Universitat Politecnica De Catalunya, BarcelonaTech
Masters in Computational Mechanics

Course
Computational Structural Mechanics and Dynamics

Assignment 9
on
Axisymmetric Shells

by

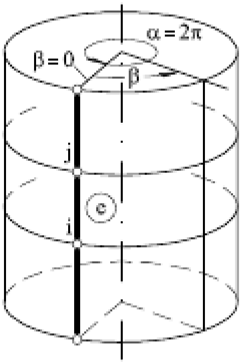
SAMADRITA KARMAKAR

1. Describe in extension how can be applied a non-symmetric load on this formulation?

The axisymmetric shell formulation can be extended for non-symmetric loading. The displacement field is split into symmetric and anti-symmetric component with respect to plane at $\beta=0$. For a n noded strip with n nodes, we have,

$$u'=\sum_{l=0}^{m}\sum_{i=1}^{n} N_i(\bar{S}^l \bar{a}'^l_i + \bar{\bar{S}}^l \bar{\bar{a}}'^l_i)$$

where u' is the displacement vector, $(\bar{\ })$ and $(\bar{\bar{\ }})$ axisymmetric and anti-symmetric components of displacements.



$$\bar{S}^l(y)=\begin{bmatrix} S^l & & & & 0 \\ & C^l & & & \\ & & S^l & & \\ & & & S^l & \\ 0 & & & & C^l \end{bmatrix}$$

$S^l=\sin(\gamma y)$ where $\gamma=l$. b is the plate length $\bar{\bar{S}}^l$ is where S^l is replaced by C^l and visa-versa.

The loads are expanded in Fourier series using harmonic functions as for the displacements.

$$t=\sum_{l=0}^{m}(\bar{S}^l\bar{t}^l+\bar{\bar{S}}^l\bar{\bar{t}}^l)$$

The local stiffness matrix for axisymmetric strip element is given by

$$[K'^{ll}_{ij}]=C\int_{a^{(e)}}[B'^l_i]^T\hat{D}'B'^l_j rds$$

where,

$$C=\begin{matrix}2\pi \; for \; l=0 \\ \pi \; for \; l\neq 0\end{matrix}$$

For symmetric case, γ is replaced by $-l$.

For anti-symmetric case, γ is replaced by l

In global stiffness matrix is,

$[K^{ll}_{ij}]=C\int_{a^{(e)}}[B^l_i]^T\hat{D}'B^l_j rds$ where $B_i=B'^l_i L^{(e)}_i$. $L^{(e)}_i$ is the transformation matrix.

2. Using thin beams formulation, describe the shape of the B(e) matrix and comment the integration rule.

Unsing thin beam formulation leads to neglecting the effect of transverse shear strains in the analysis. The formulation is therfore applicable to thin shell problems only. Making $\hat{\epsilon}_s = 0$ we get,

$$\hat{\epsilon}' = \sum_{l=1}^{m} \sum_{i=1}^{n} \hat{S}^l B_i{}'^l a_i{}'^l \quad ; \quad a_i{}'^l = [u_0{}'^l, v_0{}'^l, w_0{}'^l, \frac{\partial w_0{}'^l}{\partial s}]$$

$$\hat{\epsilon} = \begin{Bmatrix} \hat{\epsilon}'_m \\ \epsilon'_b \end{Bmatrix} \quad ; \quad \hat{\epsilon}_m = \begin{Bmatrix} \frac{\partial u_0{}'}{\partial s} \\ \frac{1}{r}\frac{\partial v_0{}'}{\partial \beta} + \frac{u_0{}'}{r} C - \frac{w_0{}'}{r} \\ \frac{\partial v_0{}'}{\partial s} + \frac{1}{r}\frac{\partial u_0{}'}{\partial \beta} - \frac{v_0{}'}{r} C \end{Bmatrix} \quad ; \quad \hat{\epsilon}_m = \begin{Bmatrix} \frac{\partial^2 w_0{}'}{\partial s^2} \\ \frac{1}{r}\frac{\partial^2 w_0{}'}{\partial \beta^2} + \frac{S}{r^2}\frac{\partial v_0{}'}{\partial \beta} \mp \frac{C}{r}\frac{\partial w_0{}'}{\partial s} \\ \frac{2}{r}\frac{\partial^2 w_0{}'}{\partial s \partial \beta} - \frac{2CS}{r^2} v_0{}' - \frac{2C}{r^2}\frac{\partial w_0{}'}{\partial \beta} + \frac{S}{r}\frac{\partial v_0{}'}{\partial s} \end{Bmatrix}$$

$$B'^l_{m_i} = \begin{Bmatrix} B'^l_{m_i} \\ B'^l_{b_i} \end{Bmatrix} \quad ; \quad B'^l_{m_i} = \begin{bmatrix} \frac{\partial N_i}{\partial s} & 0 & 0 & 0 \\ \frac{N_i}{r} C & \frac{-N_i}{r}\gamma & \frac{-H_i{}'}{r} S & \frac{-\bar{H}_i}{r} S¿ \\ \frac{N_i}{r} C & (\frac{\partial N_i}{\partial s} - \frac{N_i}{r} C) & 0 & 0 \end{bmatrix} \quad ;$$

$$B'^l_{b_i} = \begin{bmatrix} 0 & 0 & \frac{\partial^2 H_i}{\partial s^2} & \frac{\partial^2 \bar{H}_i}{\partial s^2} \\ 0 & \frac{N_i}{r^2} S\gamma & [\frac{C}{r}\frac{\partial H_i}{\partial} s - (\frac{\gamma}{r})^2 H_i] & [\frac{C}{r}\frac{\partial \bar{H}_i}{\partial} s - (\frac{\gamma}{r})^2 \bar{H}_i] \\ 0 & (\frac{S}{r}\frac{\partial N_i}{\partial s} - 2\frac{N_i}{r^2} CS) & (2\frac{\gamma}{r}\frac{\partial H_i}{\partial s} - 2\frac{H_i}{r^2} C\gamma) & (2\frac{\gamma}{r}\frac{\partial \bar{H}_i}{\partial s} - 2\frac{\bar{H}_i}{r^2} C\gamma) \end{bmatrix}$$

N_i = Langrange Shape Functions ; $H_i, \bar{H}_i$ = Hermite Shape Functions

Integration Rule: Two-point quadrature is recommended for computing the integrals. But results also can be obtained using simplest reduced one-point quadrature.