

COMPUTATIONAL STRUCTURAL MECHANICS AND DYNAMICS

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# Homework 8: Shells

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## Assignment 8.1

Analyse the concrete hyperbolic Shell under self weight presented in Figure 1. Explain the behaviour of all the stresses presented.

Material properties adopted:

$$E = 3.0 \cdot 10^{10} N/m^2, \nu = 0.2, \gamma_C = 2.5 \cdot 10^4 N/m^3, t = 0.1 m$$

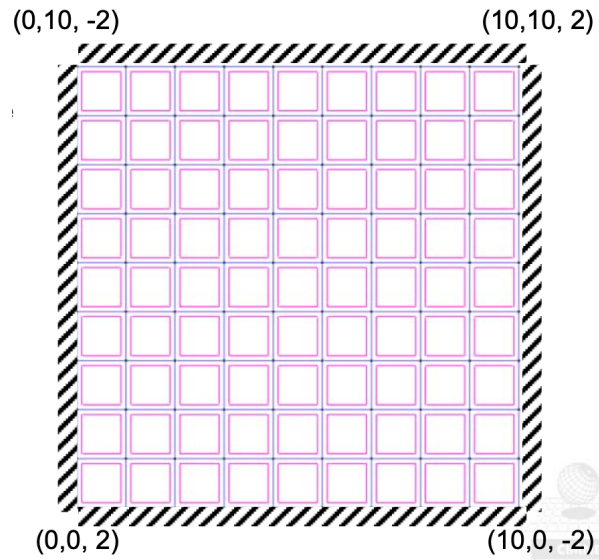


Figure 1: Hyperbolic concrete shell under self weight.

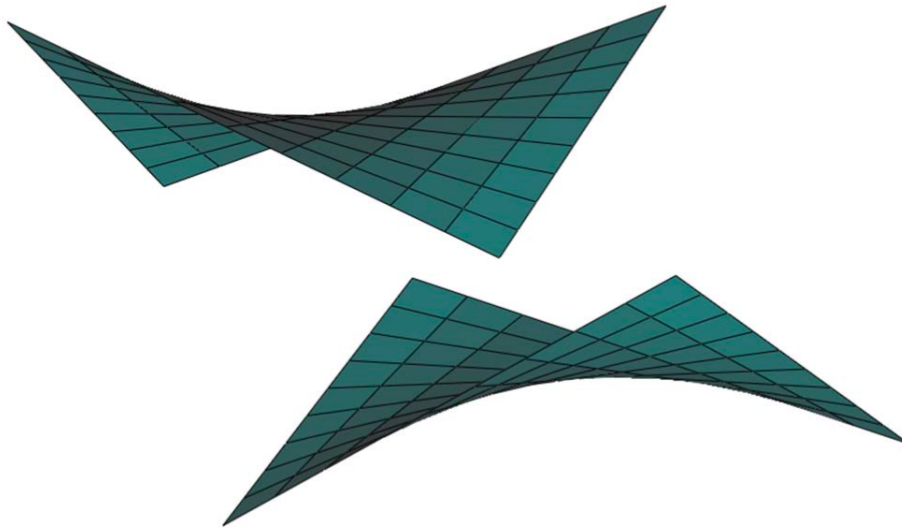


Figure 2: Hyperbolic concrete shell under self weight.

# 1 Assignment 8.1

The obtained results of the hyperbolic thick concrete shell are analysed in the following sections. To model this structure the *Shell\_T\_RM\_v1\_1.m* MATLAB file obtained from <http://www.cimne.com/mat-fem/shells.asp> allowed for the calculation of *triangular* thick shell elements instead of *quadrilateral* elements as shown in Figure 1. The adopted mesh is presented in Figure 3, composed of 162 elements and 100 nodes.

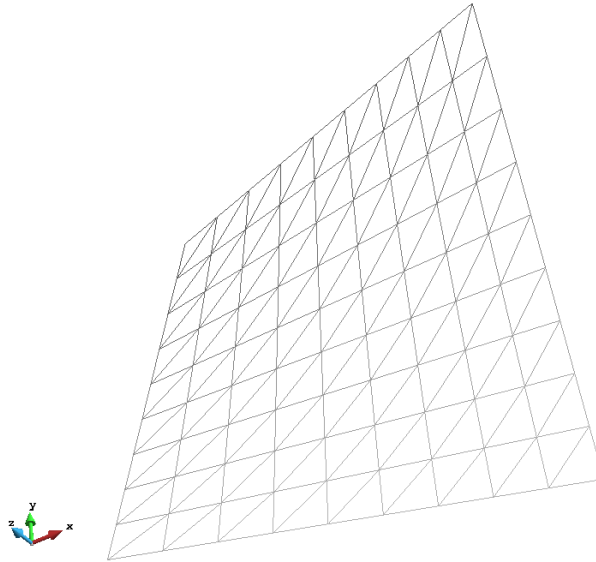


Figure 3: Hyperbolic concrete shell under self weight: Mesh.

## 1.1 Results

### 1.1.1 Displacements

The displacements obtained for the hyperbolic shell are shown in Figure 4 and Figure 5. From the figures, it is seen that the maximum displacement on X and Y direction, due to the decomposition of the gravity acceleration on the oblique plane, generates displacements 100 times smaller than maximum displacements in Z direction.

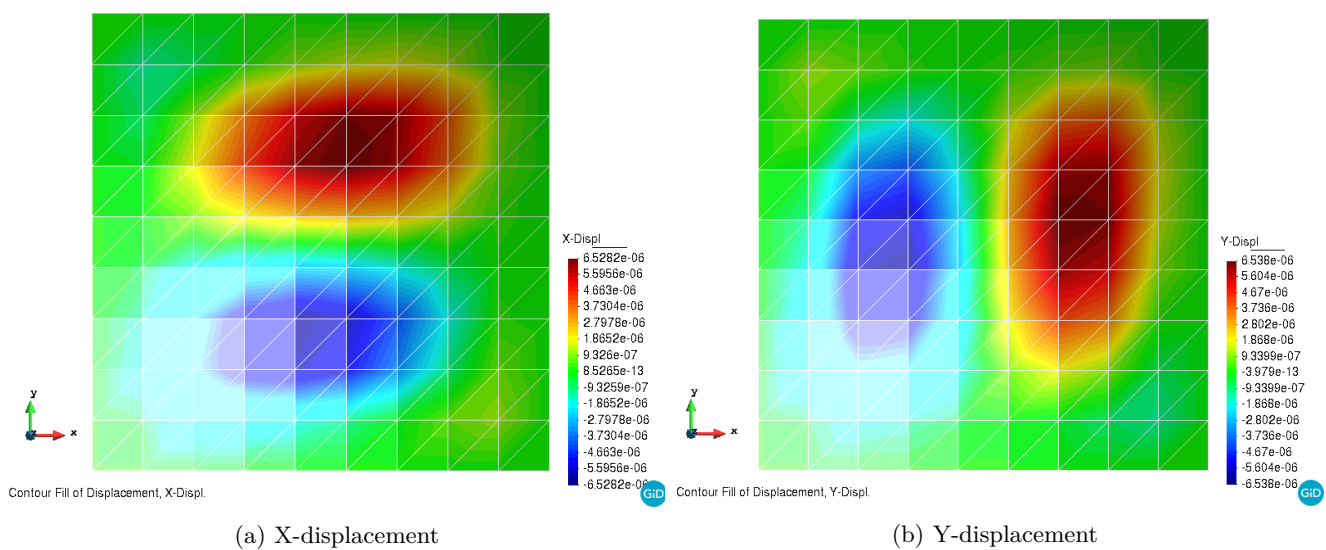


Figure 4: Displacements in X and Y directions.

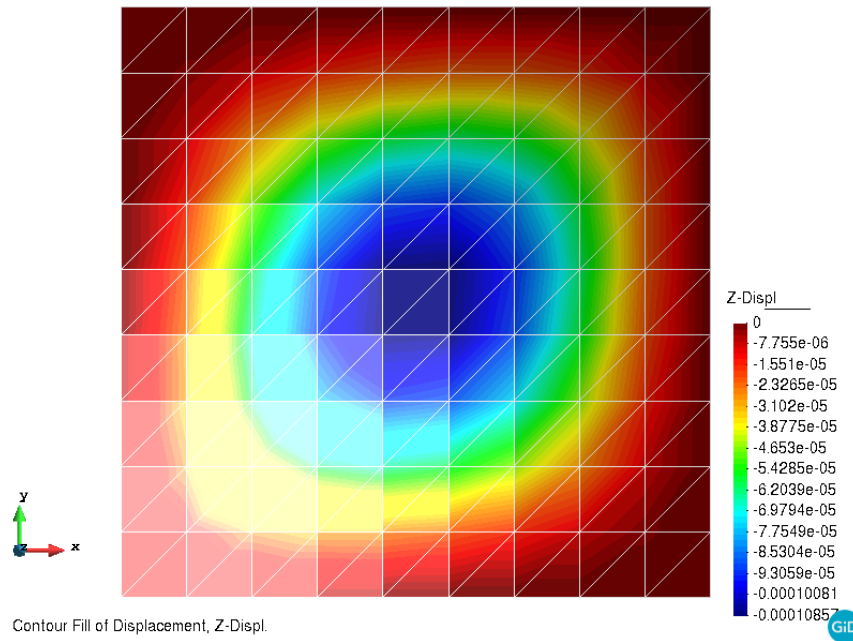


Figure 5: Displacement in Z direction.

### 1.1.2 Rotation

The rotations registered symmetric with respect to a line joining left lower edge and right upper edge. When they are splitted into X and Y direction, they take the form seen in Figure 6.

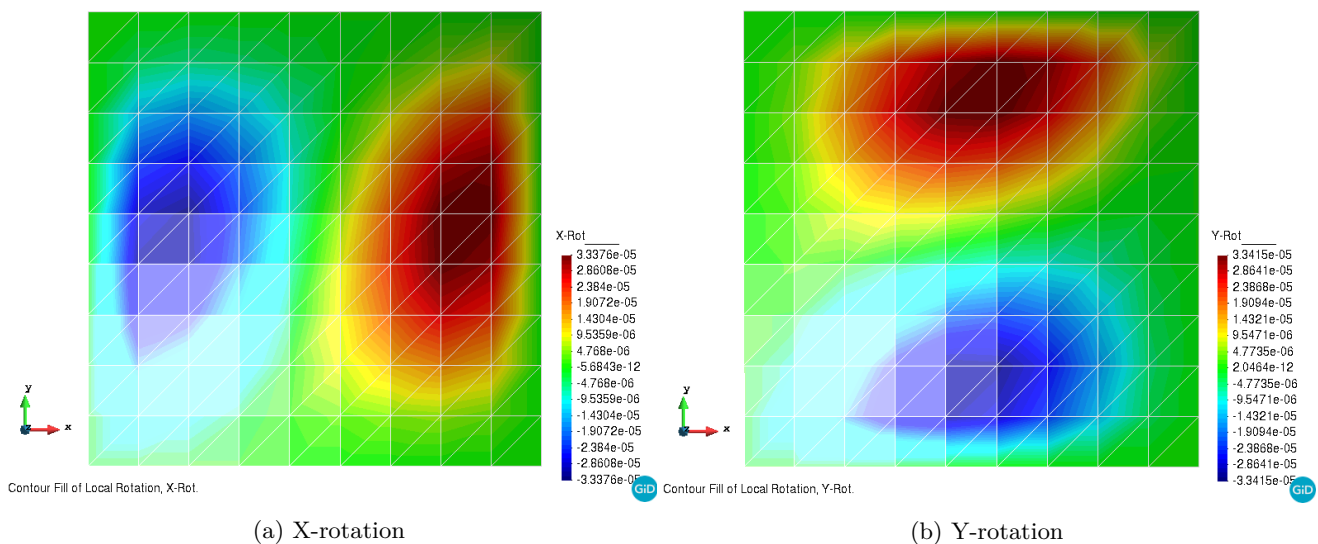


Figure 6: Rotations in X and Y directions.

### 1.1.3 Membrane stresses

The stresses associated with membrane strains are shown in Figure 7 and Figure 8. It is seen in Figure 7 how the stresses in X and Y, both in tension and compression meet their maximum near the edges, where the structure is hanging from (tension) or is mainly supported (compression). When analysing the  $T_{xy}$  stress in Figure 8 cross derivatives strains (distortions)  $(\partial u_0/\partial y$  and  $\partial v_0/\partial x)$  have a maximum in the middle of the shell due to its geometry and the redistribution of stresses.

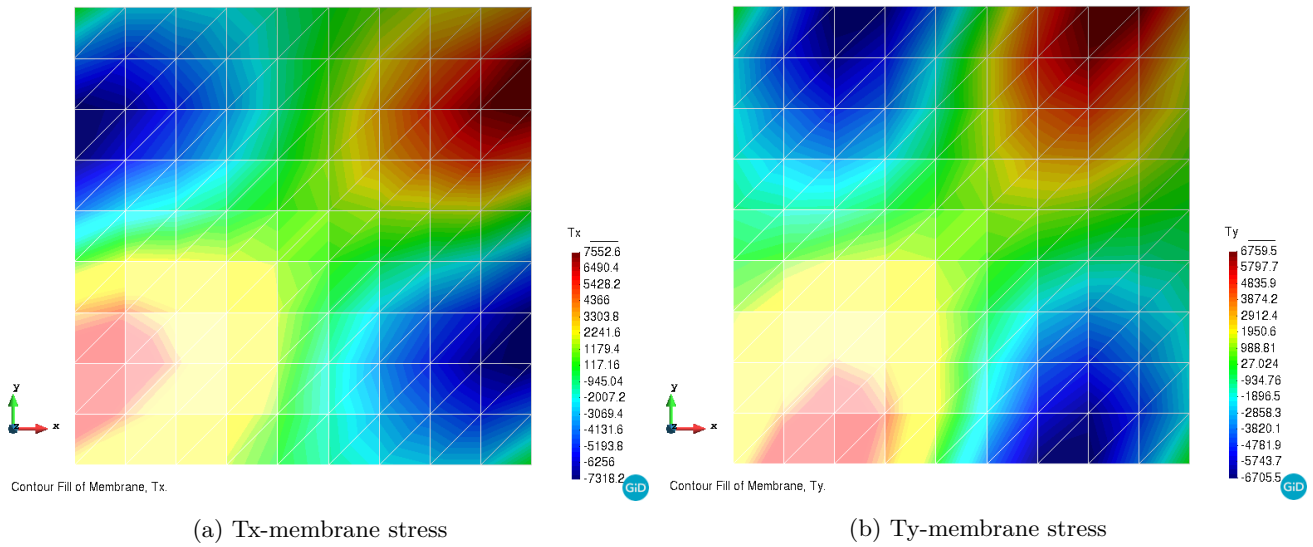


Figure 7: Membrane stresses in X and Y directions.

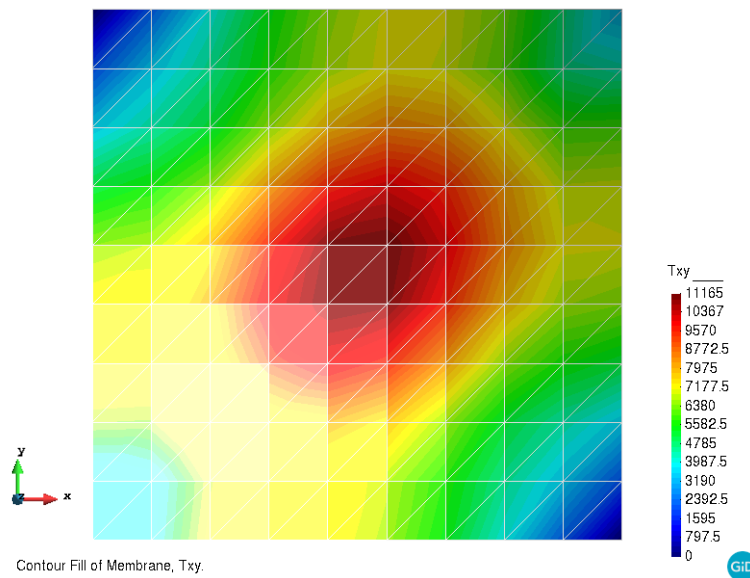


Figure 8: Membrane stress in Txy direction.

### 1.1.4 Bending moments

The registered bending are shown in Figure 9, Figure 10 and Figure 11. As expected in the middle of the shell, both X and Y bending moments are maximum and XY torsion is minimum. Also the relationship is almost double in the fixed edges (for a doubled fixed beam would be in the edge  $M = q \cdot L^2/12$  and in the middle  $M = q \cdot L^2/24$  the values of the moments do not follow this rule of course).

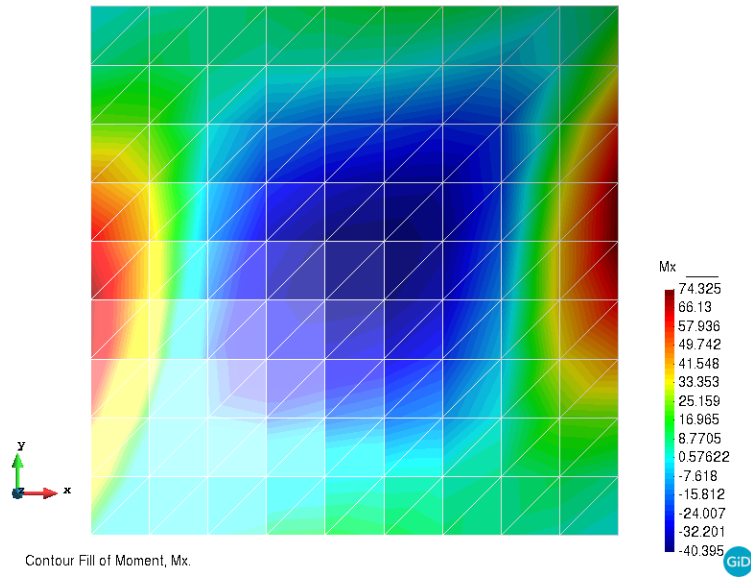


Figure 9: Mx: Bending moment in X-direction.

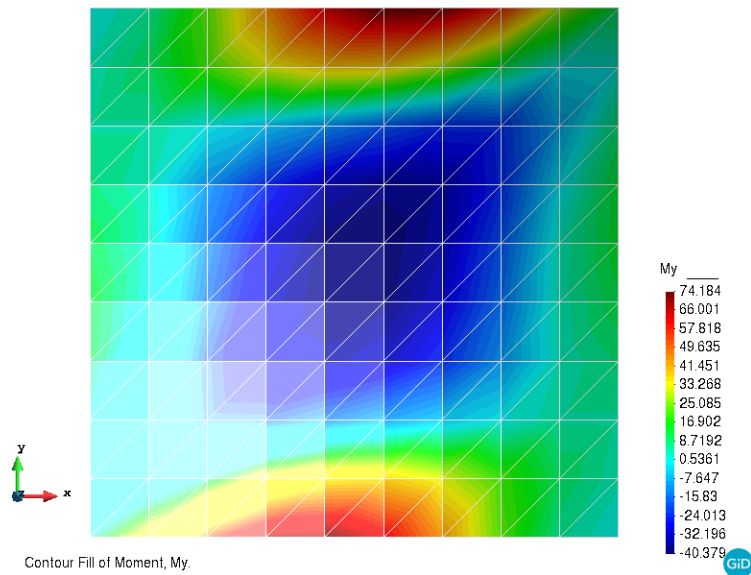


Figure 10: My: Bending moment in Y-direction.

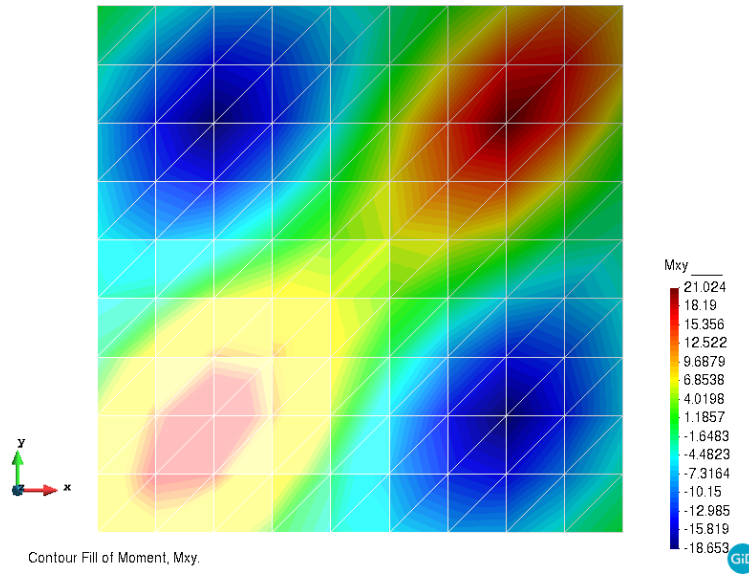
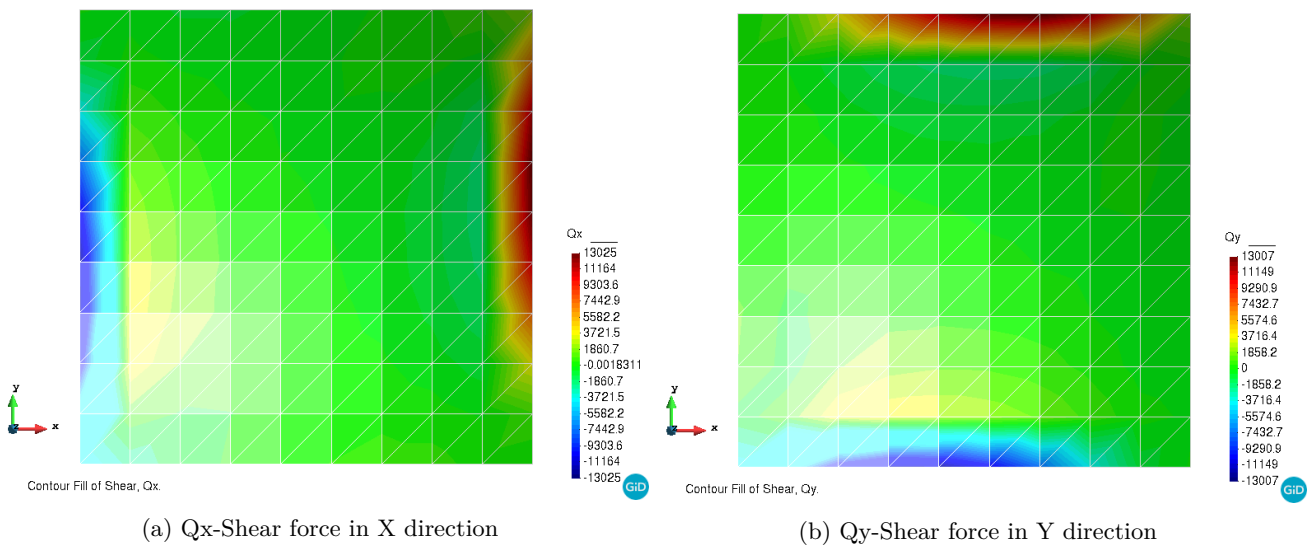


Figure 11: Mxy: Bending moment in XY-direction.

1.1.5 Shear forces

The registered shear forces are shown in Figure 12a and in Figure 12b. Again comparing the symmetry of the structure is observed for the shear forces where in both X and Y direction, the values are approximately equal with opposite signs.



(a) Qx-Shear force in X direction

(b) Qy-Shear force in Y direction

Figure 12: Shear forces in X and Y directions.