Computational Structural Mechanics and Dynamics - Assignment 7 Plates

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1 Introduction

This report describes the solution of the assignment concerning Plate theory in the subject Computational Structural Mechanics and Dynamics.

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2 Exercise 1

2.1 A1

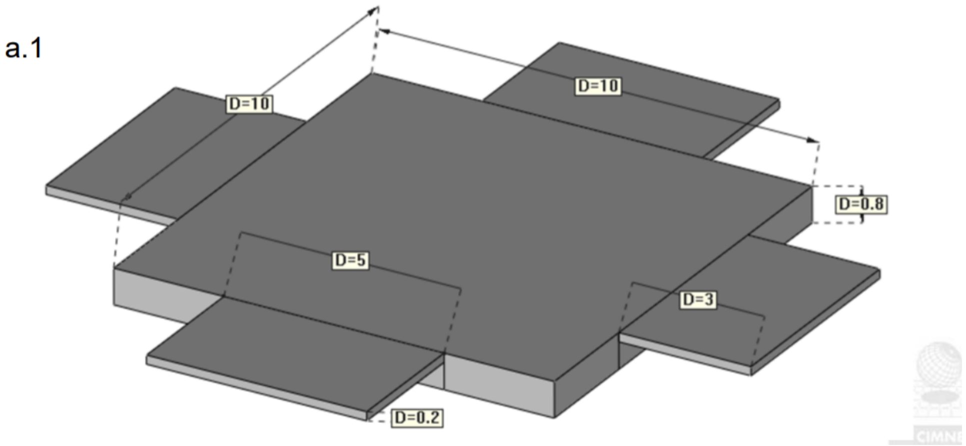

Figure 1: Plate given in the exercise with two different plate types not with corresponding midplanes.

As we could see from the figure above, the small plates connected to the big main plate, is placed in the way that both plates have corresponding height of their tops. This means that the position of the midplane for the small plates does not correspond with the position of the midplane for the big plate. Therefore, in this case, it is not possible to use plates FEM to analyze the plate. To analyze this combination of plates we would have to use 3D elements.

2.2 A2

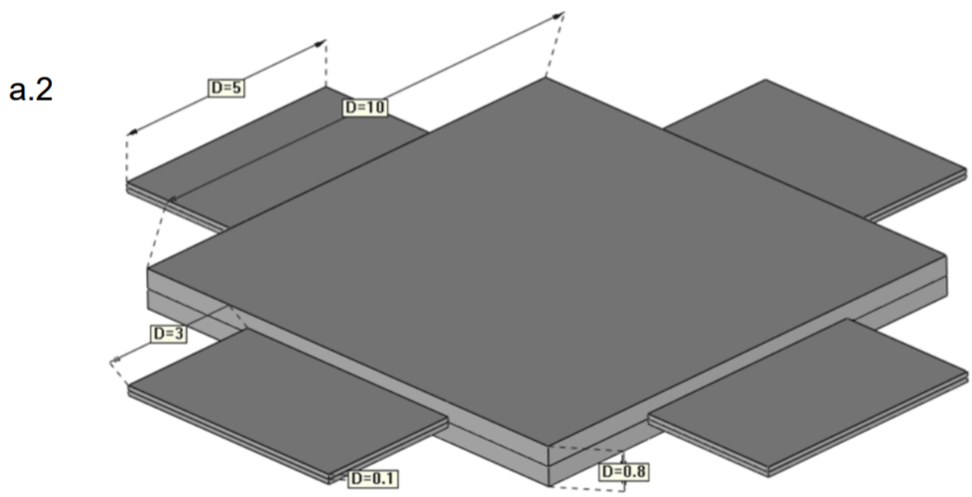

Figure 2: Plate given in the exercise with two different plate types with corresponding midplanse.

For this plate we observe that the small plates and the big plate have corresponding midplanes. This means that we could use the theory learned in class. The classical Kirchoff plate theory holds for thin plates, where the plate is thin if the ratio of $\frac{Thickness}{Width} \leq 0.10$. For the given platen the ratio is less than 0.10, and we could have used the Kirchoff theory to analyze this plate. At the same time, Reissner-Mindlin plate theory holds for both thick and thin plates, but the Kirchoff is generally the most used when the plate is considered thin.

One important observation of this is plate is that we also could use symmetry to solve the problem. Thereby, simplifying the calculations and saving machine power. When considering which type of element to be used to analyze the plate, it is suitable to use a kind of element type that provides a good enough solution with a small computation time. In this case, we could use the simplest element type with 4 integration type.

Regarding the boundary conditions, we do not have enough information to prescribe it for the problem. However, it is important to set common lines in the edges between the plates where the plates merge together to ensure compatibility of displacement.

3 Exercise 2

To define and verify the patch test for both the MCZ element and the RM element it was analyzed a quadratic clamped plate with a uniform load. The displacement of the center of a clamped plate submitted to an uniform load is given by the equation.

$$\delta = \frac{\alpha q L^4}{D} \tag{1}$$

Where for this case, $\alpha = 0.0012653$ q = -1, L = 10 and $D = \frac{Et^3}{12(1-\nu^2)}$. As we would do the patch test for a thin and thick plate to check for both elements, it is only the value of D that will differ. The patch test is verified by checking two conditions. Firstly, checking the satisfaction of rigid body by imposing a nodal displacement field corresponding to zero strain value. This was done by imposing a vertical displacement for all nodes on the boundary, and after analyzing the plate check if the total displacement of the center node was correctly according to load on the plate and the imposed displacement.

Secondly, it was checked for convergence when increasing the amount of elements. The convergence was checked by comparing the solutions when refining the mesh with the analytic one.

3.1 MCZ element

To analyze the MCZ element we will use a thickness equal to 0.01. Therefore, when calculating the displacement of the center node analytic, using Equation 1, we obtain:

$\delta = -1.2653E + 07$

Below is the table comparing the values for the analyzed plate without and with an imposed displacement on the outer nodes. In this case, it was used an imposed displacement equal to -100000.

Table 1: Table showing the displacement in the center node together with the relative error compared to the analytic solution. Also the displacement in the center is showed after given the boundaries an imposed displacement.

Elements	Nodes	Disp. in center node	Relative error	With imposed disp.
4	9	-1.5625E+07	23.49%	-1.6625E+07
16	25	-1.4077E+07	11.25%	-1.4177E+07
36	49	-1.4015E+07	10.76%	-1.4115E+07
64	81	-1.3054E+07	3.08%	-1.3154E+07
100	121	-1.2904E+07	1.98%	-1.2804E+07

As we could see from the table, the displacement of the center node, after imposing a displacement of the boundary nodes, is exactly -100000 more than the calculated displacement when not imposing the displacement. This means that the rigid body condition is satisfied. As well we see that the calculated displacements converge when increasing the amounts of elements. This is also shown in the table below.

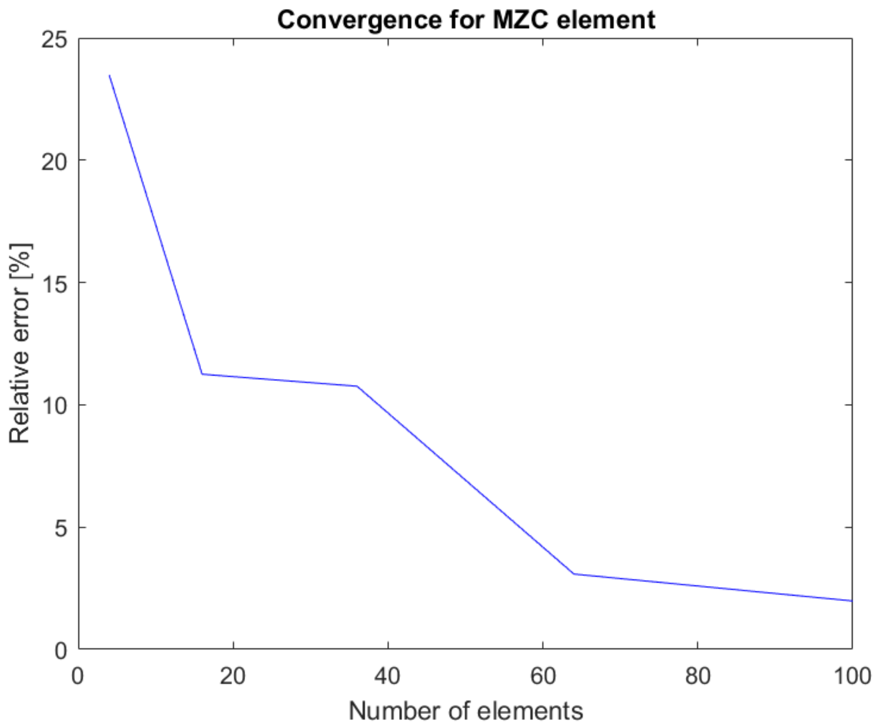

Figure 3: The convergence for the MZC element when analyzing a clamped plate and increasing the amount of elements.

3.2 RM element

To analyze the RM element we will use a thickness equal to 1. Therefore, when calculation the displacement of the center node analytic, using Equation 1, we obtain:

$\delta = -12.653$

Below is the table comparing the values for the analyzed plate without and with an imposed displacement on the outer nodes. In this case, it was used an imposed displacement equal to -1.

Table 2: Table showing the displacement in the center node together with the relative error compared to the analytic solution. Also the displacement in the center is showed after given the boundaries an imposed displacement.

Elements	Nodes	Disp. in center node	Relative error	With imposed disp.
4	9	-2.6786	78.83%	-3.6786
16	25	-6.9897	44.76%	-7.9897
36	49	-9.6242	23.94%	-10.6242
64	81	-11.519	8.96%	-12.519
100	121	-12.560	0.74%	-13.560

As we could see from the table, the displacement of the center node, after imposing a displacement of the boundary nodes, is exactly -1 more than the calculated displacement when not imposing the displacement. This means that the rigid body condition is satisfied. As well we see that the calculated displacements converge when increasing the amounts of elements. This is also shown in the table below.

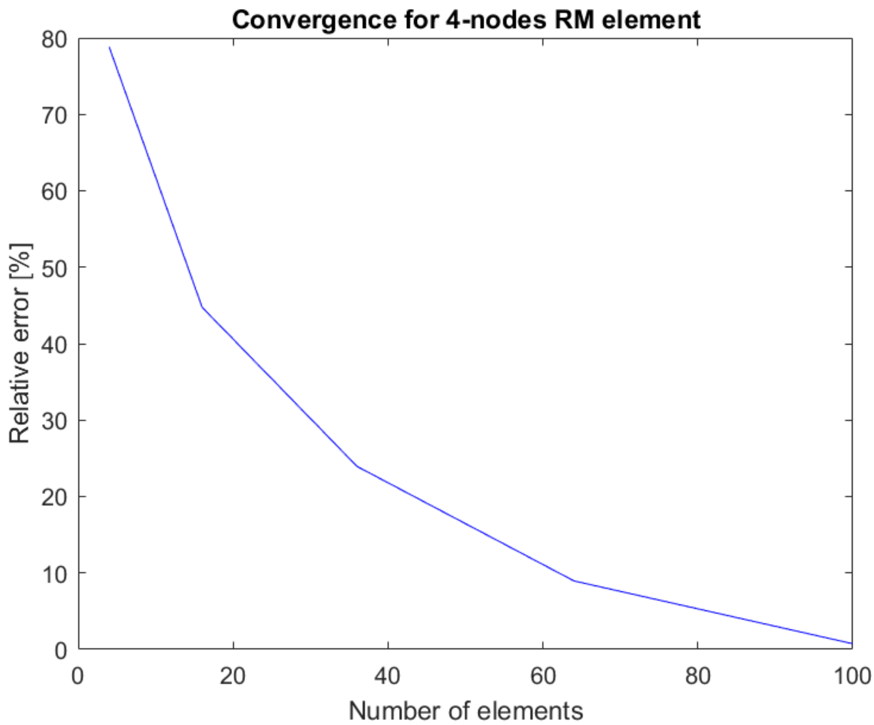

Figure 4: The convergence for the RM element when analyzing a clamped plate and increasing the amount of elements.