



INTERNATIONAL CENTRE FOR  
NUMERICAL METHODS IN ENGINEERING  
UNIVERSITAT POLITÈCNICA DE CATALUNYA  
MASTER OF SCIENCE IN COMPUTATIONAL MECHANICS

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# Computational Structural Mechanics and Dynamics

## Assignment 8

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*Submitted To:*  
Prof. Francisco Zárata

## ASSIGNMENT 8: SHELLS

*a. Analyze the following concrete hyperbolic Shell under self weight. Explain the behavior of all the Stresses presented. Consider thickness  $t = 0.1$*

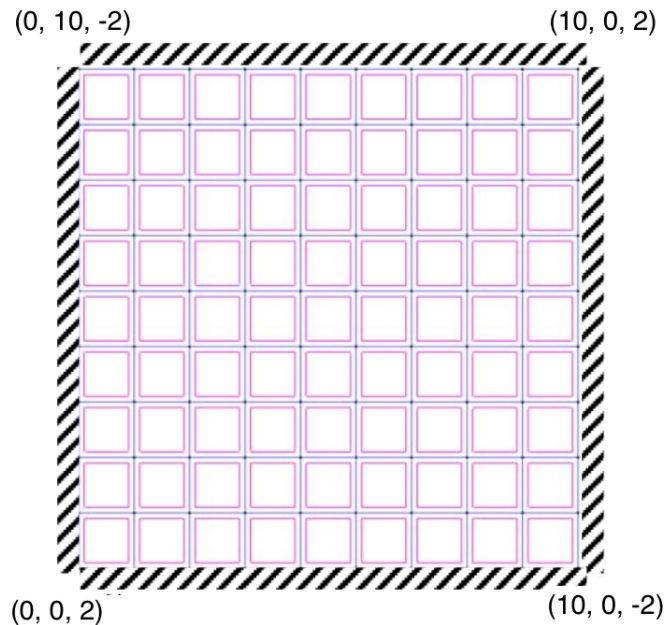


Figure 0.1: Geometry of the shell and mesh considered.

### **Solution:**

In order to analyze the behavior of the structure presented in figure 0.1, a FEM analysis is carried out by using the pre and post-processor GiD with the calculation solver of shells provided by MATFEM, which incorporates an interface, while the calculus is done in Matlab. The material properties for concrete used are:

- Young Modulus  $E = 3.0e10 \text{ N/m}^2$
- Poisson coefficient  $\nu = 0.2$
- Self weight  $\gamma = 25000 \text{ N/m}^3$

The discretization of the continuous element is given by the problem, which corresponds to a 9X9 quadrilateral mesh of 81 elements and 100 nodes (fig 0.2). The only force acting over the shells is

the concrete self weight. Then, after calculation the first result that is shown is the deformation of the structure presented in figures 0.3. In order to make easier the deformation analysis of the shell structure, the displacements of  $X$ ,  $Y$  and  $Z$  are presented using color diagrams and observing the  $XY$  plane 0.4. It is interesting to observe that the displacements in each direction are not obvious because of the shape of the geometry as an hyperboloid. Then this FEM analysis helps to understand how the shell behaves, for example the  $X$ -displacement in figure 0.4a shows how the shell is moving to positive values in the superior part and below the miplane ( $XY$ ) the displacements are opposite. Now, this effect is verified at observing the normal stress in  $X$  direction of the shell in figure 0.5a, where there will be forces in the opposite direction of displacements. The same results can be verified in the  $Y$  direction. Another interesting result is the absolute values of the moments, in which the distribution shows an interesting approximation.

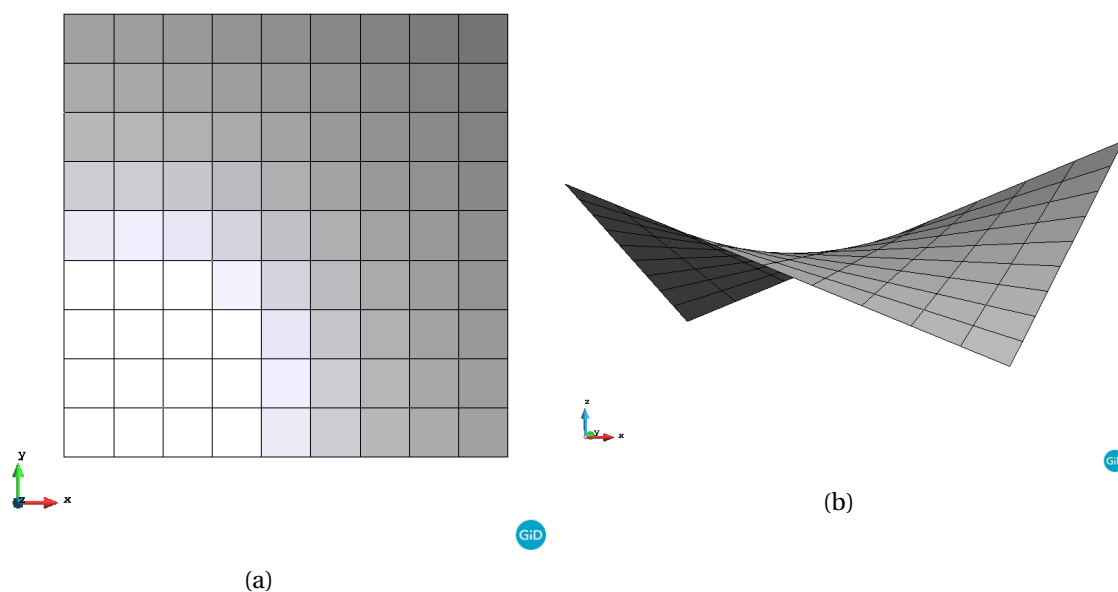


Figure 0.2: Discretization of the hyperboloid structure.

As the plate theory, the rotations of the plate can be computed after solving the system of equations. Then the figures 0.6 present the rotational variations within the shells.

Now, even though the formulation of the shell used in this work is based on the Reissner-Mindlin plate theory, the stress state is more complicated, because the computation involves the combination of bending moments and axial forces. Then, the figure 0.7 shows the moments that are acting on the shell. For example, it can be seen how the moments in  $X$  and  $Y$  have certain symmetry, and also due to the shape of the hyperboloid, the minimum values are located near the center but not in the exact center, there is an “island” shape where moments decrease, which helps in the designing process. And as expected, maximum values are located at the clamped boundaries.

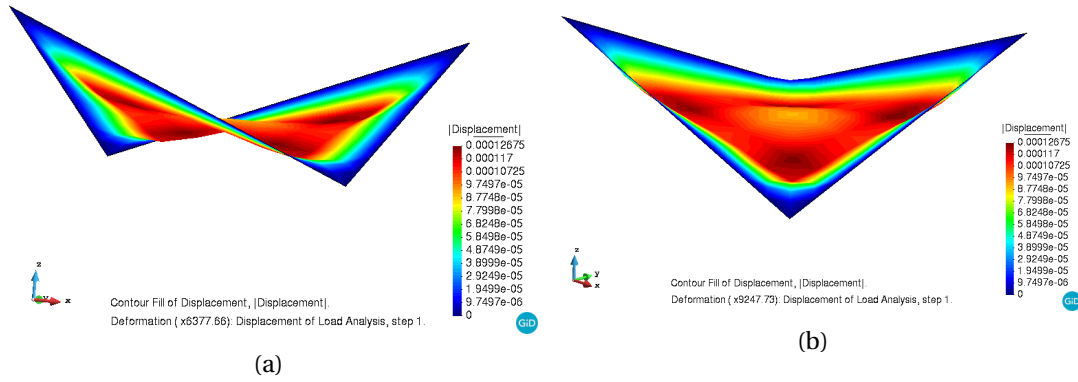


Figure 0.3: Deformed state of the structure.

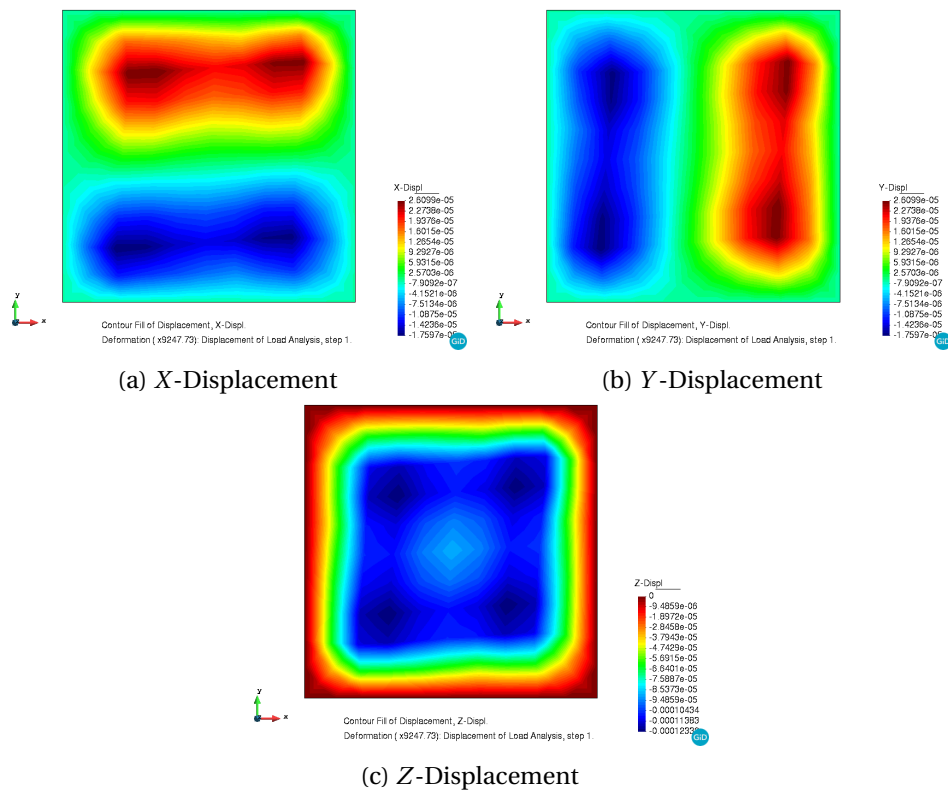
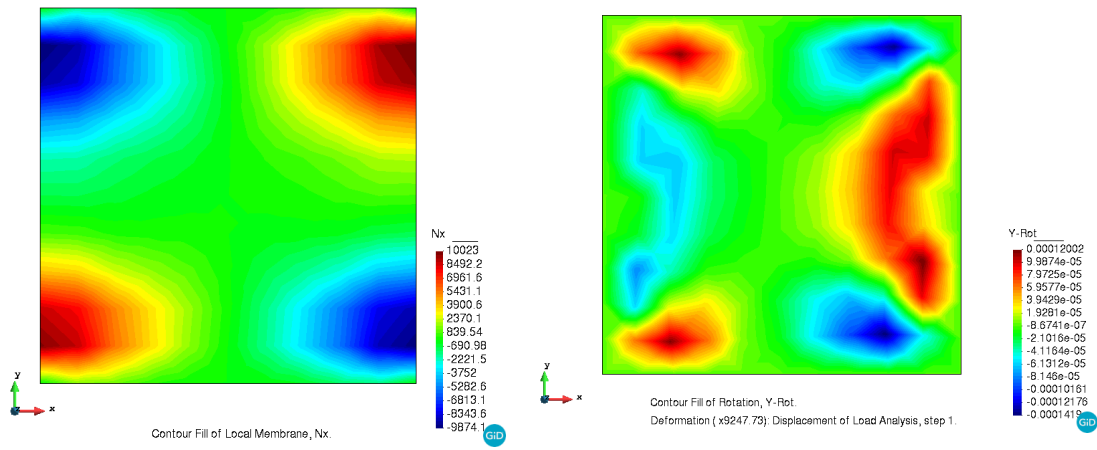
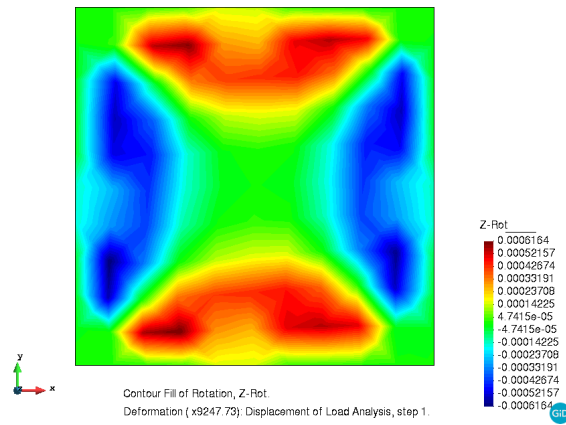


Figure 0.4: XY plane showing the displacement state in each direction.



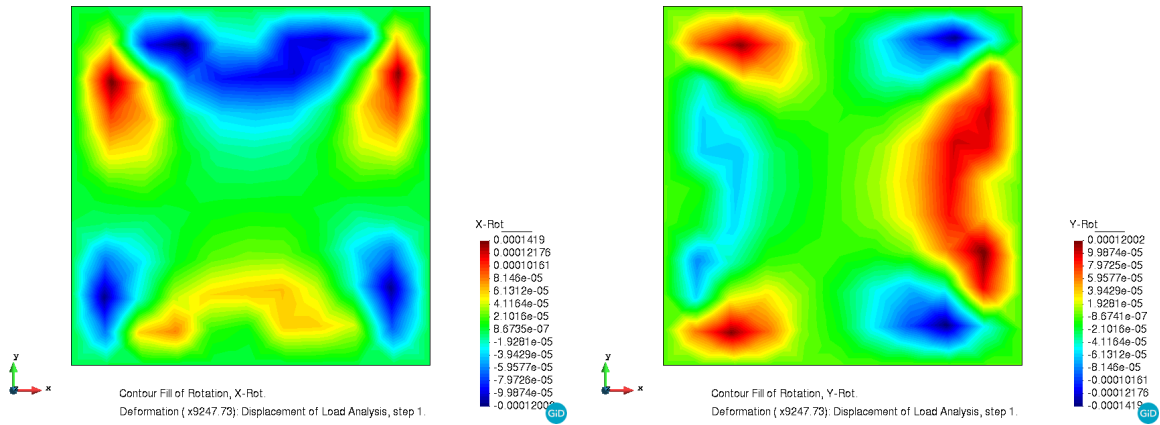
(a) Normal stress with respect to - X

(b) Normal stress with respect to - Y



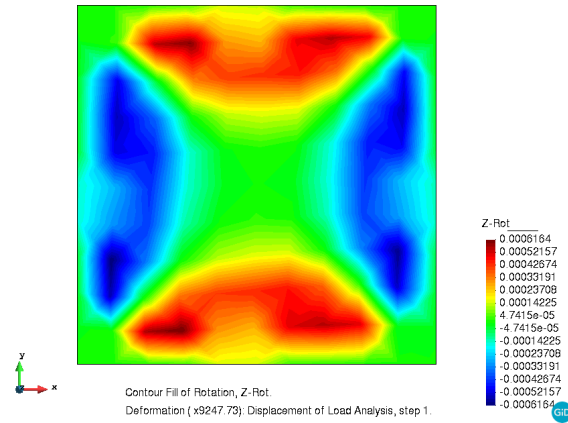
(c) Normal stress with respect to - Z

Figure 0.5: XY plane showing the normal stresses with respect to each direction.



(a) Rotation with respect to - X

(b) Rotation with respect to - Y



(c) Rotation with respect to - Z

Figure 0.6: XY plane showing the moments with respect to each direction.

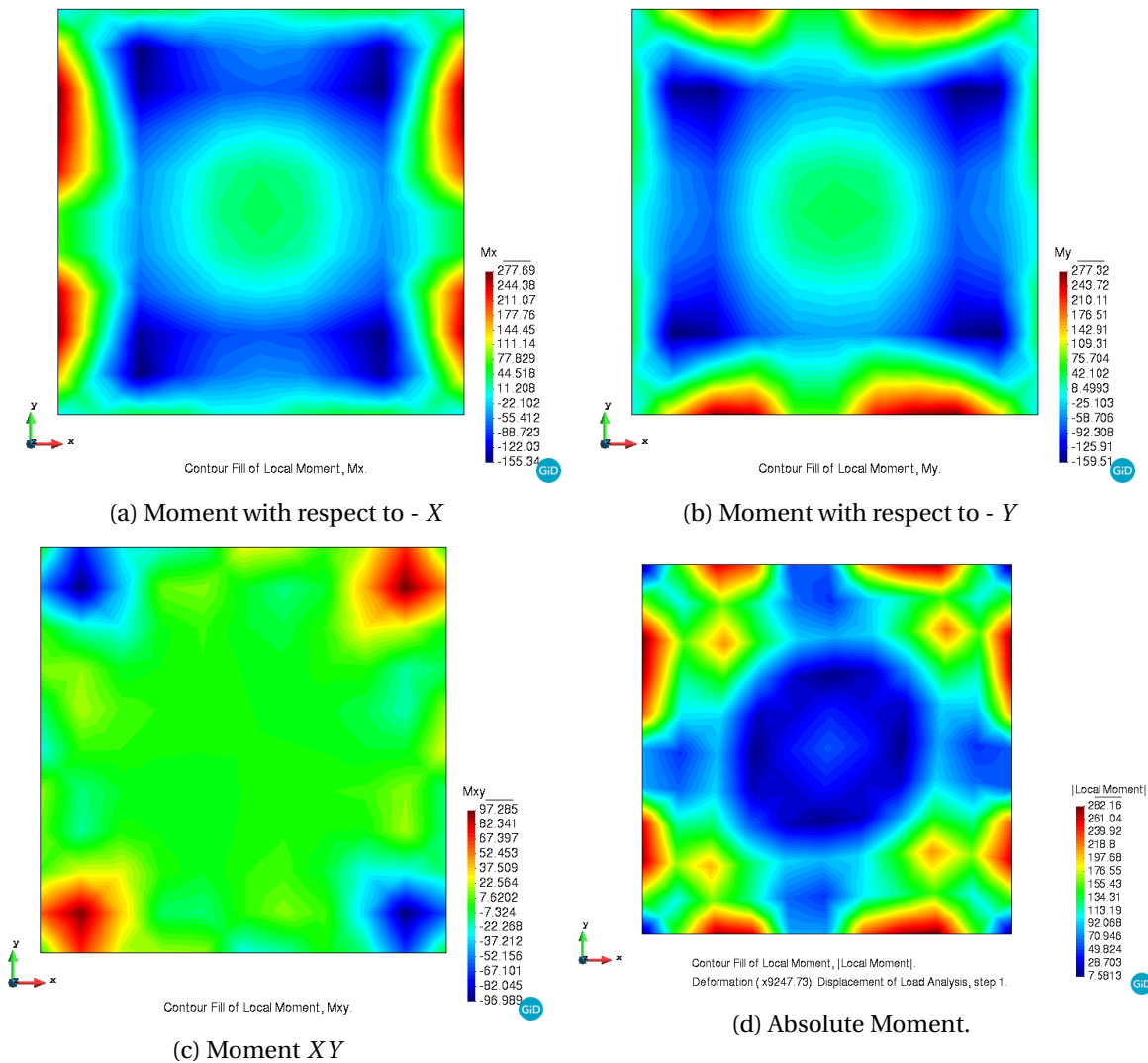


Figure 0.7:  $XY$  plane showing the moments with respect to each direction.

Finally, the shear stress state is shown in the figure 0.8. This result is interesting to observe due to the shape of the structure and the clamped boundaries. Because, each clamped direction contains a reaction, but as the shape is an hyperboloid, the clamped line boundary starts from a point in  $Z$  and finishes in other. Consequently there is a part of tension shear stress and some part of compression shear stress, both are almost symmetric. And in the figure 0.9 it is shown the reaction moments in the clamped boundary, which is interesting to observe how the maximum and minimum values are located in the opposite boundary, and how almost at the middle plate it the values tends to an almost zero moment value, which is the inflection point of the structure.

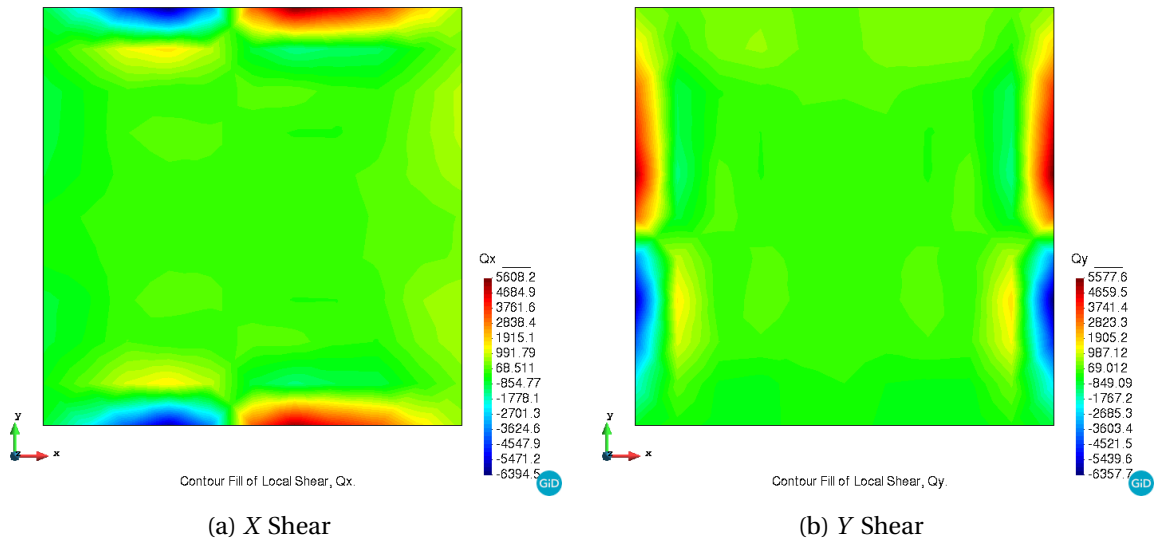


Figure 0.8: XY plane showing the shear stress with respect to each direction.

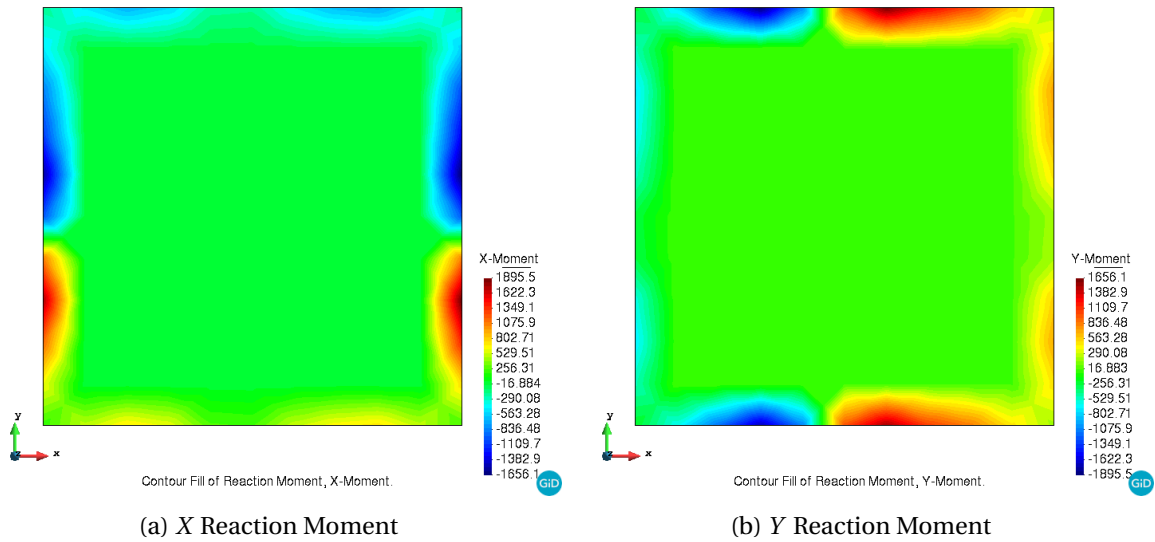


Figure 0.9: XY plane showing the reaction moments with respect to each direction.