Computational Structural Mechanics and Dynamics
MSc in Computational Mechanics
Universitat Politecnica de Catalunya

Assignment 7
Plates

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Contents

1. Abstract

In this report it is going to be analyzed the difference between two types of element in order to study the behaviours of thick and thin plates. It will be briefly introduced the difference between two main theories about plates and the respective element representation. Then the same plates will be tested for different values of thickness in order to better see the difference. Finally it will be launched a patch test.

a) Analyze the shear blocking effect on the Reissner Mindlin element and compare with the MZC element. For the Simple Support Uniform Load square plate. Use the 5x5 Mesh.

b) Define and verify a patch test mesh for the MCZ element.

2. Introduction

In order to study the bending of a plate, due to a load normal to its surface, there are two main theories referring to the thickness of the plate. The first one is mainly for thin plates, when the ratio $thickness/width < 0.1$ and is the classical Kirchoff theory. While the second theory; the Reissner-Mindlin (RM) plate theory, holds for both thick and thin plates but it has problems when the thickness $\rightarrow 0$. This is because of the shear locking effects, and it is due to the different assumptions done for each theory.

The difference here is similar to the one present in the beams bending theory. Both of the theories, the Kirchoff and the RM, agree with the three first main assumption that states the σ_z is negligible, the points along the normal to the middle plane have the same displacements and they move only vertically. The difference is in the last assumptions, in which for the Kirchoff theory, based on thin plate, states that points after deformation remain on straight lines orthogonal to the middle plane before the deformation. On the other hand, the RM theory, being based on thick plates, states that this condition can't be assumed. This will lead to a ϕ angle definition on the rotation field. This will generate the creation of two stiffness matrices, the bending stiffness matrix and the shear stiffness matrix, proportional respectively to t^3 an t. This will lead to the shear locking effect when $t \rightarrow 0$ as the bending matrix will tend to zero, resulting in a loss of accuracy.

In order to run the simulations and test the two different types of elements, the following data have been chosen:

$$\begin{aligned} E &= 10.92 \\ \nu &= 0.3 \\ Q &= 1 \end{aligned} \tag{2.1}$$

and five different values of thickness:

$$\begin{aligned} t &= 0.001 \\ t &= 0.01 \\ t &= 0.02 \\ t &= 0.1 \\ t &= 0.4 \end{aligned} \tag{2.2}$$

3. Part a)

As requested in 1, the considered plate will be a $5x5$ square plate, defined in Figure 3.1.

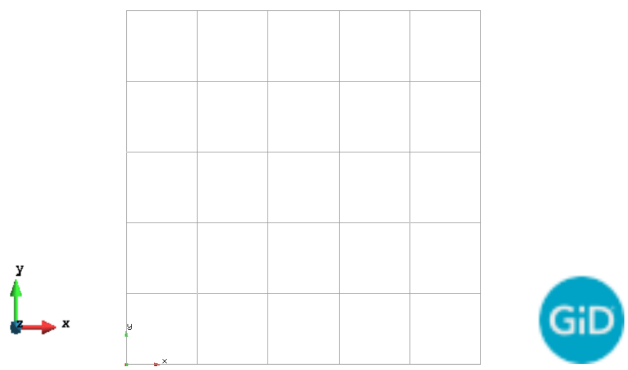

Figure 3.1: Mesh plate 5x5

In Figure 3.2 are reported the displacements of both elements MZC and MR, while changing the thickness. As can be noticed, it is clear the problem of the MR element when $t \to 0$. In order to better show the behaviours of both element types, it has been reported the plot in a logarithmic scale.

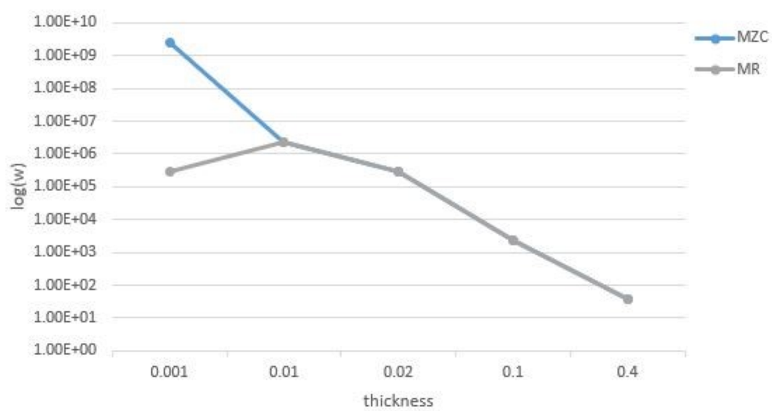

Figure 3.2: w-thickness

As can be noticed, for $t = 0.001$ the displacements is smaller than for a greater thickness. This doesn't have much sense as it is expected a greater displacement. In fact, the problem comes from the shear locking effect that doesn't take in account (for very thin plates) the stiffness bending matrix as explained in Chapter 2.

In Figure 3.3 are reported the values for the moment in function of the thickness changing.

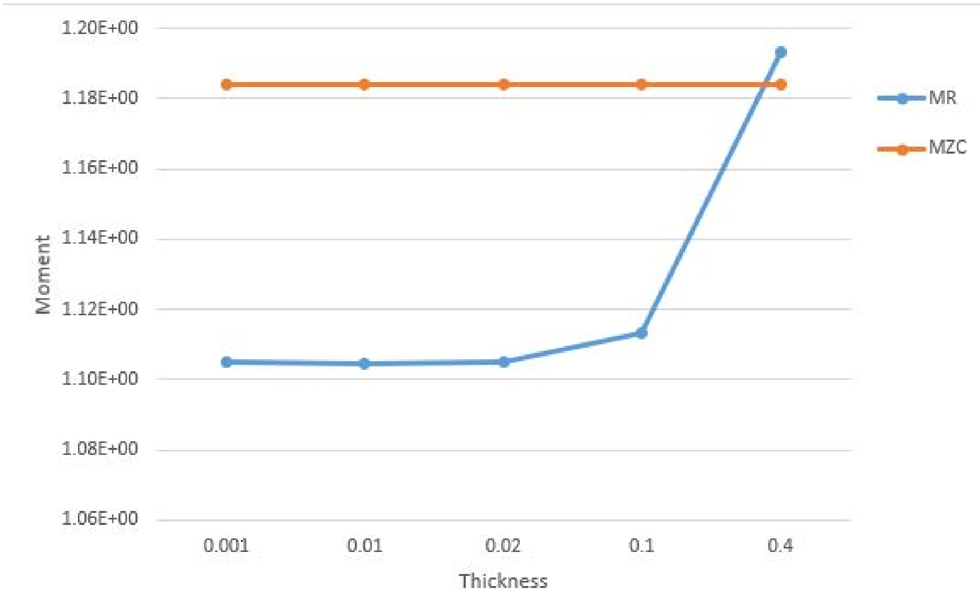

Figure 3.3: moment-thickness

It can be appreciated the implementation of the shear stiffness matrix. As can be noticed, for the MZC element, the moment is constant for every thickness. That's because, considering thin plates and the Kirchoff theory, the thickness doesn't have an important role in computing the bending moment. In fact, as explained before (2), in the MZC elements there is not a source of shear stresses. On the other hand, while increasing the thickness of the plate, the shear stresses start having an important role, determining in fact, the increasing of the bending moment as the thickness of the plate becomes bigger.

It is reported only the moment on one axis, as due to the symmetry of the plate, the two moments are the same.

4. Part b)

For the MZC element there are some problems. While choosing the rotation as a nodal variable, it doesn't guarantee the continuity of the rotation field along the boundaries. To solve this problem, there is the patch test. If the element satisfies the test, the solution will converge to the exact one.

The patch test consists in imposing to the patch boundaries (usually the elements close to the boundaries as in Figure 4.1 defined by the red dots) a displacement field that can be described by linear shape functions.

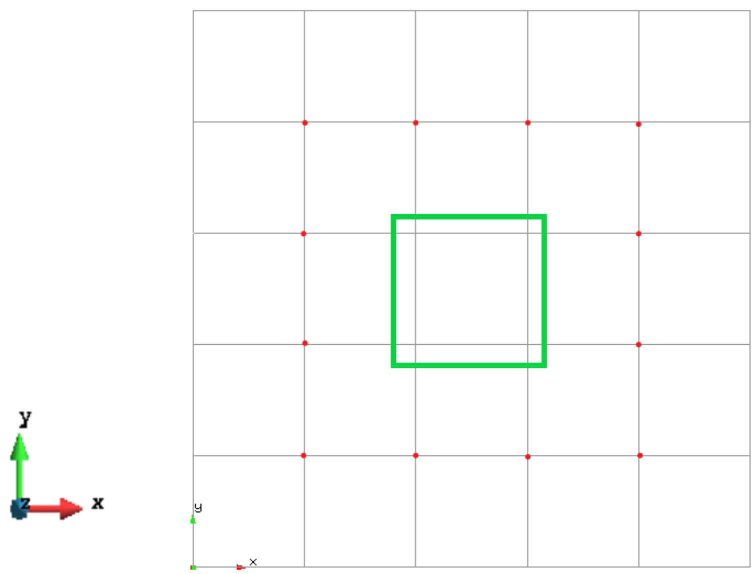

Figure 4.1: Patch boundaries

If the patch test is right, the patch boundaries have to behave like a rigid body. And the displacements of the inner nodes has to be constant with no strains.

To do so, the following Dirichlet conditions have to be imposed to the patch boundaries:

$$\begin{aligned} w &= -1 \\ \frac{\partial w}{\partial x} &= 0 \\ \frac{\partial w}{\partial y} &= 0 \end{aligned} \tag{4.1}$$

and the data are the same as in 2.1 with a thickness of 0.001.

Table 4.1: Patch test results

Displacement	Rotation$_x$	Rotation$_y$
-1	1.1996x10^{-15}	1.5122x10^{-16}
-1	3.2860x10^{-16}	2.4652x10^{-16}
-1	1.38657x10^{-16}	3.1649x10^{-16}
-1	-2.5478x10^{-16}	-3.9846x10^{-15}

The results obtained are showed in Table 4.1. These are the displacements and rotations of the inner nodes of the patch boundaries, shown by the green square in Figure 4.1.

As it can be seen, the inner nodes of the patch boundaries have the same displacement and the rotation is almost zero. So the body can be considered rigid and the patch test is fine.