CSMD: Assigment 9

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1 Non axy-simmetric loads formulation

In order to correctly represent non-symmetric loads within a 2D formulation, we can decompose loads and displacements into series of Fourier functions.

Loads can be described as

$$f(\theta) = \frac{a_0}{2} + \sum_{i=1}^{\infty}(\frac{cosn\theta}{\pi}\int_{-\infty}^{\infty} f(\theta)cosn\theta d\theta + \frac{sinn\theta}{\pi}\int_{-\infty}^{\infty} f(\theta)sinn\theta d\theta) \quad (1)$$

And displacements (axial, radial and circumferencial respectively) as

$$u = \sum_{i=1}^{\infty}(u_{a_n}cosn\theta + u_{b_n}sinn\theta) \quad (2)$$

$$v = \sum_{i=1}^{\infty}(v_{a_n}cosn\theta + v_{b_n}sinn\theta) \quad (3)$$

$$w = \sum_{i=1}^{\infty}(w_{a_n}cosn\theta + w_{b_n}sinn\theta) \quad (4)$$

These will make the strain matrices depend on circumferencial variable θ. Taking into account that Fourier series produce symmetric and non-symmetrix solutions, the approximation functions can be divided into symmetric and non-symmetric. The formulation of the stiffness matrices are (a for symmetric n=0,2,4... and b for not symmetric n = 1,2,3...)

$$\mathbf{k}_{an} = \pi\int_A \mathbf{B}_{ai}^T\mathbf{D}\mathbf{B}_{ai}dA \quad (5)$$

$$\mathbf{k}_{bn} = \pi\int_A \mathbf{B}_{bi}^T\mathbf{D}\mathbf{B}_{bi}dA \quad (6)$$

With strain matrices:

$$
\left[B_{a_n}^{i}\right] = \begin{bmatrix}
\frac{\partial N_i}{\partial r} & 0 & 0 \\
0 & \frac{\partial N_i}{\partial z} & 0 \\
\frac{N_i}{r} & 0 & +\frac{nN_i}{r} \\
\frac{\partial N_i}{\partial z} & \frac{\partial N_i}{\partial r} & 0 \\
-\frac{nN_i}{r} & 0 & \left(\frac{\partial N_i}{\partial r} - \frac{N_i}{r}\right) \\
0 & -\frac{nN_i}{r} & \frac{\partial N_i}{\partial z}
\end{bmatrix}
$$

$$
\left[B_{b_n}^{i}\right] = \begin{bmatrix}
\frac{\partial N_i}{\partial r} & 0 & 0 \\
0 & \frac{\partial N_i}{\partial z} & 0 \\
\frac{N_i}{r} & 0 & -\frac{nN_i}{r} \\
\frac{\partial N_i}{\partial z} & \frac{\partial N_i}{\partial r} & 0 \\
\frac{nN_i}{r} & 0 & \left(\frac{\partial N_i}{\partial r} - \frac{N_i}{r}\right) \\
0 & \frac{nN_i}{r} & \frac{\partial N_i}{\partial z}
\end{bmatrix}
$$

Figure 1: Strain matrices $\mathbf{B}_{ai}$ and $\mathbf{B}_{bi}$

The load vector must be also discretized taking into account the decomposition in equation (1) and shapefunctions N_i.

Our solution will be a function of θ, so we can calculate for any given (r, θ, z).

2 $\mathbf{B}^e$ matrix integration for thin beams

For thin beams, $\mathbf{B}^e$ matrix is divided into the three $\mathbf{B}$ matrices related to membrane, bending and shear.

$$\mathbf{B}_i = \begin{bmatrix} \mathbf{B}_m \\ \mathbf{B}_b \\ \mathbf{B}_s \end{bmatrix} \tag{7}$$

In order to integrate them, and to avoid shear locking effects, reduced integration must be performed over the shear matrix. Also, and if some element have radial coordinate r = 0, Lobato's integration rule must be avoided, making use of full Gauss integration for the bending and membrane matrices.