Master on Numerical Methods in Engineering

Computational Structural Mechanics and Dynamics

Assignment 7

Bending of plates 2D

- Kirchhoff Plate Theory
- Reissner-Mindlin Plate Theory
- Shear locking effects
- Patch test

Use of Matlab and GiD software

Mónica Ortega Castro

Section A

Analyse the shear blocking effect on the Reissner-Mindlin element and compare with the MZC element. For the Simple Support Uniform Load square plate.

Use the 5x5 Mesh.

E= 10.92
ν= 0,3
Q = 1.0

t = 0,001

t = 0,010

t = 0,020

t = 0,100

t = 0,400

Discuses the results observed.

Shear locking effects

The aim of this section is to evaluate the effect of the Ks matrix when using the RM element in comparison with the MZC element. Problem is solved using the full integration.

Shear blocking/locking effect occurs due to thickness impact on the inertial moment (I). Even when the most important and high value moment is the bending moment, thickness could drastically impact on the total moment by increasing the shear momentum value. It is solved by reduced integration.

Problem data

```
%SIDE LENGTH L=10m

% Material Properties
%
 young  = 10.92 ;
 poiss  = 0.3 ;
 thick  = 0.001;
 denss  = 1.000000000 ;
%
% Coordinates
%
global coordinates
coordinates = [
    10, 0;
     8, 0;
    10,2;
    8,2;
…
%
% Elements
%
global elements
elements = [
    26, 17, 18, 27;
    17, 11, 13, 18;
    11,  6,  7, 13;
     6,  2,  4,  7;
     2,  1,  3,  4; %element 5
    27, 18, 22, 29;
…

global fixdesp
```

```
%
% Fixed Nodes
%
fixdesp  = [
   26, 1,  0.0;
   26, 2,  0.0;
   26, 3,  0.0;
   17, 1,  0.0;
   17, 2,  0.0;
   17, 3,  0.0;
   11, 1,  0.0;
   11, 2,  0.0;
   11, 3,  0.0;
    6, 1,  0.0;
    6, 2,  0.0;
    6, 3,  0.0;
…
```

Pre-processing

Mesh was defined as follow:

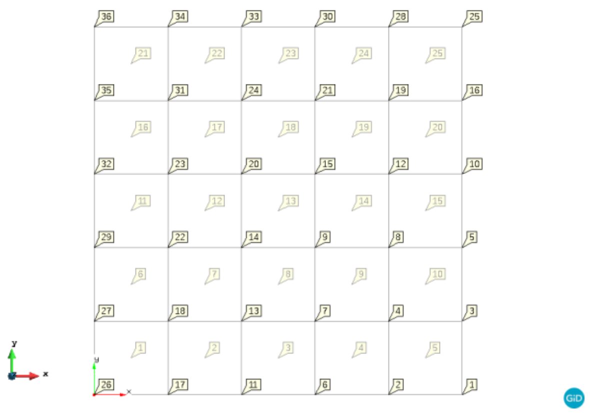


Figure 1. Mesh definition

Post-processing – Elements comparison

Maximum displacement (scalar values) from each t_i simulation as well as for element type were gathered in Table 1.

THICKNESS (T)	L/T	MZC element (y axis)	RM element (z axis)
0,001	**10000**	2,5872E+10	2,3289E+04
0,01	**1000**	2,5752E+10	2,3288E+03
0,02	**500**	3,2340E+06	1,1642E+03
0,1	**100**	2,5828E+04	2,3133E+02
0,4	**25**	4,0425E+02	5,2600E+01

Table 1. Maximum displacement comparison

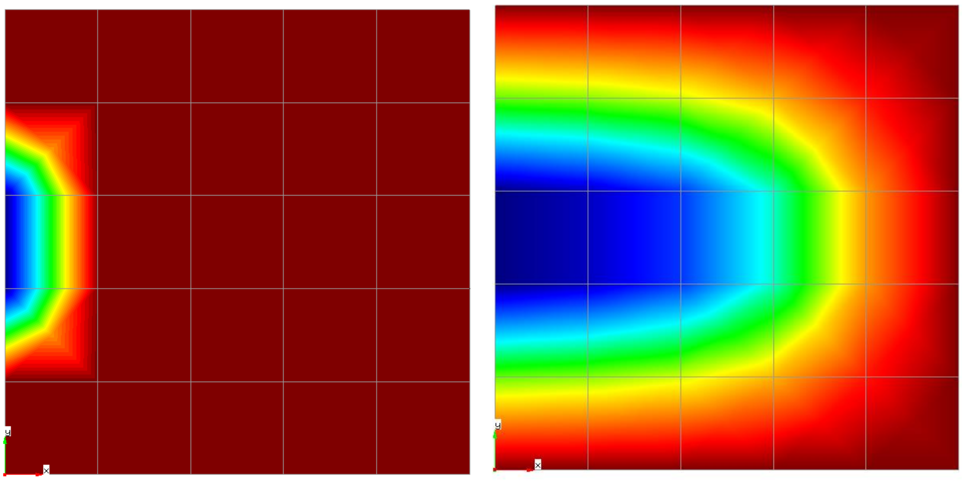


Figure 2 Displacements for MZC (left) and RM (right) elements for t=0.010 – Result surface view

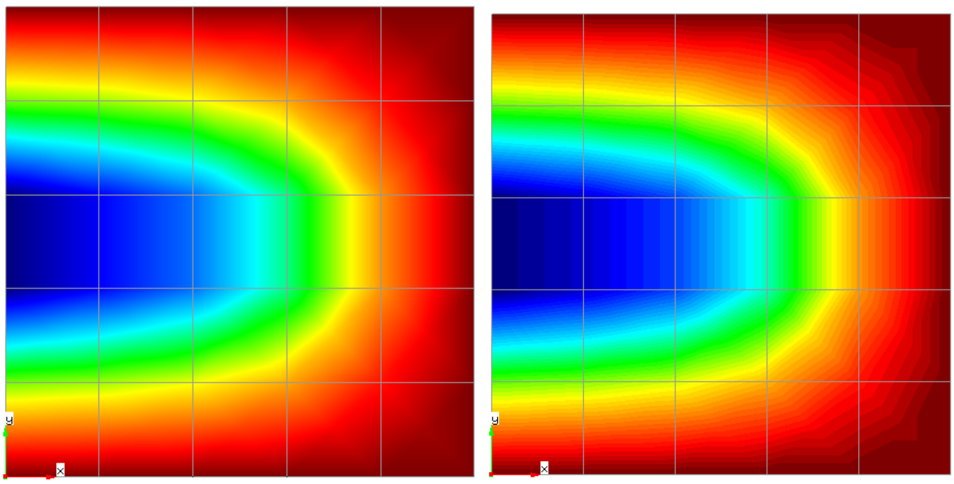


Figure 3 Displacements for MZC (left) and RM (right) elements for t=0.400 – Result surface view

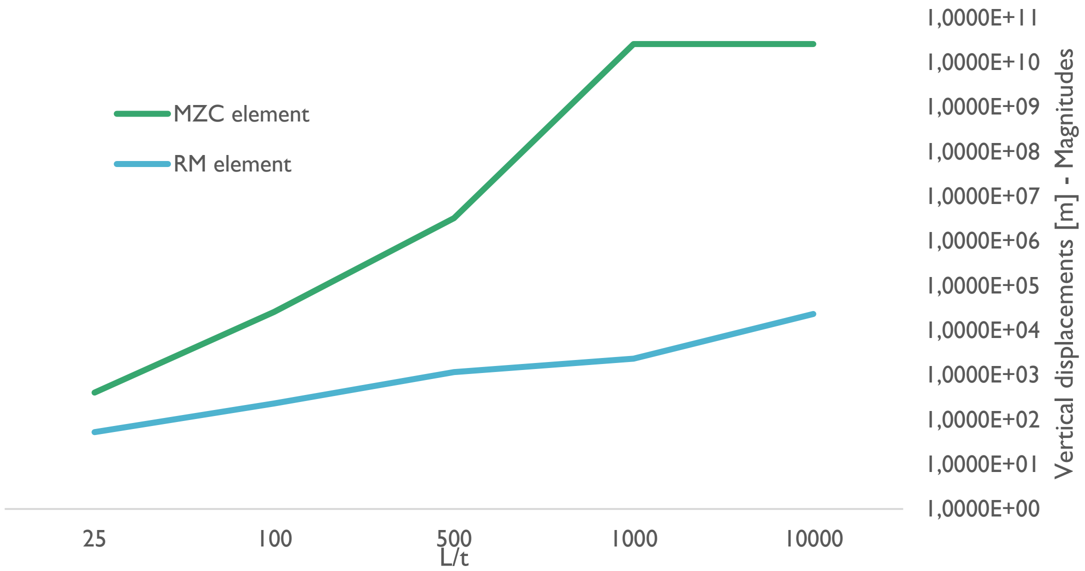


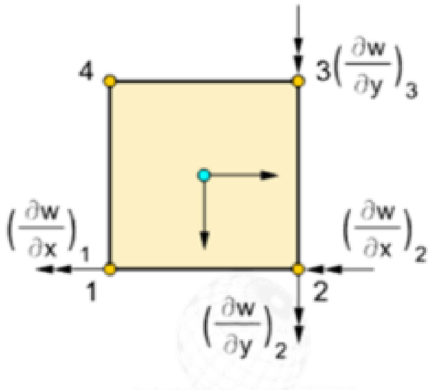
Figure 4. Maximum displacement – Comparison between elements

Given a thickness (t), Figure 2 to 4 highlight the significant difference of displacement that occur. The greater the thickness, the smaller the displacement difference. RM method adds an artificial stiffness that decreases the displacement values. A solution could be to apply the reduced integration to the RM theory. Under the current conditions, RM method is not valid for thin plates but gives reasonable results for thick plates.

Section B

Define and verify a patch test mesh for the MCZ element.

Patch test is a common problem when using MZC elements. It happens when incompatibility of the rotation fields appears.



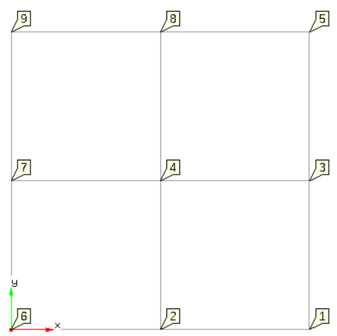
This is, when:

$$\left(\frac{d^2w}{\partial x\partial y}\right) \neq \left(\frac{d^2w}{\partial y\partial x}\right) \; at\ a\ node$$

Elements that violate this continuity condition are denominated incompatible or non-conforming elements.

Patch test verification:

4 4-noded quadrilateral elements will be tested. Linear displacement will be imposed in the boundary node (all nodes except number 4). Equation 1, 2 and 3 defined the imposted values for the fixed points.



Linear equation:

$$w = 0,3 + 0,04 * x - 0,02 * y \quad [1]$$

$$\frac{dw}{dx} = 0,04 \qquad [2]$$

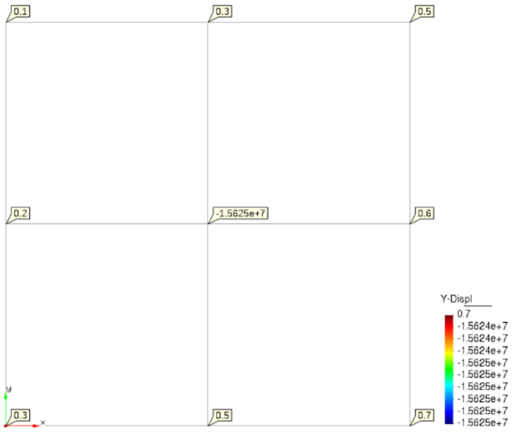
$$\frac{dw}{dy} = -0,02 \qquad [3]$$

		Node number								
		1	2	3	4	5	6	7	8	9
Coordinates	x	10	5	10	5	10	0	0	5	0
	y	0	0	5	5	10	0	5	10	10
Assigned constraints (except for node 4)	w	0,7	0,5	0,6	0,4	0,5	0,3	0,2	0,3	0,1
	$\frac{dw}{dx}$	0,04	0,04	0,04	0,04	0,04	0,04	0,04	0,04	0,04
	$\frac{dw}{dy}$	-0,02	-0,02	-0,02	-0,02	-0,02	-0,02	-0,02	-0,02	-0,02

Table 2. Displacement and rotation values for each node

Yellow cells from table 2 are the values that node 4 should take when running the simulation. However, there is a mistake in the script that is not solved due to lack of time.

Following code is an extraction of the problem description for t=0.010




```
global coordinates
coordinates = [
10, 0;
5,0;
10,5;
5,5;
10,10;
0,0;
0,5;
5,10;
0,10;%node
];

%
% Elements
%
global elements
elements = [
     6,   2,   4,   7;
     2,   1,   3,   4;
     7,   4,   8,   9;
     4,   3,   5,   8;
];
 %
fixdesp
%
global fixdesp
%
% Fixed Nodes
%
fixdesp  = [
     1, 1,  0.7;
    1, 2,  0.04;
    1, 3,  -0.02;
    2, 1,  0.5;
    2, 2,  0.04;
    2, 3,  -0.02;
    3, 1,  0.6;
    3, 2,  0.04;
    3, 3,  -0.02;
    5, 1,  0.5;
    5, 2,  0.04;
    5, 3,  -0.02;
    6, 1,  0.3;
    …
```