UNIVERSITAT POLITÈCNICA DE CATALUNYA, BARCELONA

MSc. COMPUTATIONAL MECHANICS ERASMUS MUNDUS

ASSIGNMENT 7: PLATES
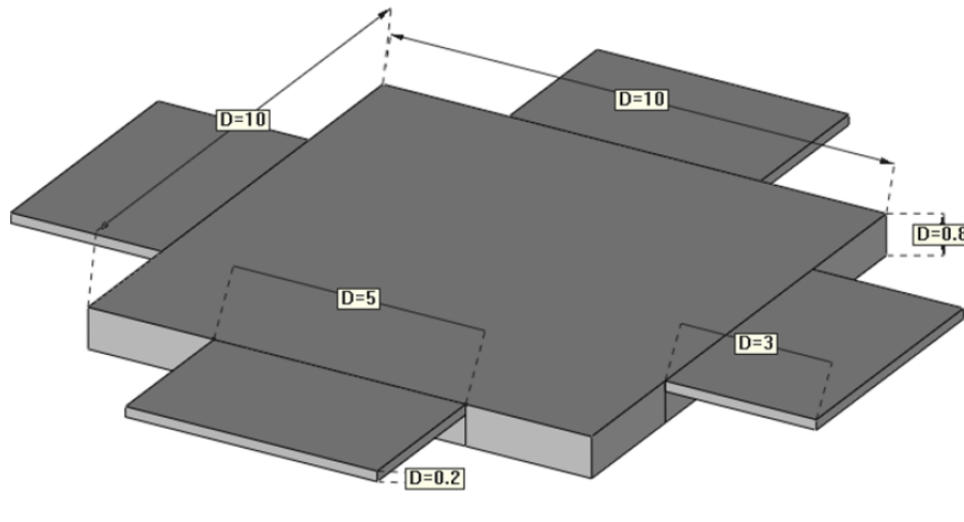
Computational Structural Mechanics & Dynamics

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a.1)

What kind of strategy (theory, elements, integration rule, boundary conditions, etc.) will you use for solving the following problem?

Figure 1: Problem 1

Solution:

The strategy suggested for the first problem is given below:

- **Theory:** Firstly, we check the thickness/average side ratio to see if we are dealing with thick or thin plates. We notice that the central plate dimensions are different than the other four connected plates.

$$\frac{t}{L} = \frac{0.8}{10} = 0.08 < 0.1 \Rightarrow \text{thin plate (central)}$$

$$\frac{t}{L} = \frac{0.2}{4} = 0.05 < 0.1 \Rightarrow \text{thin plate (sides)}$$

Since all of them fit into the criteria of thin plates, the Kirchhoff theory is valid. Although, the non-alignment of all the plates at the mid-plane surface may cause shear and moments which makes the use of Reissner-Mindlin theory a favourable assumption. Importantly, these shear forces and moments due to the mismatch of the mid-line surface of the plates should be taken into consideration while solving the problem as neglecting these may lead to design failures.

- **Elements:** The given geometry of the problem is simple and using quadrilateral elements is a fair choice. Due to complexities on the interface of all plates, we would prefer a higher order mesh for generating better results. To this regard, the use of Clough and Felippa elements is suggested.

- **Integration rule:** Using a higher order mesh for the problem leads us into using more integration points in the Gaussian quadrature which is governed by the order of the shape function polynomial used for the element.

- **Boundary conditions:** We assume that the plates on the sides require support and therefore a fixed displacement boundary condition needs to be applied at all ends. The difference in the mid-line surface of the plates leads us to consider the problem of the central and side plates separately and apply a boundary condition at all the interfaces to curb the rigid body motion condition. A reasonable assumption would be to ensure the continuity of displacement and slope across the interfaces. Furthermore, due to the symmetric nature of the problem, we could analyse only a quarter of a problem and impose a symmetry condition on the mirror axes, leading to a reduction in computational cost, given as,

$$\theta_x = 0 \text{ on the y axis}; \qquad \theta_y = 0 \text{ on the x axis}$$

a.2)

What kind of strategy (theory, elements, integration rule, boundary conditions, etc.) will you use for solving the following problem?
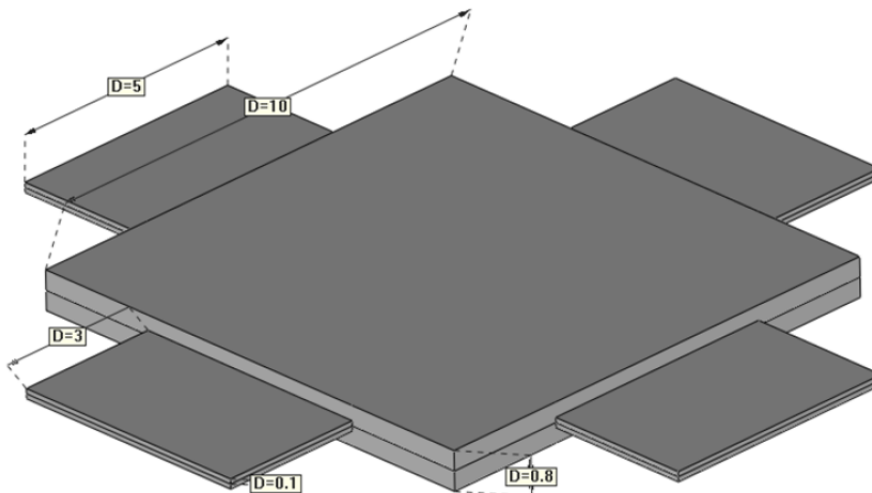

Figure 2: Problem 2

Solution:

The strategy suggested for the second problem is given below:

- **Theory:** Similar to the first problem, we fit into the criteria of thin plates, so the Kirchhoff theory is valid. Since all the plates are aligned at the mid-line surface, the problem is simplified and we do not encounter the effects considered in the previous problem.

- **Elements:** The uniformity of the geometry leads us to use quadrilateral elements. Since the problem is simple, using MCZ elements instead of higher order elements is a fair choice for saving computational cost.

- **Integration rule:** The decision of working with linear elements directs us to use a Gaussian quadrature with 4 points (2 in each direction).

- **Boundary conditions:** Similar to the last problem, we assume that the plates on the sides require support and therefore a fixed displacement boundary condition needs to be applied at all ends. The continuity condition is not needed because we are not considering different problems here. Although, due to the symmetric nature of the problem, we could again analyse only a quarter of a problem and impose a symmetry condition on the mirror axes.

b)

Define and verify a patch test mesh for the MCZ element. Discuss the results observed.

Solution:

To define a patch test for the MZC elements, we consider a test node surrounded by a patch of elements and impose a displacement field upon. An example [1] of a patch of elements in shown in Figure 3. This test is useful in assessing the stability of the finite element solution and also in providing a necessary and sufficient condition for convergence.
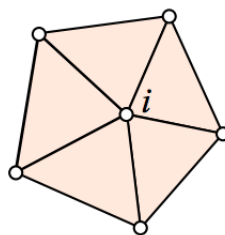
Figure 3: Example [1] of a patch of elements sourced from Colorado University Advanced FEM Chapter 15 - The Patch Test

The process involves prescribing the known condition on the external nodes and computing the solution at the internal nodes numerically. The condition can be displacement, strain, forces. Here we perform the Displacement Patch Test (DPT) using the FE code provided. The numerical solution is then compared with the analytical result for the internal node using the expression of the imposed displacement field. Figure 4 shows an example [1] setup for a Displacement Patch Test (DPT) used in Finite Element Analysis.
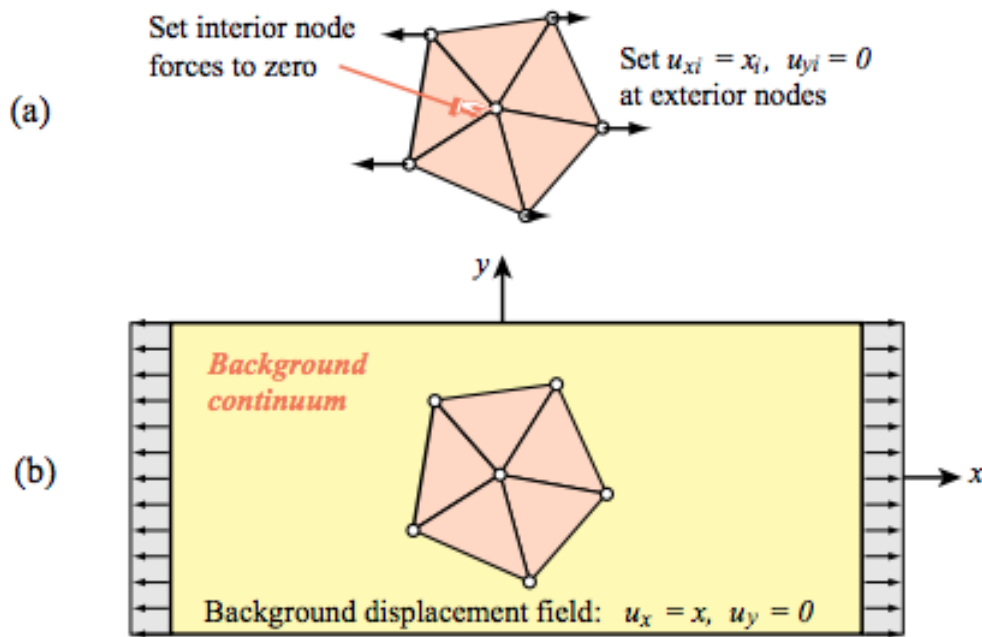
Figure 4: Example [1] setup of a Displacement Patch Test (DPT) sourced from Colorado University Advanced FEM Chapter 15 - The Patch Test

In this problem, we perform a patch test using MCZ elements for two cases of applied displacement field.

Case 1: Firstly, a linear displacement field is applied of the form,

$$w(x, y) = a_1 + a_2 x + a_3 y$$

with the rotations, θ_x and θ_y about the x and y axis respectively given as,

$$\frac{\partial w(x, y)}{\partial x} = \theta_x = a_2; \qquad \frac{\partial w(x, y)}{\partial y} = \theta_y = a_3$$

Case 2: Next, for the second case a quadratic displacement field is considered of the form,

$$w(x, y) = 1 + x + y + xy + x^2 + y^2$$

and rotations given as,

$$\theta_x = \frac{\partial w(x, y)}{\partial x} = 1 + 2x + y; \qquad \theta_y = \frac{\partial w(x, y)}{\partial y} = 1 + 2y + x$$

In our analysis we consider a patch of 4 square elements defined by 9 nodes as shown in Figure 5,
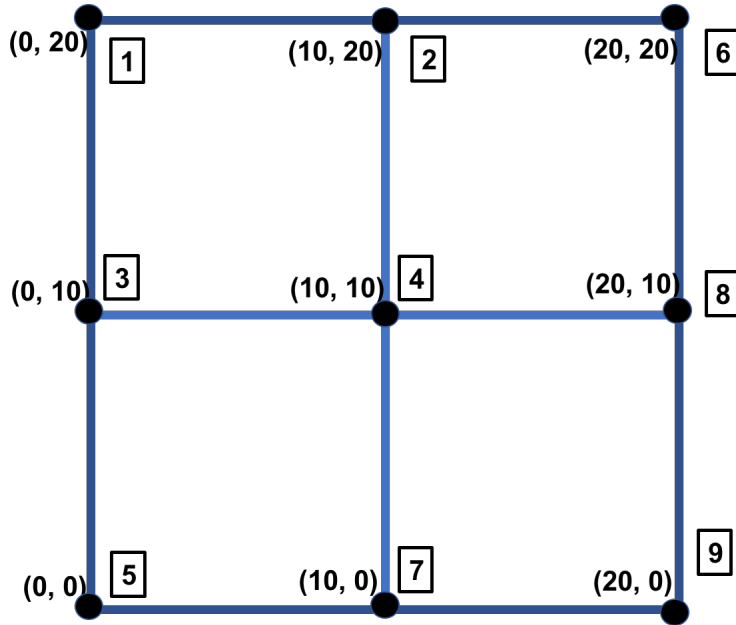

Figure 5: Considered patch of elements

The coordinates of these elements are given in Table 1.

Node #	Coordinates	
	x_i	y_i
1	0	20
2	10	20
3	0	10
4	10	10
5	0	0
6	20	20
7	10	0
8	20	10
9	20	0

Table 1: Coordinates of the nodes on the patch

Case 1: Linear displacement field
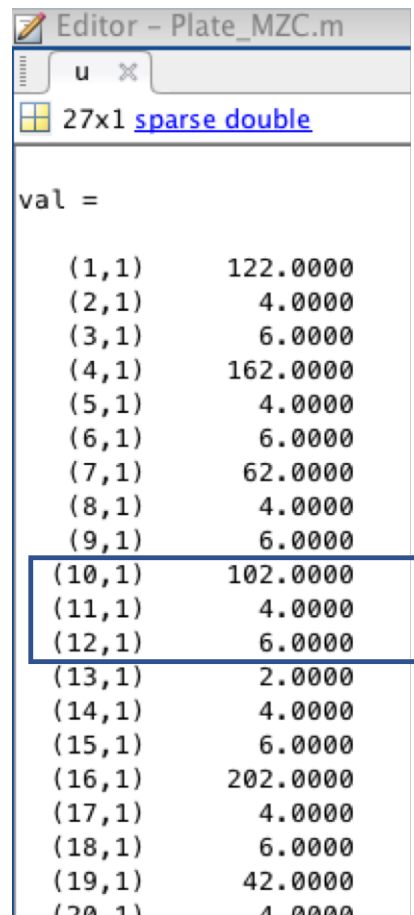
For the linear displacement field we impose,

$$w = 2 + 4x + 6y; \qquad \theta_x = 4; \qquad \theta_y = 6$$

which gives us the nodal values on the boundary as shown in Table 2,

Node	$w(x,y)$	θ_x	θ_y
1	122	4	6
2	162	4	6
3	62	4	6
5	2	4	6
6	202	4	6
7	42	4	6
8	142	4	6
9	82	4	6

Table 2: Nodal values imposed on the boundary for Case 1

Now, we use the FEM code provided to obtain the displacement and rotations at the interior node i.e. node number 4. Analytical solution for node 4 gives a value of $w = 102$, $\theta_x = 4$ and $\theta_y = 6$. The results obtained from the MATLAB file are shown in Figure 6.

Figure 6: Results: Case 1 - Linear displacement field for MCZ elements

The results show the expected values for node 4, thus verifying the stability of the MCZ mesh elements for a linear behaviour.
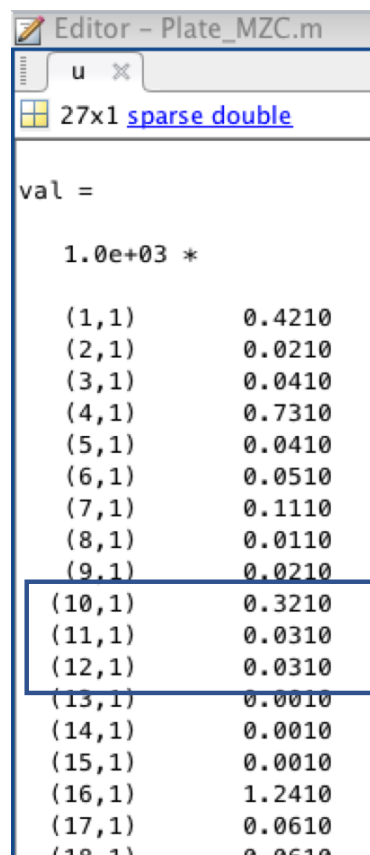
Case 2: Quadratic displacement field

The defined quadratic displacement field gives us the nodal values on the boundary as shown in Table 3,

Node	$w(x,y)$	θ_x	θ_y
1	421	21	41
2	731	41	51
3	111	11	21
5	1	1	1
6	1241	61	61
7	111	21	11
8	731	51	41
9	421	41	21

Table 3: Nodal values imposed on the boundary for Case 2

Similar to the last case, we compute the displacement and rotations of the internal node number 4. The expected values using for the defined quadratic displacement field are $w = 321$, $\theta_x = 31$ and $\theta_y = 31$. The results obtained using the provided code are shown in Figure 7.

Figure 7: Results: Case 2 - Quadratic displacement field for MCZ elements

It is seen from the results that the expected values for the internal node are exactly matching the numerical results. Hence, the patch test verifies that MCZ elements are stable with both linear and quadratic displacement fields.
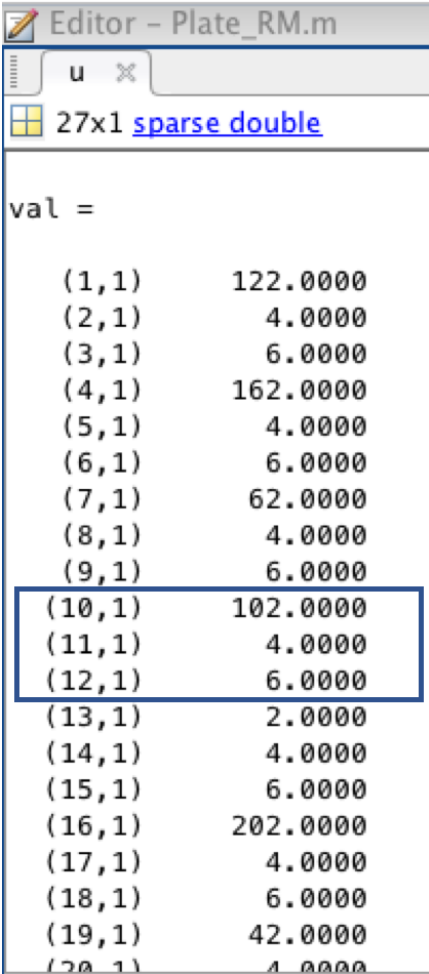
c)

Define and verify a patch test mesh for the RM element. Discuss the results observed.

Solution:

A similar analysis is performed for RM elements.

Case 3: Linear displacement field using RM elements

Now, we use the FEM code provided for RM elements to obtain the displacement and rotations at the interior node i.e. node number 4. Analytical solution for node 4 gives again a value of $w = 102$, $\theta_x = 4$ and $\theta_y = 6$. The results obtained from the MATLAB file for RM elements are shown in Figure 8.


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Editor – Plate_RM.m

  u   ✖

  27x1 sparse double


val =

     (1,1)        122.0000
     (2,1)          4.0000
     (3,1)          6.0000
     (4,1)        162.0000
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     (7,1)         62.0000
     (8,1)          4.0000
     (9,1)          6.0000
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    (11,1)          4.0000
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    (13,1)          2.0000
    (14,1)          4.0000
    (15,1)          6.0000
    (16,1)        202.0000
    (17,1)          4.0000
    (18,1)          6.0000
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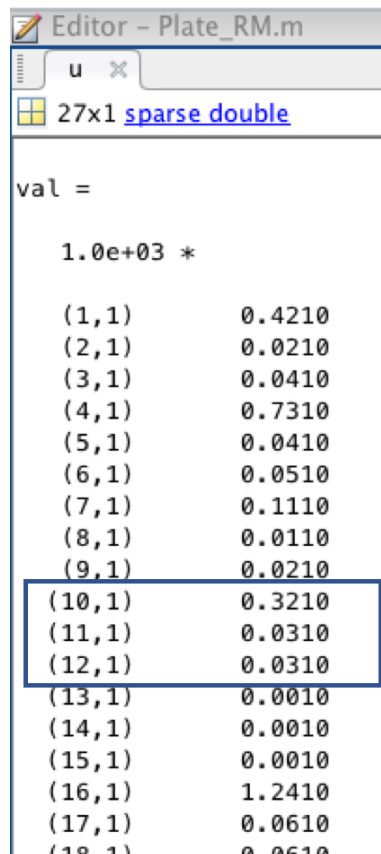
Figure 8: Results: Case 3 - Linear displacement field for RM elements

The results show the expected values for node 4, thus verifying the stability of the RM mesh elements for a linear behaviour.

Case 4: Quadratic displacement field using RM elements

Similar to the last case, we compute the displacement and rotations of the internal node number 4 for RM elements. The expected values using for the defined quadratic displacement field are $w = 321$, $\theta_x = 32$ and $\theta_y = 32$. The results obtained using the provided code for RM elements are shown in Figure 9.

Figure 9: Results: Case 4 - Quadratic displacement field for RM elements

It is seen from the results that the expected values for the internal node are exactly matching the numerical results. Hence, the patch test verifies that RM elements are stable with both linear and quadratic displacement fields.

References

[1] Colorado University Advanced FEM Chapter 15 The Patch Test. https://www.colorado.edu/engineering/CAS/courses.d/AFEM.d/AFEM.Ch15.d/AFEM.Ch15.pdf, Dec 2017. Accessed on 2018-29-03. pages 4, 5