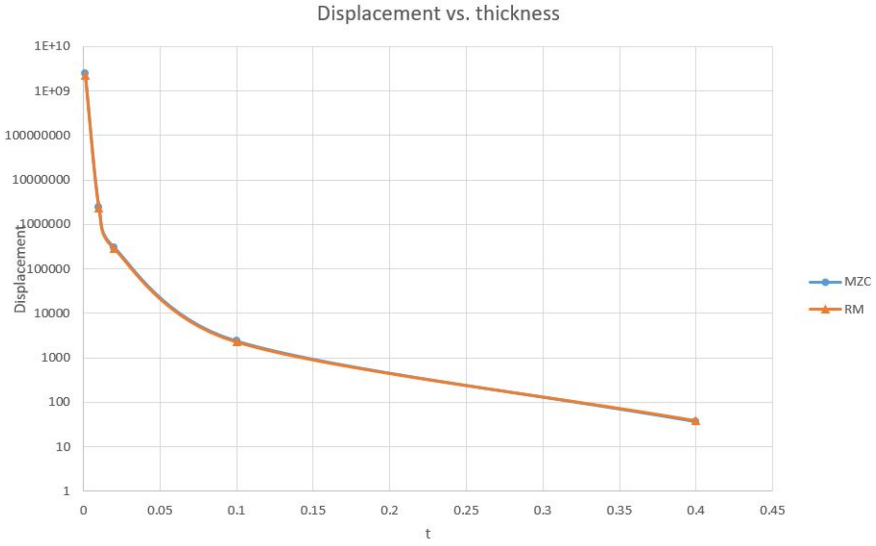

Figure 2: Max. Displacement vs. Plate thickness

Due to the large magnitude of the displacements (10^9 order), the difference between the results obtained with MZC and RM methods cannot be easily appreciated in the graphic despite the use of logarithmic scale. In order to make the difference visible, the results are also presented in the following table:

t	**MZC**	**RM**
0.001	2.4134e9	2.2760e9
0.01	2.4134e6	2.2762e6
0.02	3.0168e5	2.8461e5
0.1	2.4134e3	2.2958e3
0.4	3.771	3.9298

Table 1: Max. Displacements

We may then draw the following conclusions from Table 1:

- Analytically, the expression for deflection of a beam or plate depends on the inertia of the cross section of the structure (displacements decrease at a rate of 10^3 as thickness increases). The MZC model fulfills this relation since the maximum displacements for the plate with t=0.1 are exactly 10^3 times smaller than those of the plate with t=0.01 and 10^6 times smaller than those of the plate with t=0.001. However, this relation does not hold exactly for the RM model due to the shear blocking phenomenon.
- Due to the shear blocking effect, the results for displacement in the RM model are slightly smaller than the ones obtained for the MZC model. This difference is of about 5.6%, except for the case of the plates with t=0.4, where the shear blocking effect is reduced and the difference is of around 4.8%.

1.2 Bending Moments

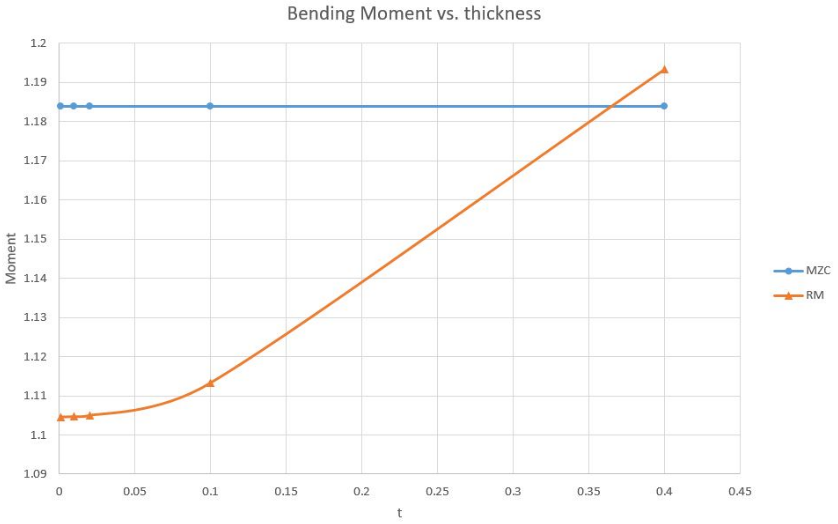

Figure 3: Max. Moment vs. Plate thickness

While the maximum moments obtained for the MZC model are constant for all plate thicknesses, the results for the RM vary for each analysis, becoming closer to the results of the MZC model as thickness becomes larger, due to a reduction of shear locking effect. Just like it was the case in the displacements analysis, we may observe that the moments of the two models become very similar for t=0.4.

2 Patch Test Implementation for MZC Element

According to Oñate (see reference 1), an alternative way to perform a patch test is to prescribe a known linear displacement field at the boundary nodes of the patch. The displacements at the interior nodes of the patch should coincide with the exact values of the boundary and hence, a constant strain field is obtained throughout the patch. In other words, the patch should displace as a rigid body.

Using the same mesh designed for Section 1, the following elements were selected as the patch for the test:

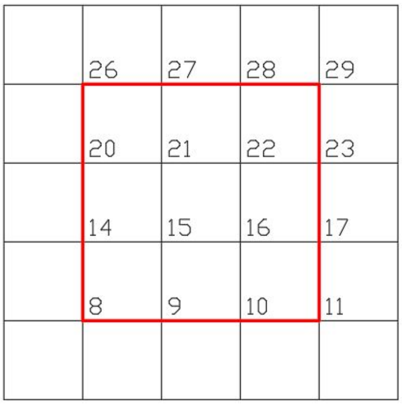

Figure 4: Patch

Applying the same Dirichlet boundary conditions used in section one and prescribing a unitary positive vertical displacement for the nodes at the boundary of the patch, while restraining both rotational degrees of freedom, we may perform a patch test. The passing criteria is therefore that the displacements of the nodes inside the patch (nodes 15, 16, 21 and 21) are identical to the one prescribed for the nodes at the boundary of the patch. Appendix 1 contains the input file written to simulate these conditions.

The displacements obtained for the inner patch nodes on the performed analysis are:

Node	w	θ_x	θ_y
15	1	-1.21417e-16	-1.56555e-16
16	1	-3.78102e-17	-2.68874e-16
21	1	-1.41744e-16	-3.33829e-17
22	1	2.68824e-17	5.09933e-17

Table 2: Patch Displacements

All vertical displacements are exactly equal to those at the boundary of the patch and rotations are equal to zero in any practical sense. Therefore, we may conclude that the patch is displacing as a rigid body and the patch test was successful.

3 References

[1] Oñate, E., Diez, P, Zárate, F, Larese, A. Introduction to the Finite Element Method. 2008

A Appendices

A.1 Patch Test Input file for $Plate\_MZC\_v1\_4$ MatFEM code

```
%
% Material Properties
%
 young  = 10.92 ;
 poiss  = 0.3 ;
 thick  = 0.001;
 denss  = 0.000000000 ;
%
% Coordinates
%
global coordinates
coordinates = [
    0.0,      0.0 ;
    1.0,      0.0 ;
    2.0,      0.0 ;
    3.0,      0.0 ;
    4.0,      0.0 ;
    5.0,      0.0 ;
    0.0,      1.0 ;
    1.0,      1.0 ;
    2.0,      1.0 ;
    3.0,      1.0 ;
    4.0,      1.0 ;
    5.0,      1.0 ;
    0.0,      2.0 ;
    1.0,      2.0 ;
    2.0,      2.0 ;
    3.0,      2.0 ;
    4.0,      2.0 ;
    5.0,      2.0 ;
    0.0,      3.0 ;
    1.0,      3.0 ;
    2.0,      3.0 ;
    3.0,      3.0 ;
    4.0,      3.0 ;
    5.0,      3.0 ;
    0.0,      4.0 ;
    1.0,      4.0 ;
    2.0,      4.0 ;
    3.0,      4.0 ;
    4.0,      4.0 ;
    5.0,      4.0 ;
    0.0,      5.0 ;
    1.0,      5.0 ;
    2.0,      5.0 ;
    3.0,      5.0 ;
    4.0,      5.0 ;
    5.0,      5.0];
```

```
%
% Elements
%
global elements
elements = [
      1,  2,  8,  7;
      2,  3,  9,  8;
      3,  4,  10,  9;
      4,  5,  11,  10;
      5,  6,  12,  11;
      7,  8,  14,  13;
      8,  9,  15,  14;
      9,  10,  16,  15;
      10,  11,  17,  16;
      11,  12,  18,  17;
      13,  14,  20,  19;
      14,  15,  21,  20;
      15,  16,  22,  21;
      16,  17,  23,  22;
      17,  18,  24,  23;
      19,  20,  26,  25;
      20,  21,  27,  26;
      21,  22,  28,  27;
      22,  23,  29,  28;
      23,  24,  30,  29;
      25,  26,  32,  31;
      26,  27,  33,  32;
      27,  28,  34,  33;
      28,  29,  35,  34;
      29,  30,  36,  35];
%
%
global fixnodes
%
% Fixed Nodes
%
fixnodes  = [
    1, 1,  0.0;
    2, 1,  0.0;
    3, 1,  0.0;
    4, 1,  0.0;
    5, 1,  0.0;
    6, 1,  0.0;
    7, 1,  0.0;
    12, 1,  0.0;
    13, 1,  0.0;
    18, 1,  0.0;
    19, 1,  0.0;
    24, 1,  0.0;
    25, 1,  0.0;
    30, 1,  0.0;
    31, 1,  0.0;
    32, 1,  0.0;
    33, 1,  0.0;
    34, 1,  0.0;
    35, 1,  0.0;
    36, 1,  0.0
    8, 1,  1;       % patch test bcs
    9, 1,  1;
    10, 1,  1;
```

```
    11, 1,  1;
    14, 1,  1;
    17, 1,  1;
    20, 1,  1;
    23, 1,  1;
    26, 1,  1;
    27, 1,  1;
    28, 1,  1;
    29, 1,  1;
    8, 2,  0;
    9, 2,  0;
    10, 2,  0;
    11, 2,  0;
    14, 2,  0;
    17, 2,  0;
    20, 2,  0;
    23, 2,  0;
    26, 2,  0;
    27, 2,  0;
    28, 2,  0;
    29, 2,  0
    8, 3,  0;
    9, 3,  0;
    10, 3,  0;
    11, 3,  0;
    14, 3,  0;
    17, 3,  0;
    20, 3,  0;
    23, 3,  0;
    26, 3,  0;
    27, 3,  0;
    28, 3,  0;
    29, 3,  0];

%
% Point loads
%
pointload = [ ];
%
uniload = sparse(length(elements),1);
```