Universitat Politècnica de Catalunya

Escola Tècnica Superior d'Enginyers de Camins, Canals i Ports

MASTER EN INGENIERÍA ESTRUCTURAL Y DE LA CONSTRUCCIÓN

Course:

# COMPUTATIONAL STRUCTURAL MECHANICS AND DYNAMICS

## Assignment 7

## On "Plates"

**By**

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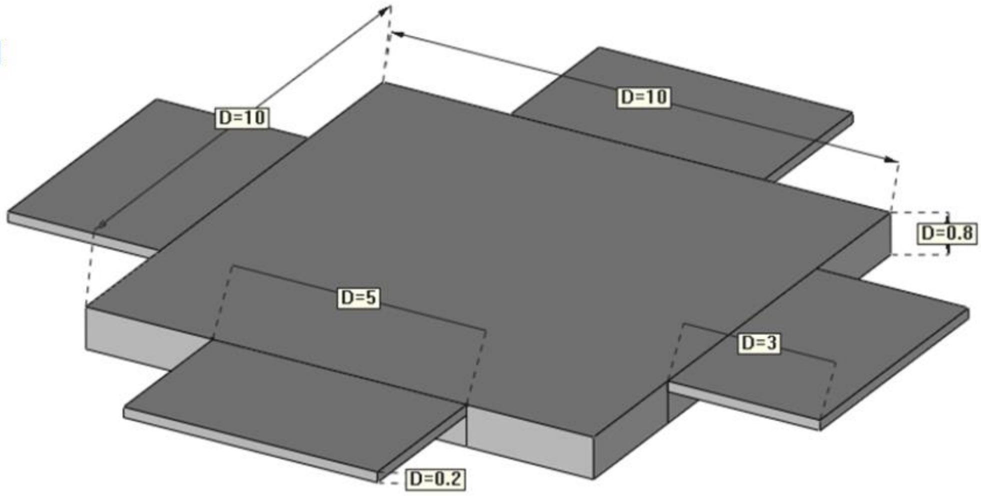
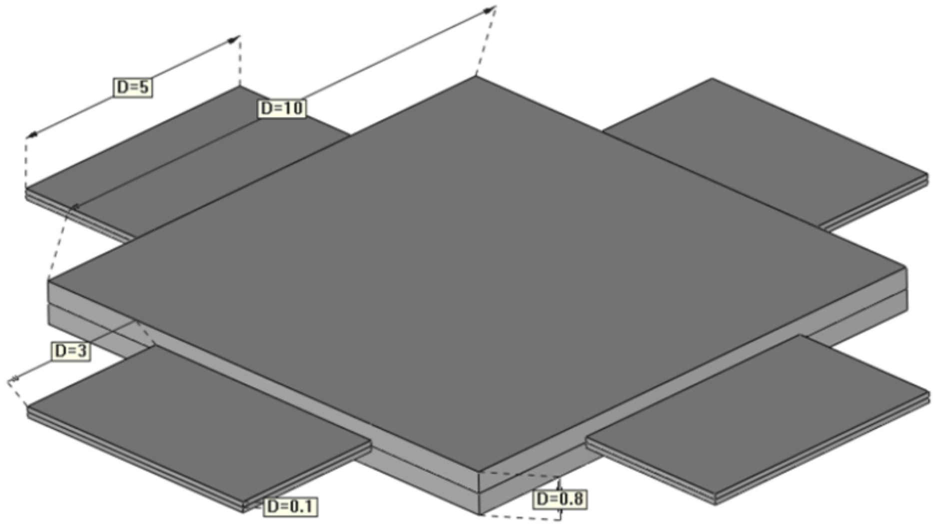
# Assignment 7:

a) Think first and answer later. What kind of strategy (theory, elements, integration rule, boundary conditions, etc) will you use for solving the following problems:

a.1




a.2




b) Define and verify a patch test mesh for the MCZ element.

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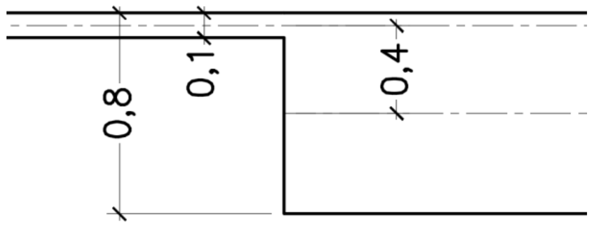
a) In both cases the relations between the thickness and the width of the plates are

$$\frac{h}{b} = \frac{0.1}{3} = 0.0333 < 0.1$$

$$\frac{h}{b} = \frac{0.8}{10} = 0.08 < 0.1$$

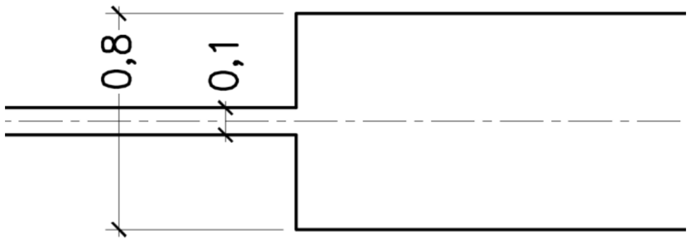
So, we can consider the both plates as thin plates. The differences between a.1 and a.2 are the position of the midsurfaces.

In a.1 there is an eccentricity between both planes

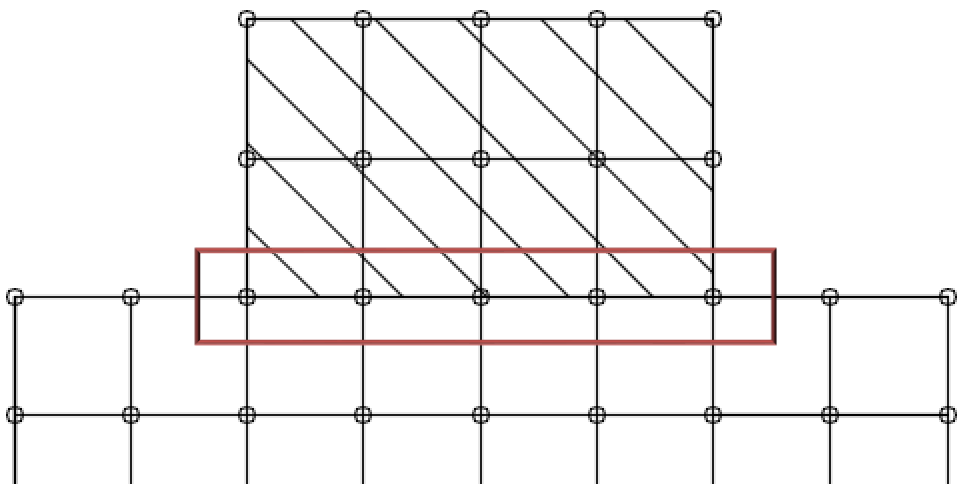



$$e = 0{,}40\ m$$

Instead, in the a.2 the midsurfaces are in the same plane.




In the next figure the MEF model is schematized.



The hatched zone represents the thinner slab. Both theories seen in the course, Classical Kirchhoff and Reissner – Mindlin, consider the plate represented by his middle plane. In the a.2 case the model represents all the plates by his middle plane, instead in the a.1 case if the middle plane of the thinner slab is adopted as representation plane, in the thick slab this plane is $0{,}4\ m$ above his middle plane. So, when PVW is applied, the integration limits are not correct. Both theories integrates along $-\frac{t}{2}$ and $\frac{t}{2}$.

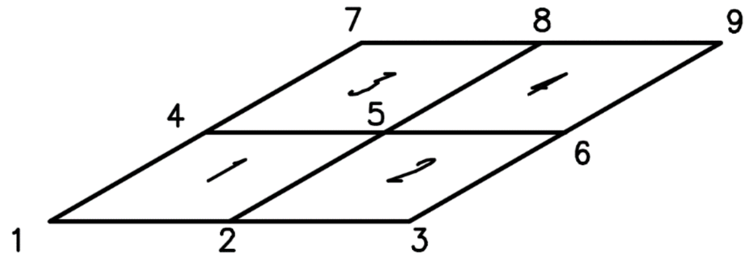
Concluding, in a.2 I would use Kirchhoff theory, the elements, integration rule, boundary conditions depends on the loads, supports of the slabs and the error I am searching for. For the a.1 should be developed especial elements type with a theory that accept the eccentricity between the slabs to use in the connected elements. In the rest of the elements is the same case than a.2. Must use integration rules and order of shape functions in the elements compatibles with the others.

b) A non-conforming plate element can still converge to the correct solution of it satisfies the patch test. This is based in imposing at the boundary of a patch of element a displacement field which can be exactly reproduced by the shape functions. The patch test is satisfied if the displacements and strains within the patch coincide with the exact values deduced from the prescribed displacement field.

For non-conforming rectangular MZC element, the approximation for w is

$$w = \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 x^2 + \alpha_5 xy + \alpha_6 y^2 + \alpha_7 x^3 + \alpha_8 x^2 y + \alpha_9 xy^2 + \alpha_{10} y^3 + \alpha_{11} x^3 y + \alpha_{12} xy^3$$

The approximation guarantees that $w$ varies as a cubic polynomial along the sides. Considering a rectangular plate formed by 4 elements and 9 nodes as is shown in the next figure




The length of the sides of the rectangular shape are 1. The boundary condition in the four edges are SS. So, imposing a quadratic displacement field (can be represented with the MCZ)

$$w(x, y) = \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 x^2 + \alpha_5 xy + \alpha_6 y^2$$

$$w = (2x^2 + 4y^2 + xy)$$

$$\theta_x = \frac{\partial w}{\partial x} = 4x + y$$

$$\theta_y = \frac{\partial w}{\partial y} = x + 8y$$

The displacement imposed in the MCZ are exposed in the next table. The node 5 is the free node to compare.

| Node | X | Y | w | $\theta_x$ | $\theta_y$ |
|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0.5 | 0 | 0.5 | 2 | 1 |
| 3 | 1 | 0 | 2 | 4 | 1 |
| 4 | 0 | 0.5 | 1 | 1 | 4 |
| 6 | 1 | 0.5 | 3.5 | 5 | 5 |
| 7 | 0 | 1 | 4 | 1 | 8 |
| 8 | 0.5 | 1 | 5 | 3 | 9 |
| 9 | 1 | 1 | 7 | 5 | 9 |

In the next table are the comparison about the values obtained by the analytical formulation and with the MCZ plate elements

| | w | $\theta_x$ | $\theta_y$ |
|---|---|---|---|
| **Analytical** | 1.75 | 2.50 | 4.50 |
| **MCZ Plate** | 1.73 | 2.31 | 4.31 |
| **Error** | 0.97% | 7.59% | 4.21% |

The maximum error obtained is in the $\theta_x$, with a 7,6% error. This error is not depreciable, so the MCZ plate element doesn't pass the patch test. We can test with a linear displacement field and observe if the element passes the patch.

$$w = 2x + 4y$$

$$\theta_x = \frac{\partial w}{\partial x} = 2$$

$$\theta_y = \frac{\partial w}{\partial y} = 4$$

The imposed displacements

| Node | X | Y | w | $\theta_x$ | $\theta_y$ |
|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 2.00 | 4.00 |
| 2 | 0.5 | 0 | 1 | 2.00 | 4.00 |
| 3 | 1 | 0 | 2 | 2.00 | 4.00 |
| 4 | 0 | 0.5 | 2 | 2.00 | 4.00 |
| 6 | 1 | 0.5 | 4 | 2.00 | 4.00 |
| 7 | 0 | 1 | 4 | 2.00 | 4.00 |
| 8 | 0.5 | 1 | 5 | 2.00 | 4.00 |
| 9 | 1 | 1 | 6 | 2.00 | 4.00 |

And the results

| | w | $\theta_x$ | $\theta_y$ |
|---|---|---|---|
| **Analytical** | 3.00 | 2.00 | 4.00 |
| **MCZ Plate** | 2.98 | 2.00 | 4.00 |
| **Error** | 0.57% | 0.00% | 0.00% |

With the linear displacement field, the errors obtained are negligible. So, the MCZ element passes the patch test.